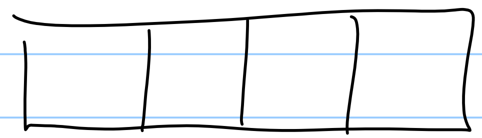
CESENATICO - FARB BDITION - V.1

Titolo nota 03/04/2007

1

4×4 $\quad$ $1, -1$



$\binom{4}{2} = 6$

$$\binom{n}{k} = \frac{n!}{k!\,(n-k)!}$$

$k! = 1 \cdot 2 \cdot 3 \cdot \ldots \cdot k$

1
1

1° caso: 2^a riga $= 1^a$ riga 6 Tabelle

2° caso: 2^a riga = opposto 1^a riga

3^a riga a piacere $\binom{4}{2} = 6$

4^a riga obbligata 36 Tabelle

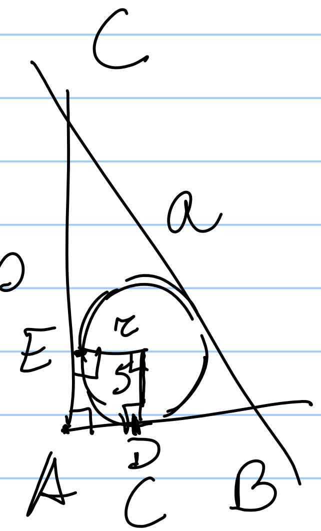
3° caso: | | 4 modi x scegliere la 2^a riga

$$\begin{array}{cc|c}
1 & 1 & \\
1 & -1 & \\
-1 & 1 & -1 \\
-1 & -1 & 1
\end{array}$$

2 scelte x 3^a e 4^a $\Rightarrow$ 48 Tabelle

$6 + 36 + 48 = \underline{90 \text{ Tabelle}}$

2



•) $a^2 = b^2 + c^2$

•) $5 = r$

$$r = \frac{S}{p} \qquad p = \frac{a+b+c}{2}$$

$$5 = AD = r = \frac{b+c-a}{2} = \frac{b+c-\sqrt{b^2+c^2}}{2}$$

$$10 - b - c = \sqrt{b^2+c^2}$$

$100 + \cancel{b^2} + \cancel{c^2} - 20b - 20c + 2bc = \cancel{b^2} + \cancel{c^2}$

