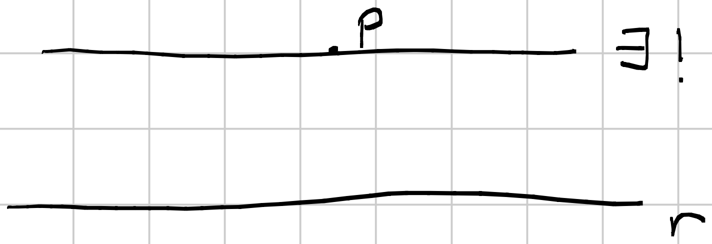
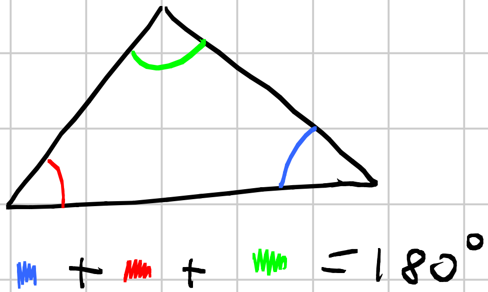
GEOMETRIA PIANA

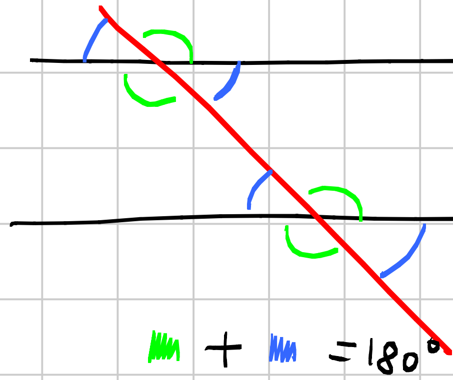
Titolo nota 21/02/2006

Geometria Euclidea

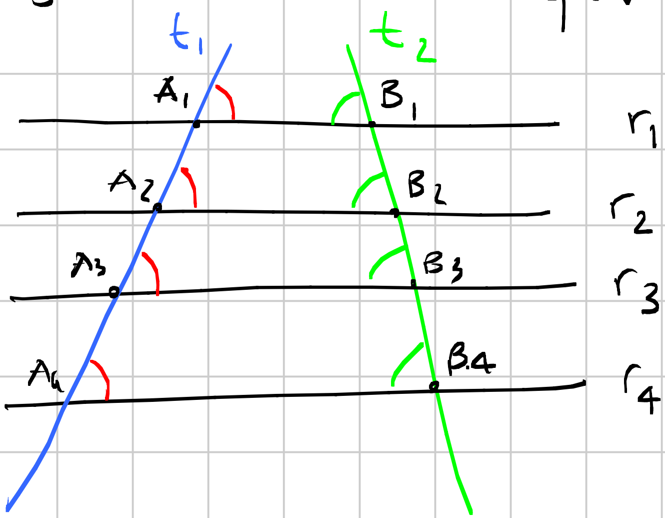


$\exists !$



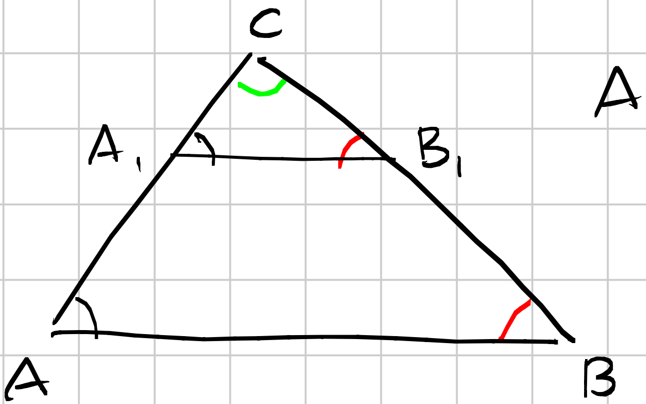


Questi 3 fatti sono equivalenti



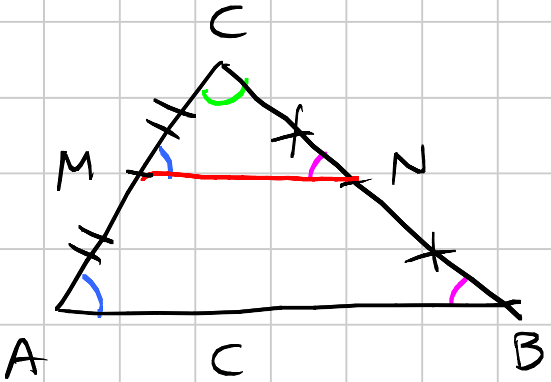
TEOREMA DI TALETE

$$\frac{A_1A_2}{B_1B_2} = \frac{A_2A_3}{B_2B_3} = \frac{A_3A_4}{B_3B_4} = \frac{A_1A_3}{B_1B_3}$$



