CESENATICO - FAKE EDITION - V.3

Titolo nota 04/04/2007

1. $N = p_1^{a_1} \cdot \ldots \cdot p_k^{a_k}$ $\quad (a_1+1)\cdot \ldots \cdot (a_k+1)$

$20 = 2^2 \cdot 5$

$1 \cdot 20 \Rightarrow$ a) $N = x^{19}$

$2 \cdot 10 \Rightarrow$ b) $N = xy^9$

$4 \cdot 5 \Rightarrow$ c) $N = x^3y^4$

$2 \cdot 2 \cdot 5 \Rightarrow$ d) $N = xyz^4$

x, y, z primi distinti

a) $x \geq 2 \Rightarrow x^{19} \geq 2^{19} \geq 2^{10} = 1024 \geq 1000$ no

b) $y \geq 2$ se $y = 3$ $\quad 3^9 > 1000 \Rightarrow$ No

se $y = 2$ $\quad x \geq 3$ $\quad y^9 x \geq 512 \cdot 3 > 1000$ $\Rightarrow$ no

c) $y \leq 3$ se $y = 3, x = 2$ $\quad N = 648$

se $y = 2, x = 3$ $\quad N = 432$

d) $z \leq 5$ se $z = 5$, $xy \leq 1 \Rightarrow$ no

se $z = 3$, $xy \leq 12 \Rightarrow N \leq 810$

se $z = 2$, $xy \leq 62 \Rightarrow \begin{matrix} x = 3 \\ y = 19 \end{matrix} \Rightarrow \boxed{N = 912}$

2

$$\left(n^{2006} + n + 1\right)\frac{(n-1)}{(n-1)} = \left[\begin{matrix} \text{se } n = 1 \quad n^2+n+1 = 3 \\ \text{è primo.} \\ \Rightarrow n \neq 1 \end{matrix}\right]$$

$$= \frac{n^{2007} + n^2 + n - n^{2006} - n - 1}{n-1} = \frac{n^{2007} - 1 - n^2\left(n^{2004} - 1\right)}{n-1}$$

$$\left(n^3 - 1 \mid n^{3k} - 1 \quad \forall k > 0 \right.$$
naturale

k dip.

$$(m^3)^k - 1^k = (m^3 - 1)(\underline{\qquad\qquad})$$

$k = 2^a \cdot h$ h dip.

$$\left((m^3)^h\right)^{2^a} - 1^{2^a} = \underbrace{\left((m^{3h})^{2^{a-1}} - 1^{2^{a-1}}\right)}\left((m^{3h})^{2^{a-1}} + 1^{2^{a-1}}\right)$$

$$\frac{(m^{2007} - 1) - m^2(m^{2004} - 1)}{m - 1} = \frac{(m^3 - 1)(\underline{\qquad\qquad})}{m - 1} =$$

$$= (m^2 + m + 1)(\underline{\qquad\qquad}) = m^{2006} + m + 1$$

•) $n^2+n+1=1 \Rightarrow n=0$ $\left(n^{2006}+n+1=1 \text{ non è primo} \right)$

$n=-1 \Rightarrow$ non acc.

$n^2+n+1=-1 \Rightarrow n^2+n+2=0$ non ha sol.

•) $\left(\text{———} \right) = -1$ $\Big|\ n^{2006}+n+1>0$ se $n \geq 0$

$n^2+n+1=-n \Rightarrow n^2+n+1+n=0$

$n>0$

non è possibile

Se m è tale che

$\left(\text{———} \right) = 1 \Rightarrow m^{2007}-m^{2006}+m^2-\cancel{1}=m^3-\cancel{1}$

$m^{2005}-m^{2004}+1=m$ se $m \neq 0,1$

$\Rightarrow$ non è possibile

(3) $p(\cancel{x}) = a^3x^3 + a^2x^2 + ax + a \qquad a > 0$

$p(0) = a > 0$

$m > 0 \qquad p(m) = \underset{\substack{\vee \\ 0}}{a^3m^3} + \underset{\substack{\vee \\ 0}}{a^2m^2} + \underset{\substack{\vee \\ 0}}{am} + \underset{\substack{\vee \\ 0}}{a} > 0$