$50 - 10b - 10c + bc = 0$

$(b-10)(c-10) - 50 = 0$

$(b-10)(c-10) = 50$

$50 = 5^2 \cdot 2$

I $50 \cdot 1$

II $25 \cdot 2$

III $10 \cdot 5$

I) $b - 10 = 50$ $\quad$ $c - 10 = 1$

$b = 60$ $\quad$ $c = 11$

$c = 60$

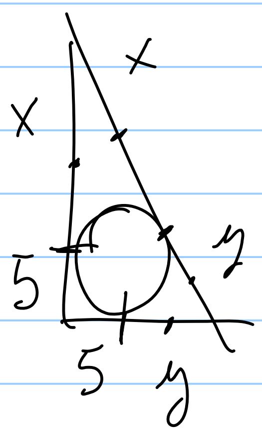
$b = 11$

II) $b-10=25$ $\quad c-10=2$ $\quad c=35$

$b=35$ $\quad c=12$ $\quad b=12$

III) $b-10=10$ $\quad c-10=5$ $\quad c=20$

$b=20$ $\quad c=15$ $\quad b=15$



$$(x+5)^2+(y+5)^2=(x+y)^2$$

$$\cancel{x^2}+10x+25+\cancel{y^2}+10y+25=\cancel{x^2+y^2}+2xy$$

$$50+10x+10y-2xy=0$$