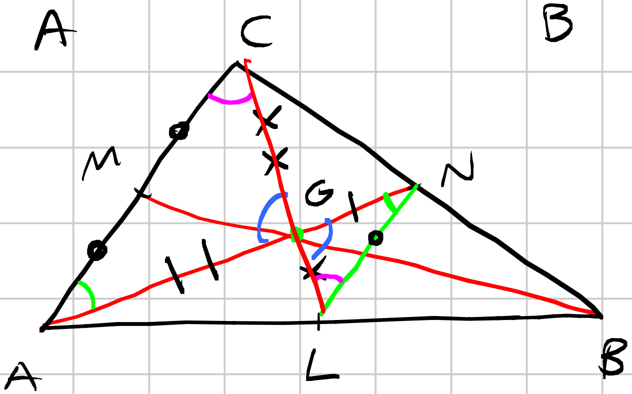
ABC $\quad A_1B_1C \quad$ sono triangoli simili

$$\frac{CA_1}{CA} = \frac{CB_1}{CB} = \frac{A_1B_1}{AB}$$



CMN è simile a CAB

MN è parallelo ad AB

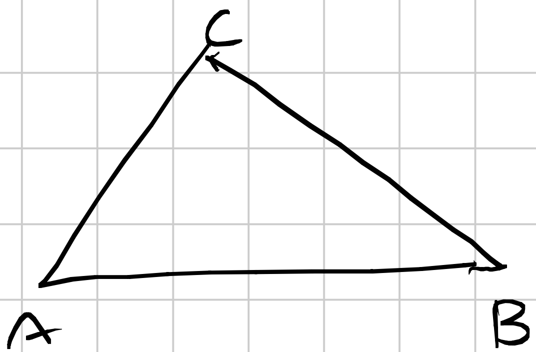


G = BARICENTRO

LBN $\quad$ ACB

GAC $\simeq$ GLN (sono simili)

