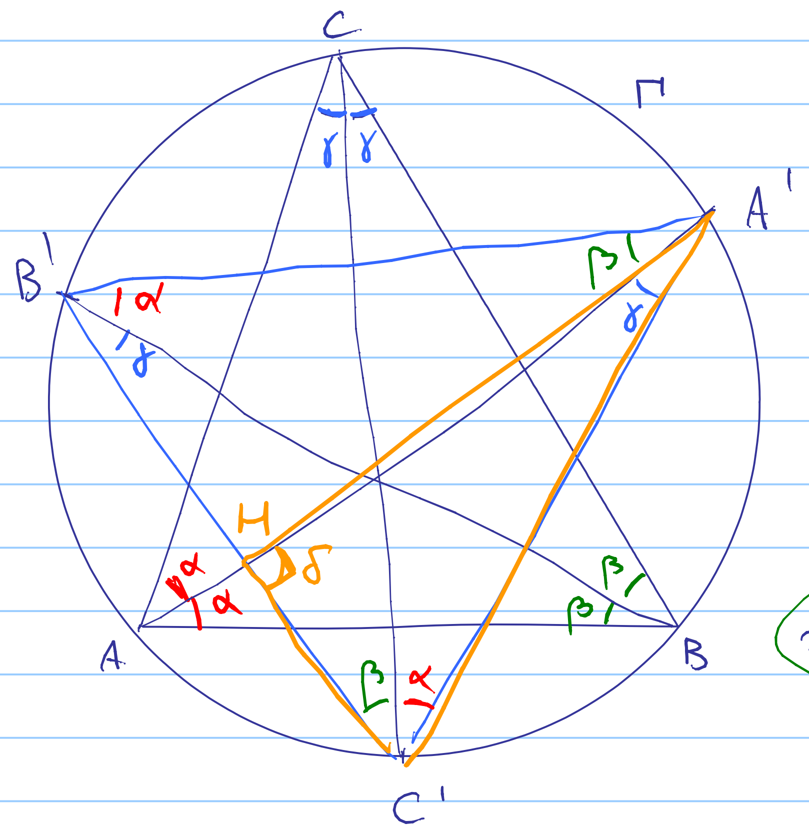
CESENATICO 1991

Titolo nota 05/04/2007

(TRANNE IL PROBLEMA 5)



x, y, z gli angoli in A', B', C' del triangolo $A'B'C'$

(1) $x = \beta + \gamma \qquad y = \gamma + \alpha \qquad z = \alpha + \beta$

$2(\alpha + \beta + \gamma) = \pi$

$\alpha + \beta + \gamma = \pi/2$

$x = \beta + \gamma = \frac{\pi}{2} - \alpha \qquad x < \frac{\pi}{2}$

$y < \frac{\pi}{2} \qquad z < \frac{\pi}{2}$

Mostro che se x, y, z sono angoli $< \frac{\pi}{2}$ con somma π, esistono α, β, γ che soddisfano le (1)

$$\begin{cases} x - y = \beta - \alpha \\ z = \beta + \alpha \end{cases} \quad \ldots \qquad \alpha = \frac{\pi}{2} - x \qquad \beta = \frac{\pi}{2} - y \quad \gamma = \frac{\pi}{2} - z$$

b)

$\boxed{2}$ $a^3 + 3a^2 + a \qquad a \geq 1$

$$k^2 = a^3 + 3a^2 + a = a(a^2 + 3a + 1)$$

$$MCD(a; a^2 + 3a + 1) = 1 \qquad \forall a$$

$$a^2 + 3a + 1 = a(a+3) + 1$$

$d = MCD(n, m) \qquad n = \alpha m + \beta \qquad d \mid \beta$

a e $a^2 + 3a + 1$ sono quadrati perfetti

$$\underbrace{a^2+2a+1}_{(a+1)^2} < a^2+3a+1 < \underbrace{a^2+4a+4}_{(a+2)^2} \quad ↯$$

③ $+(k+1)^2 - k^2 = 2k+1$

$-\left[(k-1)^2-(k-2)^2\right] = -\left[2k-3\right]$

$+ \; - \; - \; + \; = 4$

$1-4-9+16=4$

$4-9-16+25=4$

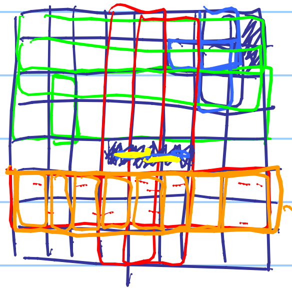
Quindi se si può ottenere il numero $k = \pm1\pm4 \ldots \pm n^2$

si può ottenere $4+k = \pm1\pm4 \ldots \pm n^2 + \ldots - \ldots - \ldots + \ldots$

$1 = +1$ $\quad 3 = -1+4$

$2 = -1-4-9+16$ $\quad 4 = + - - +$

④



$$= \left[\begin{array}{lll} \# \text{ coppie nere} & n & 2n \text{ nere} \\ \# \text{ coppie bianche} & b & 2b \text{ bianche} \end{array}\right.$$

$\rightarrow$ # coppie miste $\quad k \quad$ k bianche k nere

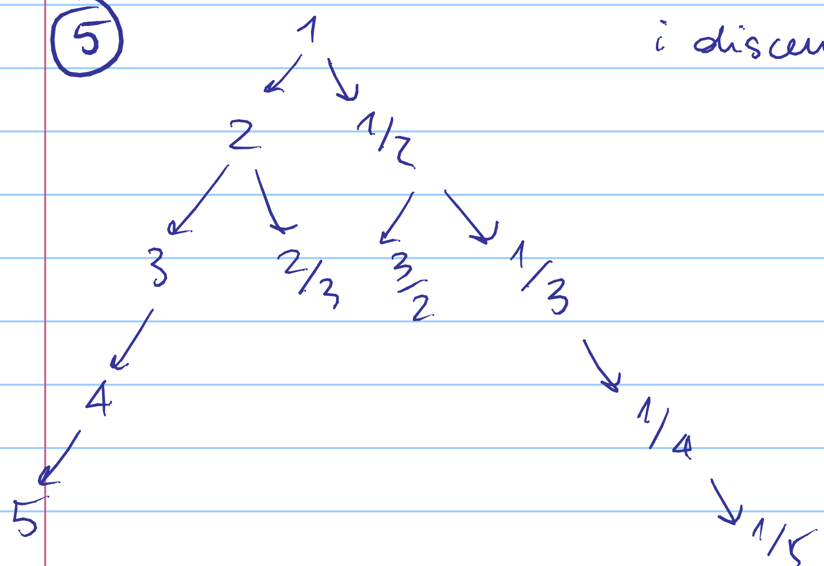
$2n+k$ nere $\qquad 2b+k$ bianche

8 bianche + 8 nere

$7 \cdot 8 \cdot 2 = 112$ coppie

divise in 14 strisce 2×8

⑤



i discendenti sono $\mathbb{Q} \cap (0, +\infty)$

$\frac{m}{n} > 1$ (è di sinistra) $\frac{m}{n} - 1 = \frac{m-n}{n}$

$\frac{m}{n} < 1$ (è di destra)

$1 - \frac{1}{x+1} = \frac{x}{x+1} = \frac{m}{n} \Leftrightarrow \frac{1}{1-\frac{m}{n}} - 1 = \frac{m}{n-m}$