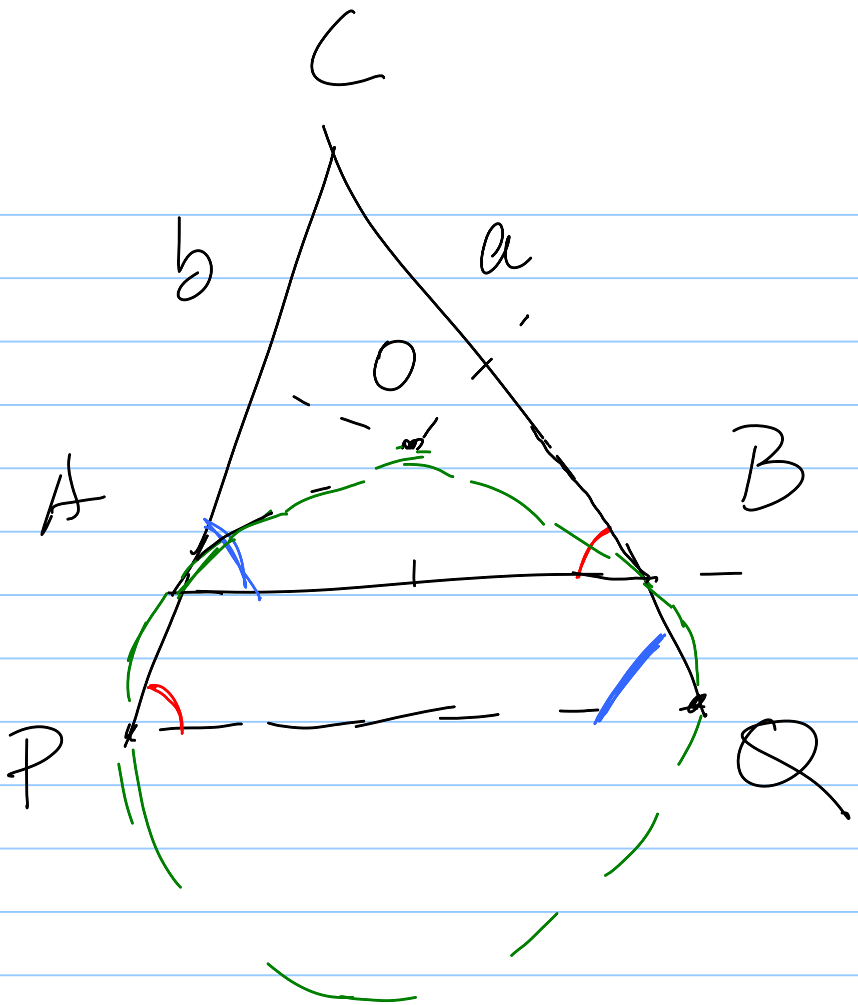
$$2(5-x)(y-5)=-100$$

$$x(10-2y) = -10y-50$$

$$x = \frac{-10y-50}{10-2y} = \frac{25+5y}{y-5} =$$

$$= \frac{5(y-5)+50}{y-5} = 5 + \boxed{\frac{50}{y-5}}$$

3

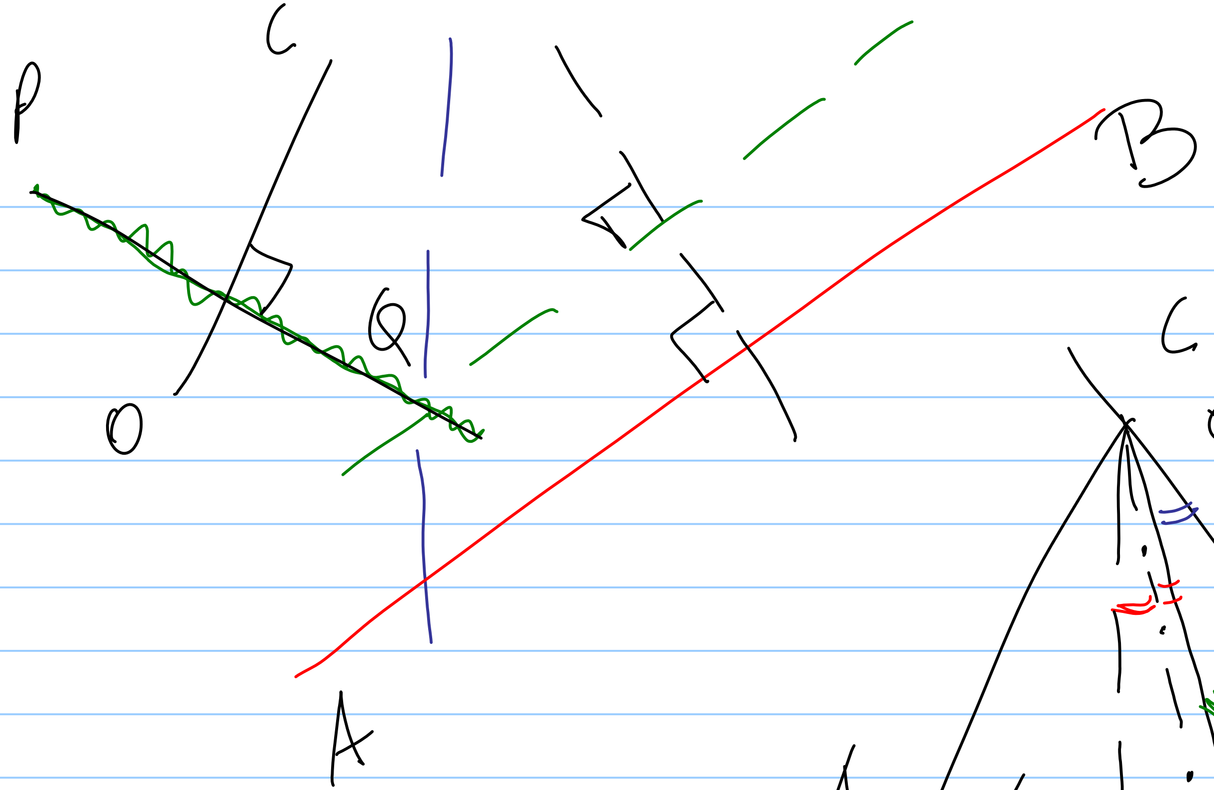


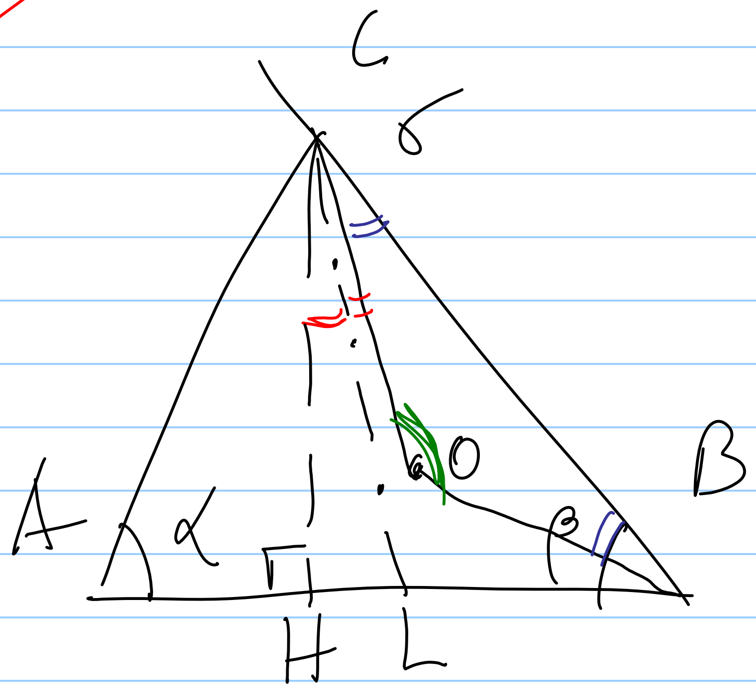
$CO \perp PQ$

$\triangle ABC \sim \triangle PQC$

PQ è parallela al simmetrico rispetto alla bisettrice di C di AB

$CO \perp PQ \iff$ simm di CO risp. alla bisett di C è $\perp AB$





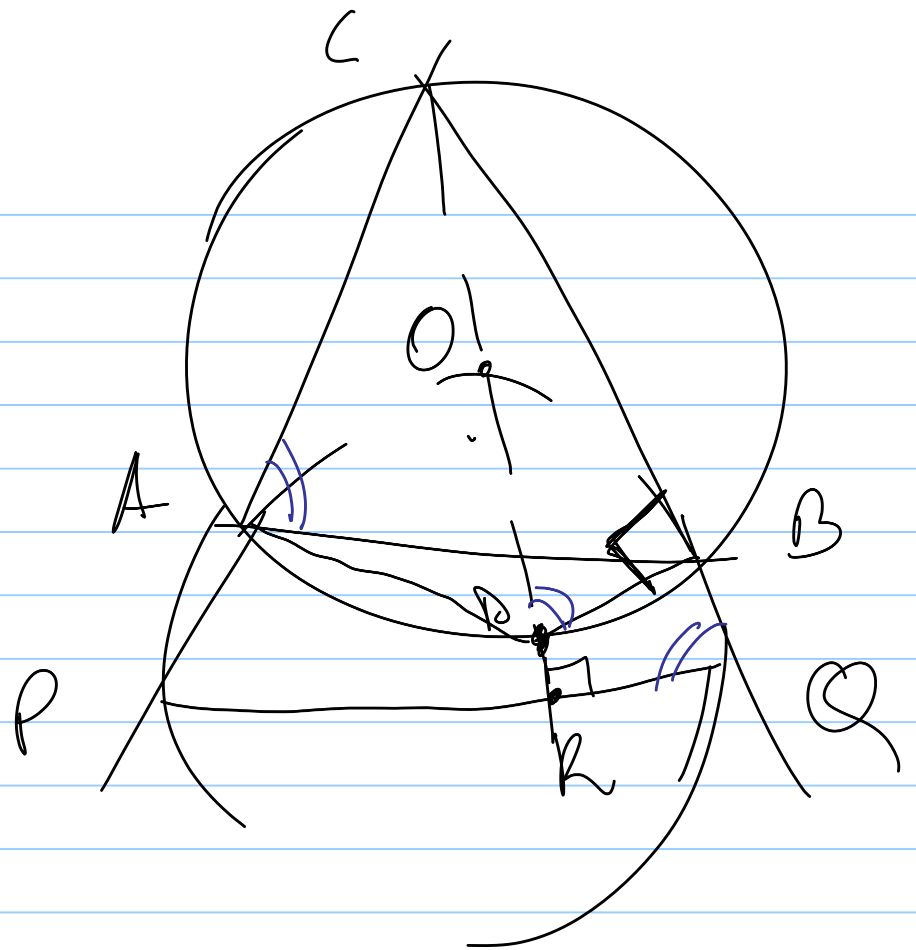
$\hat{HCL} = \hat{ACL} - \hat{ACH} =$

$= \hat{LCB} - \hat{BCO} = \hat{OCL}$

$\hat{ACH} = 90 - \alpha$

$\hat{BCO} = 90 - \alpha$

$\hat{ACL} = \hat{LCB} = \frac{\gamma}{2}$



ABQP ciclico

ADBC ciclico

$O\hat{C}B + C\hat{D}B = 90^\circ$

$\triangle$ CKQ

4 $\quad p(x)$ coeff. interi $\qquad 2 \mid 4 \quad x \mid x^2$

n radici ~~reali~~ intere distinte

$a_1 < a_2 < \dots < a_n$

t.c. $\quad p(a_1) = p(a_2) = \dots = p(a_n) = 0$

$q(x) = (p(x))^2 + 1$ $\qquad$ Dimm che esiste $h(x)$

t.c. $\quad h(x) \mid q(x)$

$\deg p(x) =$ "grado di $p(x)$" $\qquad$ $\deg h(x) \geq \left\lfloor \frac{n+1}{2} \right\rfloor$