CG = 2GL e così via



$AB < AC + BC$ diseguaglianza triangolare

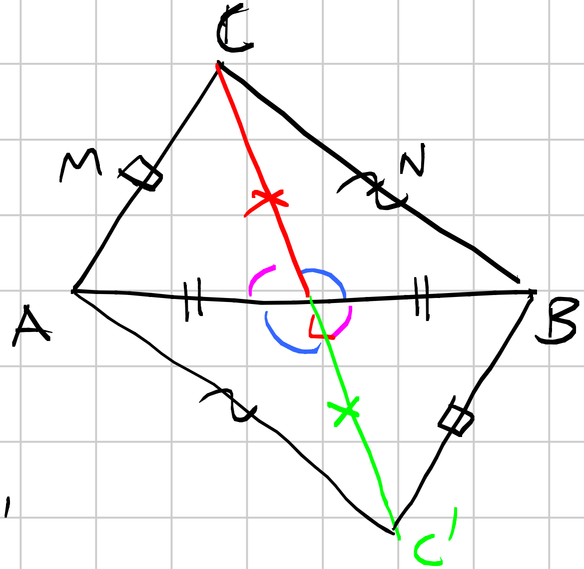
$CB < CA + AB$

$AC < AB + BC$

$AC > |AB - BC|$

$AB > |AC - BC|$

$CB > |AC - AB|$



$AC = CB$

$C'L = CL$

CLC' allineati

$AC'C$ $\quad CC' < AC + AC'$

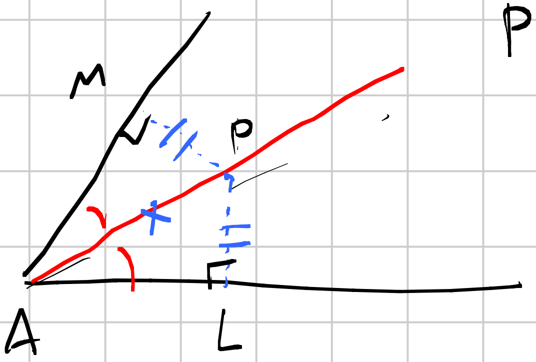
$2CL < AC + CB$

$2AN < AB + AC$

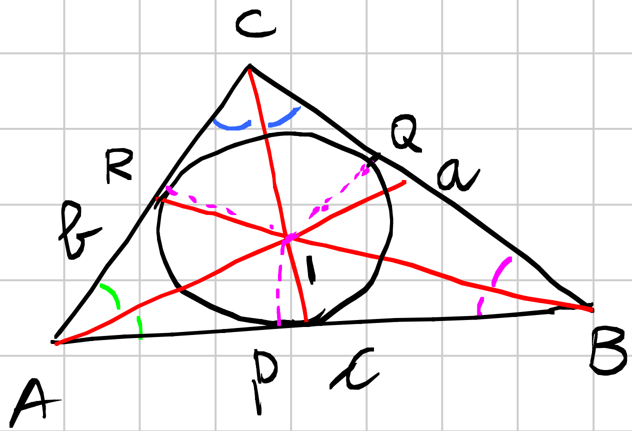
$2BM < AB + BC$

$2(CL + AN + BM) < 2(AB + AC + BC)$

somma delle mediane < perimetro



$PL = PM$



$IP = IQ = IR$

$$S = \sqrt{p(p-a)(p-b)(p-c)}$$

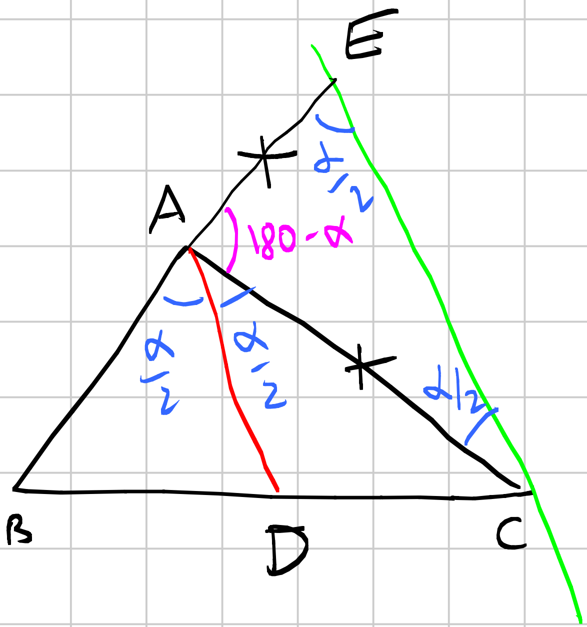
FORMULA DI ERONE

$$S(ABC) \overset{r}{=} S(AIB) + S(BIC) + S(AIC)$$

$$= \frac{AB \cdot r}{2} + \frac{BC \cdot r}{2} + \frac{AC \cdot r}{2} = p \cdot r$$

$$p = \frac{AB + BC + AC}{2}$$

$$r = \frac{S}{p}$$



$$\frac{BD}{DC} = \frac{AB}{AE}$$

ACE è isoscele

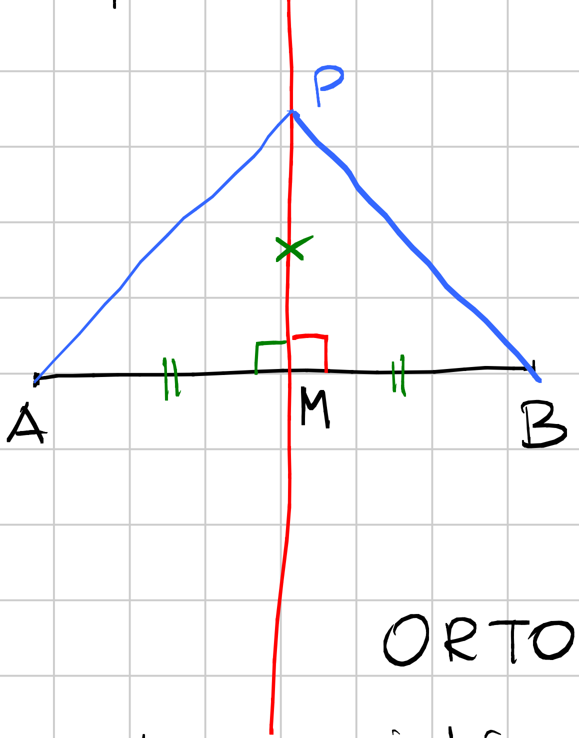
$AE = AC$

$$\frac{BD}{DC} = \frac{AB}{AC}$$

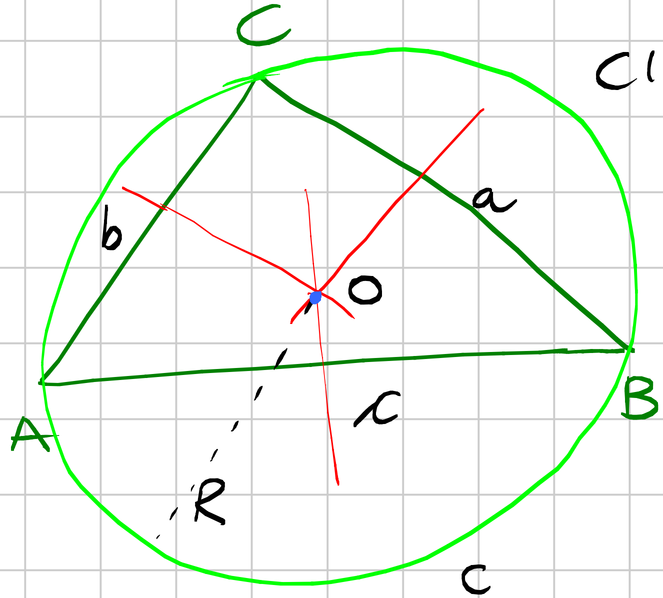
TEOREMA DELLA BISETTRICE

ASSE DEL SEGMENTO

AM = MB

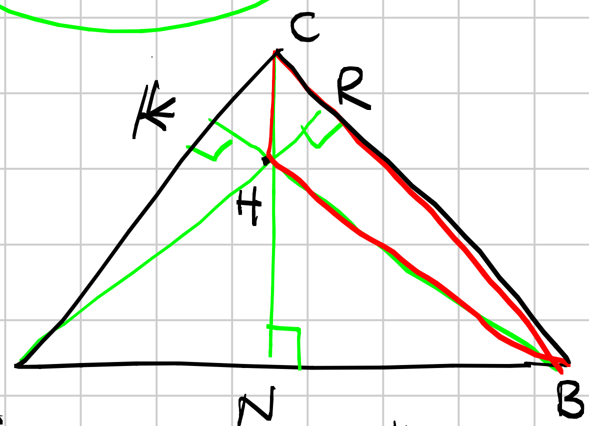


CIRCOCENTRO



$$R = \frac{abc}{4S}$$

ORTOCENTRO



H è interno per i triangoli acutangoli

≡ vertice angolo retto per i triangoli rettangoli

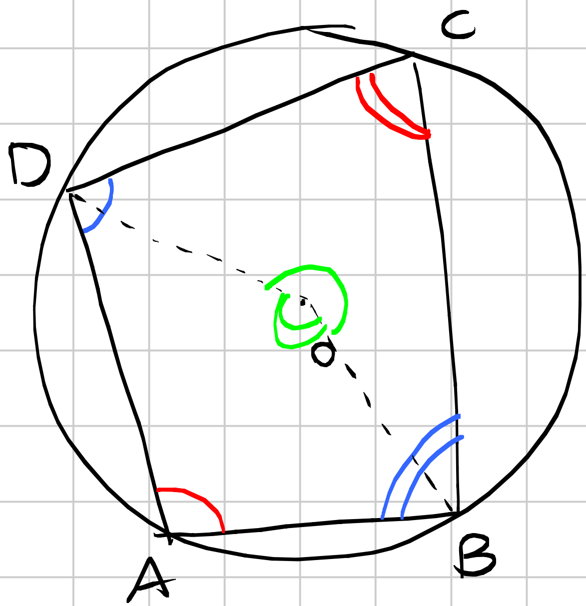
<u>esterno</u> triangoli ottusangoli

$\triangle$ BCH costituiscono un sistema ortocentrico

ognuno dei 4 punti è l'ortocentro del triangolo formato con gli altri 3 punti

H G ed O sono allineati (retta di Eulero)

$HO = 2HG$



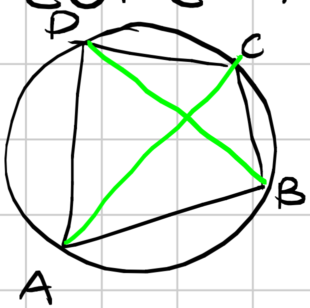
ABCD è ciclico

$\Updownarrow$

$\hat{D}+\hat{B} = \hat{A}+\hat{C} = 180^\circ$

(L'angolo alla circonferenza è la metà dell'angolo al centro)