degree $\qquad$ e h non ha fattori propri a coeff. interi

$$x^2 - x - 1 \qquad x_{1,2} = \frac{1 \pm \sqrt{1+4}}{2} = \frac{1 \pm \sqrt{5}}{2}$$

$$\left(x - \frac{1+\sqrt{5}}{2}\right)\left(x - \frac{1-\sqrt{5}}{2}\right)$$

Sia $h(x)$ pol. a coeff interi che divide $\varphi(x)$

e non ha fattori propri a coeff. interi

$$h(x) \cdot r(x) = q(x) = (p(x))^2 + 1$$

$$h(a_1) \cdot r(a_1) = 1 \qquad h(a_i) = \pm 1 \qquad i = 1, \dots, n$$

$\Rightarrow$ ci sono almeno $\left\lfloor \frac{n+1}{2} \right\rfloor$ punti in cui

$$h(X) = 1 \Rightarrow \deg h \geq \left\lceil \frac{n+1}{2} \right\rceil$$

5

$$3^k - 1 = y^n \qquad (n, k, y) \text{ naturali}$$

$(n, 0, 0)$ $n > 0$ funziona

$(1, \cancel{1}, 2)$ $(1, k, 3^k - 1)$

$(3, 2, 2)$

$n \geq 2,\ y, k > 0$

$3^k = y^n + 1$

$n = 3h + r$

$r = 0, 1, 2$

$m = 3h + r$

$\ell = 3j + s$

$m\ell = \underbrace{9jh + 3hs + 3jr}_{} + rs$