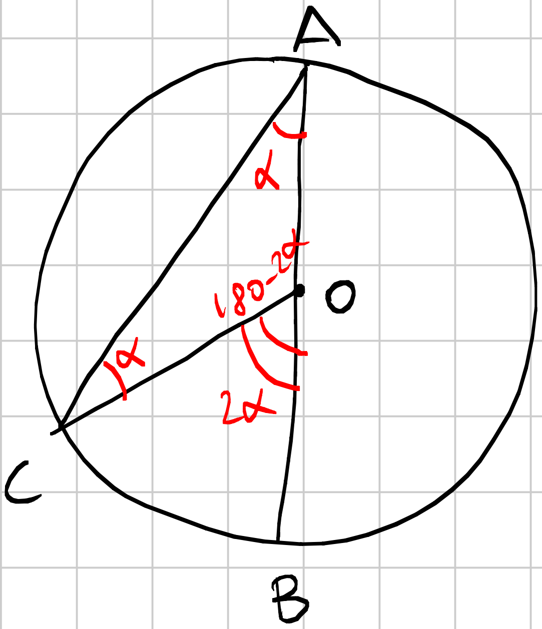
TEOREMA DI TOLOMEO



$AB \cdot DC + AD \cdot BC = AC \cdot BD$

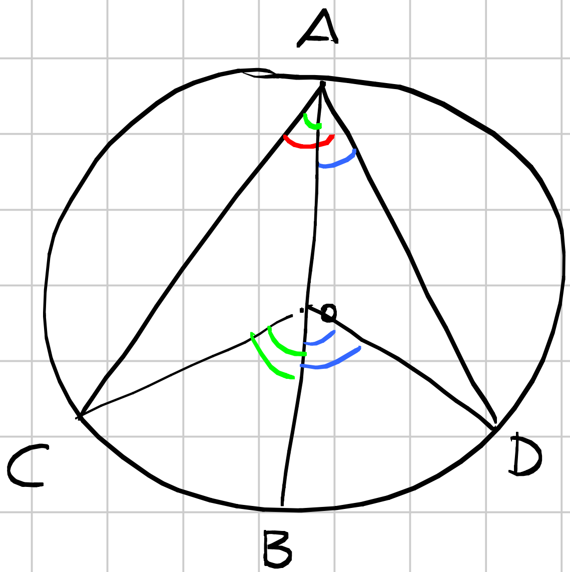
$\Updownarrow$

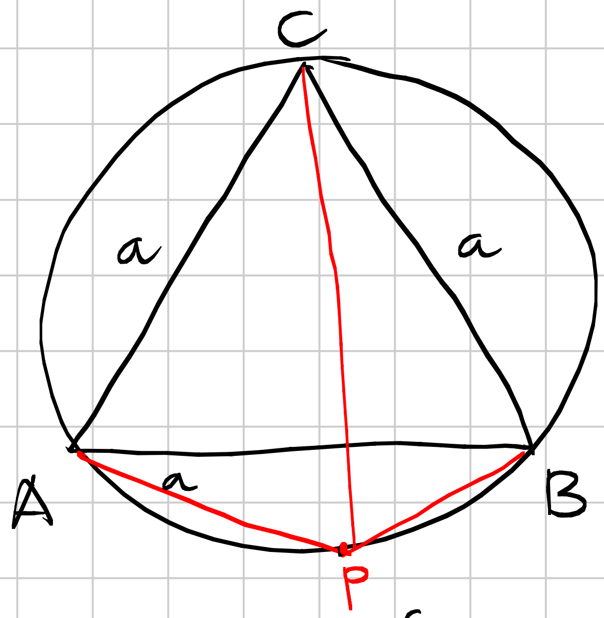
ABCD è ciclico

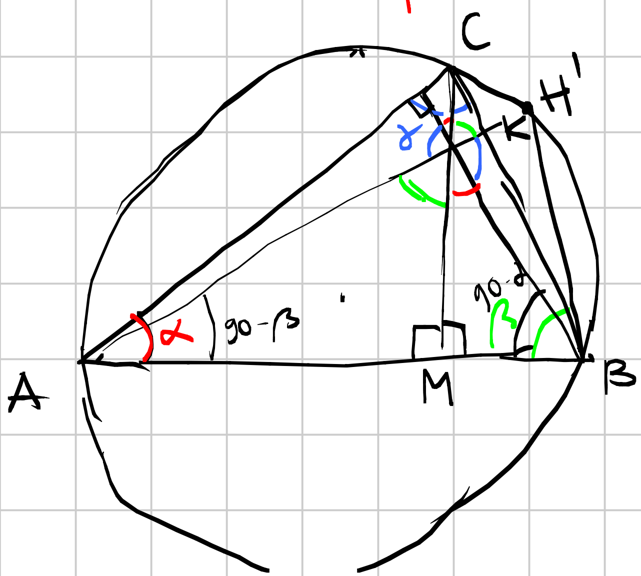


AOC è isoscele

$C\hat{O}B = 2\,C\hat{A}B$







$\forall P \quad CP = AP + PB$

APBC è ciclico

$\cancel{a}\cdot PB + \cancel{a}\cdot AP = \cancel{a}\cdot PC$

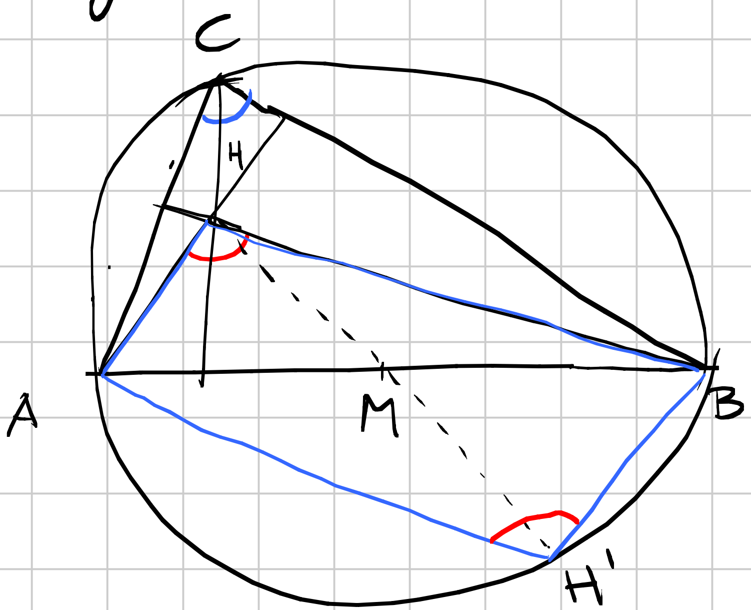
$PB + AP = PC$

$C\hat{H'}B = \beta + \gamma$

$CH'B$ è il simmetrico di CHB rispetto a CB

$ABH'C$ è un quadrilatero ciclico $\qquad \hat{A} + \hat{H'} = \alpha + \beta + \gamma = 180^\circ$

I punti simmetrici dell'ortocentro rispetto ai lati stanno sulla circonferenza circoscritta al triangolo