$m^2 = 9h^2 + 6hr + r^2$

$= 3(3h^2 + 2hr) + r^2$

$r^2 = 0, 1, 4$

$m^{2k} = 3X + 1$

	0	1	2
0	0	0	0
1	0	1	2
2	0	2	1

$3^k = y^n + 1$ se $n = 2z$

$y^{2z} = 3x + 1$

$3^k = 3x + 2$ non si può

$\Rightarrow n$ dispari $\quad y = -1 = n$

(*) $3^k = y^n + 1^n = (y+1)(y^{n-1} - y^{n-2} + y^{n-3} - \dots + 1)$

(*) $y^{n-1} - \dots + 1 = (y+1) \cdot q(y) + r$

se $y = -1$ diventa $n = r$

$y+1 = 3^{a}$ $\quad (y^{n-1} - y^{n-2} + \dots + 1) = 3^{b}$

$3 \mid r = n \Rightarrow 3 \mid n \Rightarrow n = 3m$

$3^{k} = (y^{m})^{3} + 1 \qquad z = y^{m}$

$3^{k} = z^{3} + 1 = (z+1)(z^{2} - z + 1)$

$z+1 = 3^{c} \qquad z^{2} - z + 1 = 3^{d}$

$z = 3^c - 1$ $\quad 3^{2c} - 2 \cdot 3^c + 1 - 3^c + 1 + 1 = 3^d$

$3^{2c} - 3^{c+1} + 3 = 3^d$

$3^{2c-1} - 3^c + 1 = 3^{d-1}$

$d - 1 = 0$ $\quad d = 1$ $\quad z = 3^c - 1 = 2$

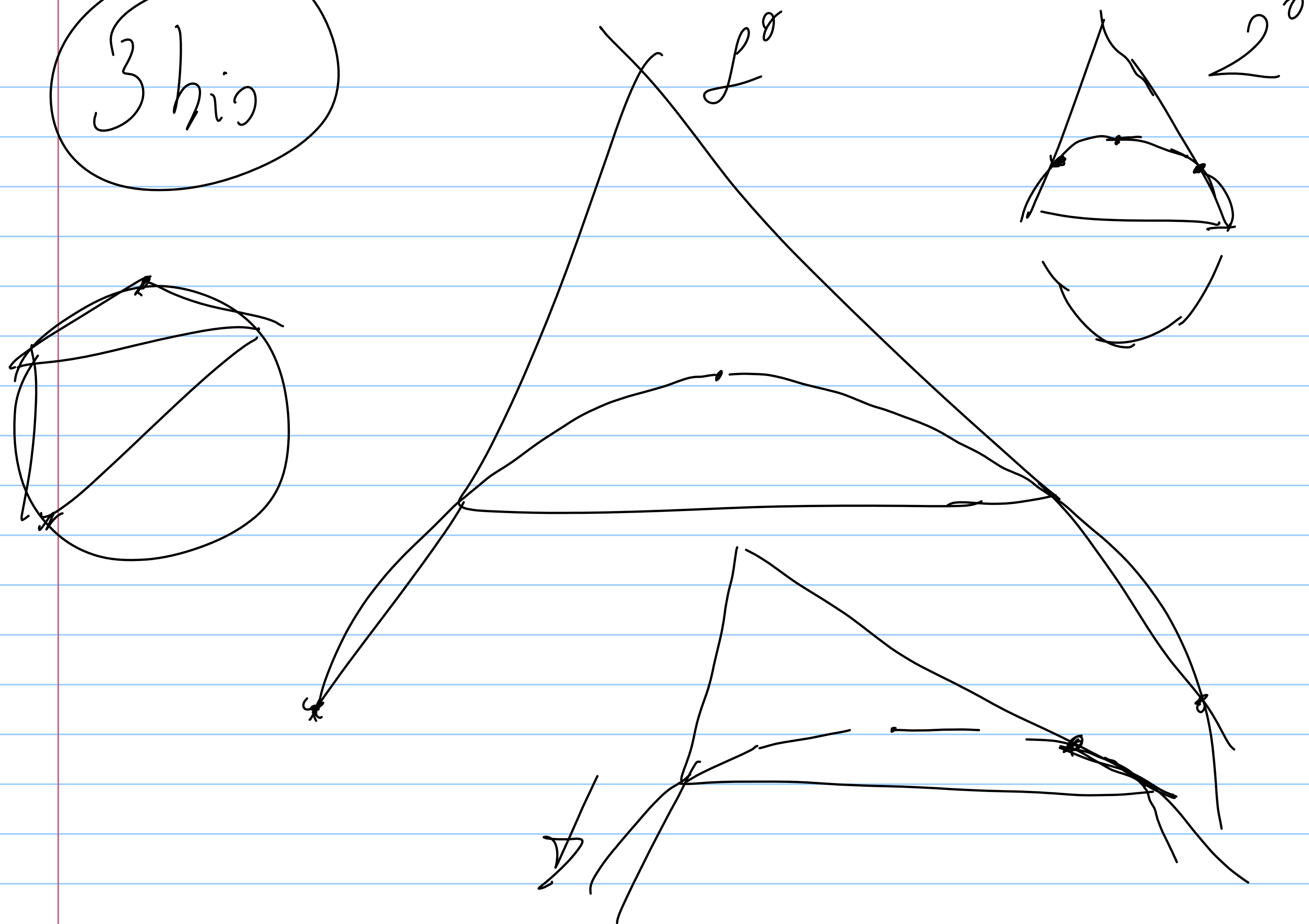
$\cancel{2}c - 1 = \cancel{0}$ $\quad c = 1$