$AH'BH$ è un parallelogramma

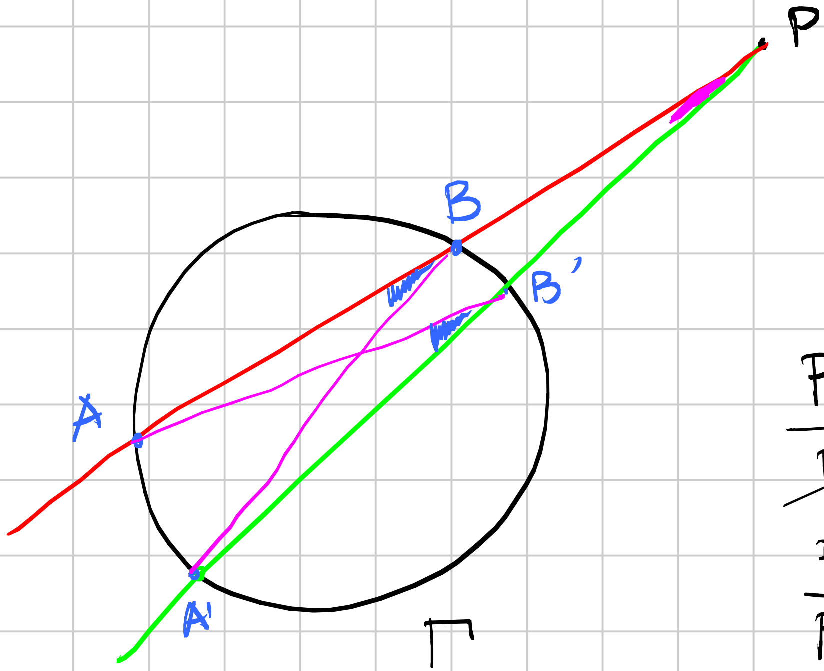
$+ \quad = 180^\circ$

$AH'BC$ è ciclico

$H' \in$ circonferenza circoscritta

H' è il simmetrico di H rispetto al punto medio di AB

POTENZA DI UN PUNTO RISPETTO A UNA CIRCONFERENZA



$$PA \cdot PB = \mathrm{pow}_{\Gamma}(P)$$

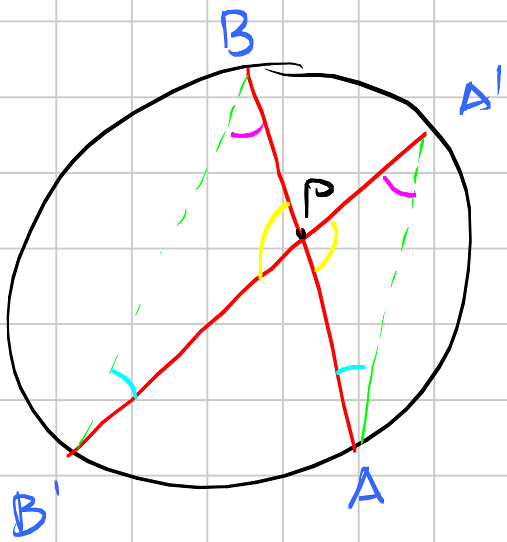
$$\parallel$$

$$PA' \cdot PB'$$

$$\frac{PA \cdot \cancel{PB}}{\cancel{PB} \cdot PA'} = \frac{\cancel{PA'} \cdot PB'}{PB \cdot \cancel{PA'}}$$

$$\frac{PA}{PA'} = \frac{PB'}{PB}$$

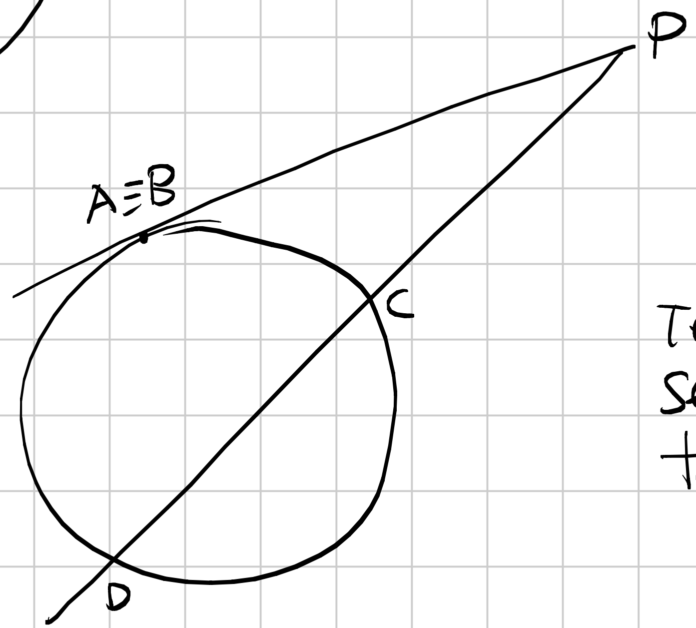
PAB' è simile a $PA'B$



$PA \cdot PB = PA' \cdot PB'$

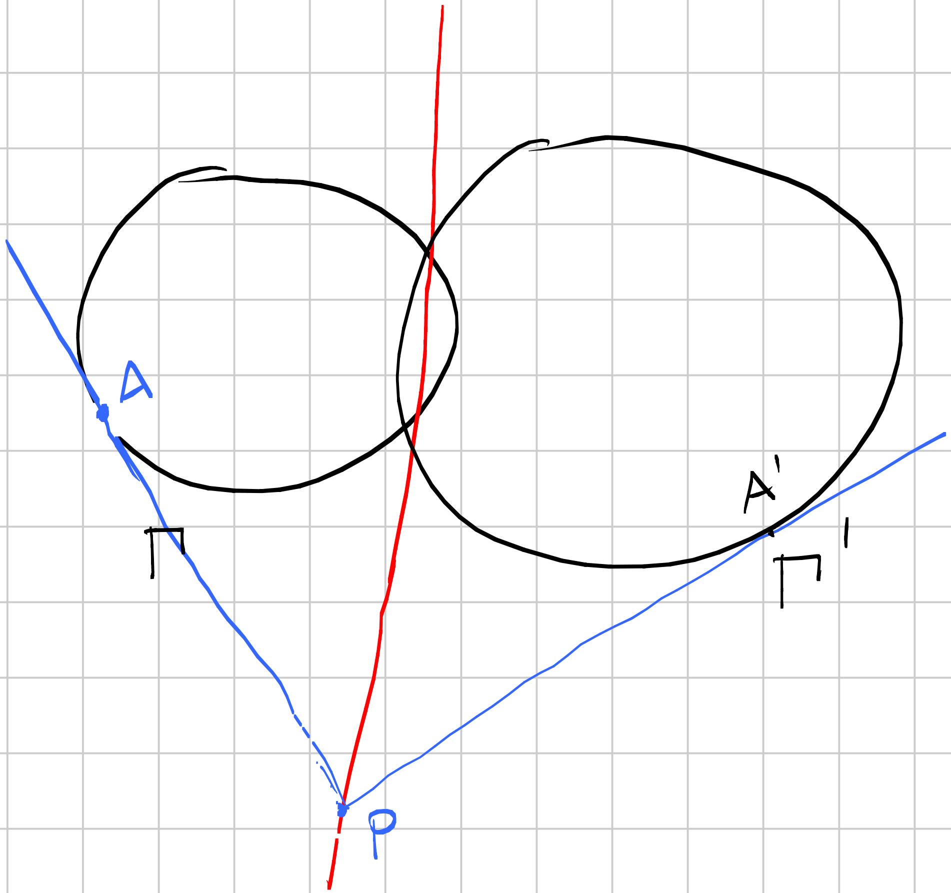
$\Downarrow$

Teorema delle 2 corde



$PA^2 = PC \cdot PD$

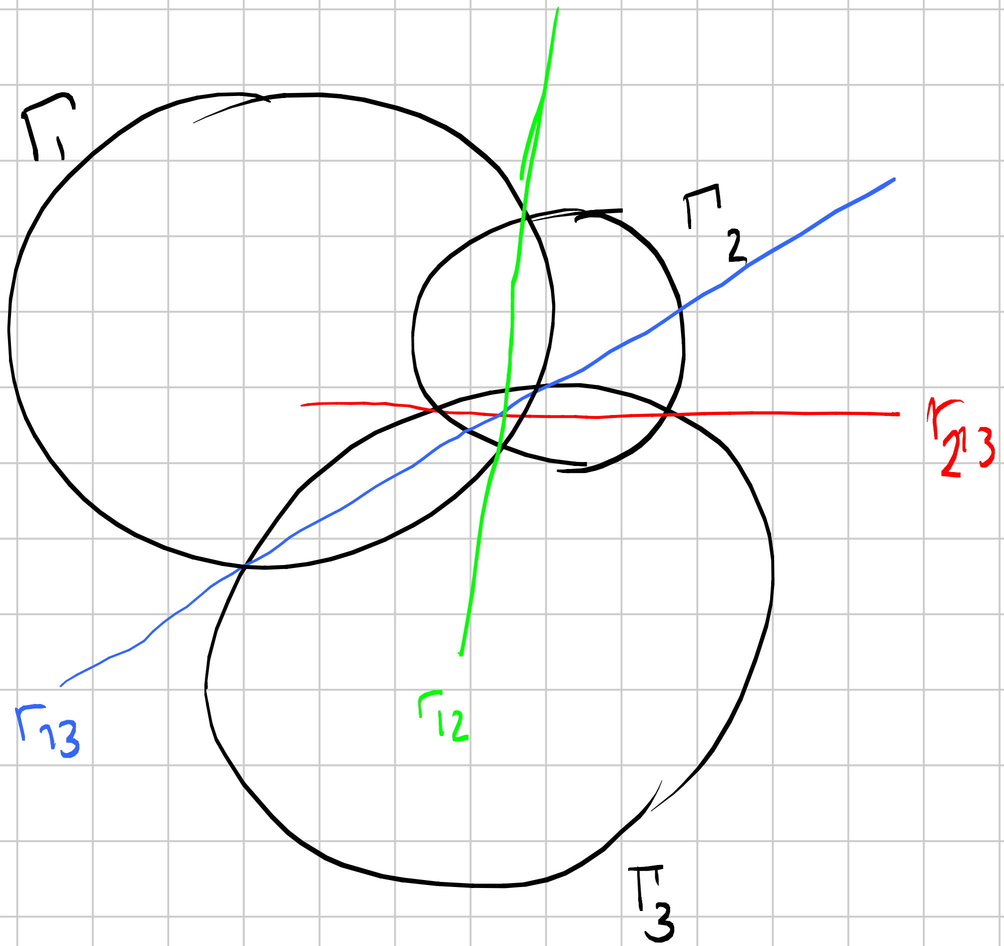
Teorema della secante e della tangente



$$\mathrm{pow}_{\Gamma}(P) = \mathrm{pow}_{\Gamma'}(P)$$

$PA \quad PA'$

$$\begin{aligned} \mathrm{pow}_{\Gamma}(P) &= PA^2 \\ \| \\ \mathrm{pow}_{\Gamma'}(P) &= PA'^2 \end{aligned} \Rightarrow PA = PA'$$



Γ_1
Γ_2
Γ_3
Γ_{12}
Γ_{13}
Γ_{23}