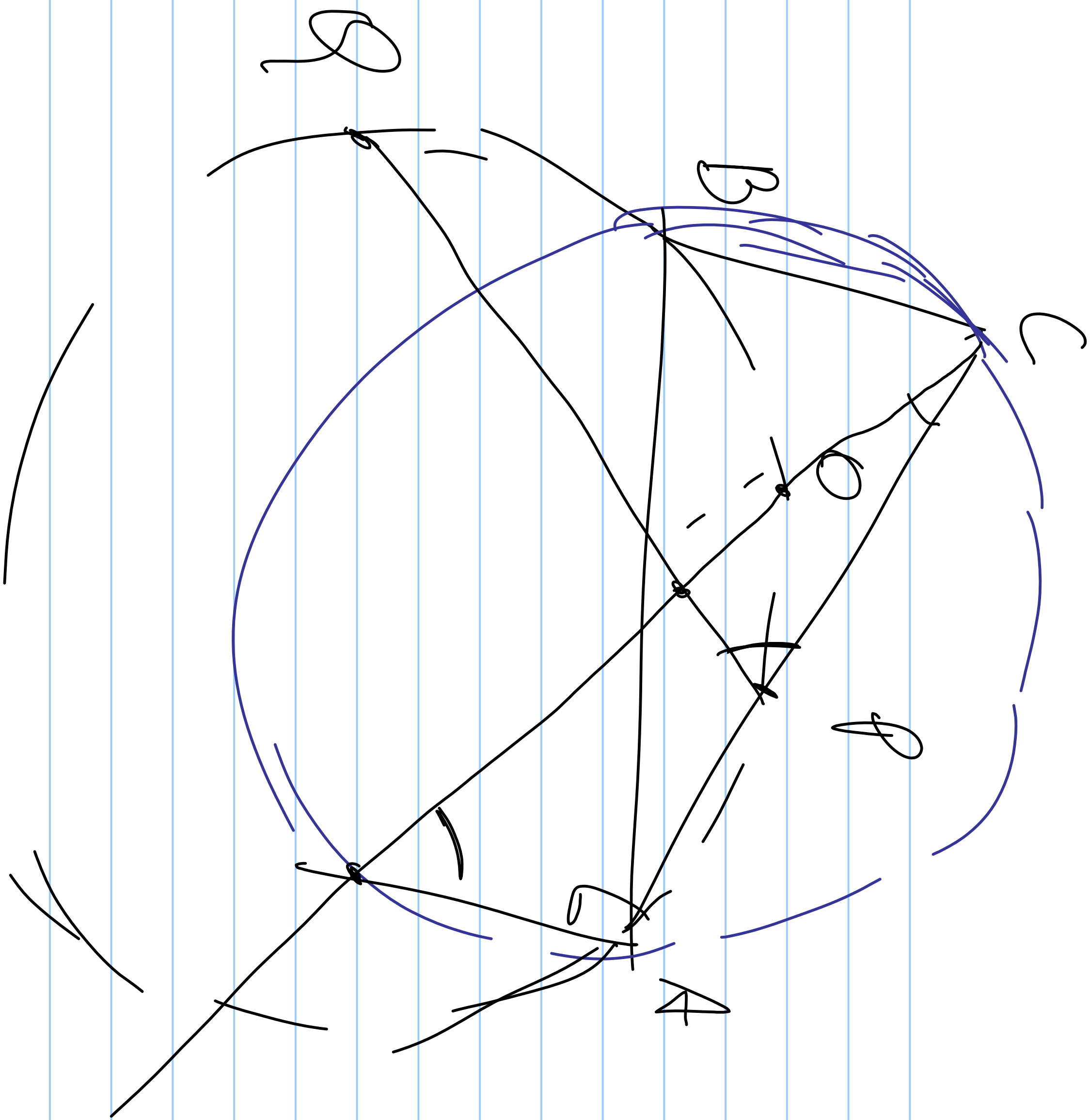
$y^m = z = 2 \Rightarrow y = 2$

$m = 1$

$n = 3m = 3$ $\quad 3^k - 1 = 2^3 \Rightarrow k = 2$

כוח 3




B
C
O
A