$p(-m) = -a^3m^3 + a^2m^2 - am + a =$

$= \underset{\substack{\wedge \\ 0}}{-a}\,(a^2m^3 - am^2 + w - 1)$

$q_m(y) = y^2m^3 - ym^2 + m - 1$

$q_m(0) = m - 1 \qquad \Delta_m = m^4 - 4m^4 + 4m^3 =$

$= -3m^4 + 4m^3$

$\Delta_m < 0 \qquad -3m^4 < -4m^3$

$3m^4 > 4m^3$

$\cancel{m-1 > 0}$

$\cancel{\text{se } m \geq 2}$

$3m > 4 \qquad m \geq 2$ <u>vero</u>

$q_m(y) \neq 0 \quad \forall y \in \mathbb{R} \quad \forall m \geq 2$

$-a\, q_m(a) \neq 0 \quad \forall a > 0 \quad$ se $\quad m \geq 2$

$\Rightarrow p(-m) \neq 0 \quad \forall a > 0 \quad$ e $\quad m \geq 2$

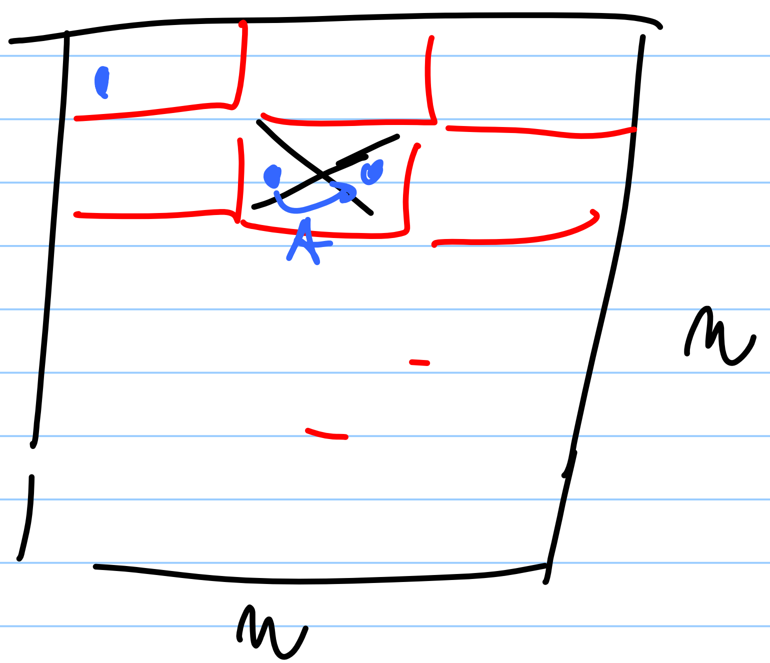
Devo controllare $\quad p(-1)$

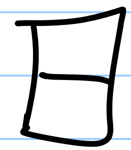
$$p(-1) = -a^3 + a^2 - \cancel{a} + \cancel{a} = a^2(1-a)$$

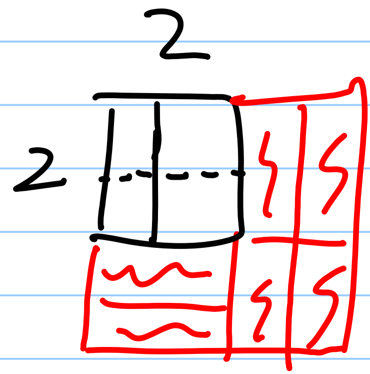
$p(-1)=0 \iff \begin{matrix} a=0 \\ a=1 \end{matrix} \to$ non acc.

$\Rightarrow$ Se $p(x)$ ha rad. intere $\Rightarrow a=1$

4 Sia $n = 2k \Rightarrow$ Posso riempire la scacchiera con tessere del domino (2×1)







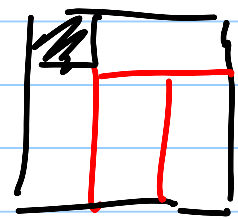
Se A ha mosso finora "rimanendo sulle stesse tessere" allora può continuare a farlo. ("induzione")

$\Rightarrow$ Se qualcuno perde, è B.

Ma qualcuno perde: dopo n^2-1 mosse avremo toccato tutte le caselle $\Rightarrow$ non si potrà più muovere.

$\Rightarrow$ perde B.

$\underline{n = 2K+1}$



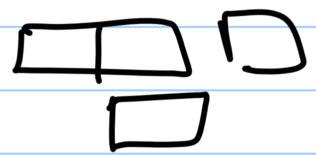
Possiamo ricoprire la scacchiera meno un angolo di tessere del domino.
Da qui come prima invertendo i ruoli.

$\Rightarrow$ perde A.

Idea :

I

$\bar{I}$



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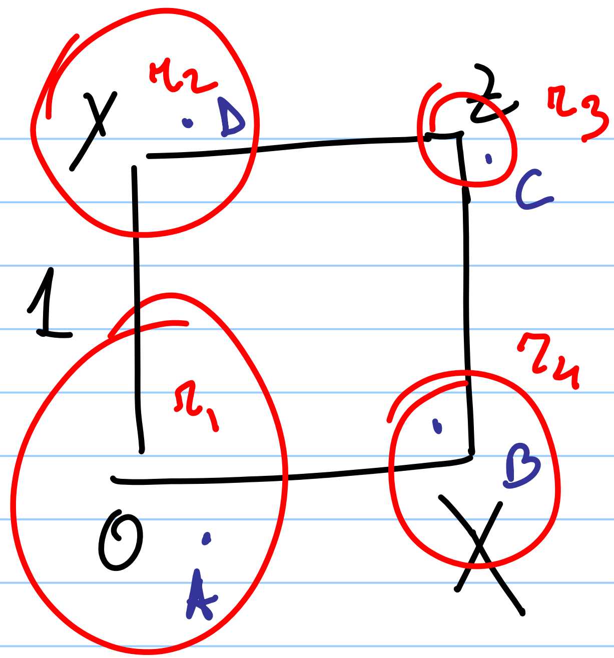
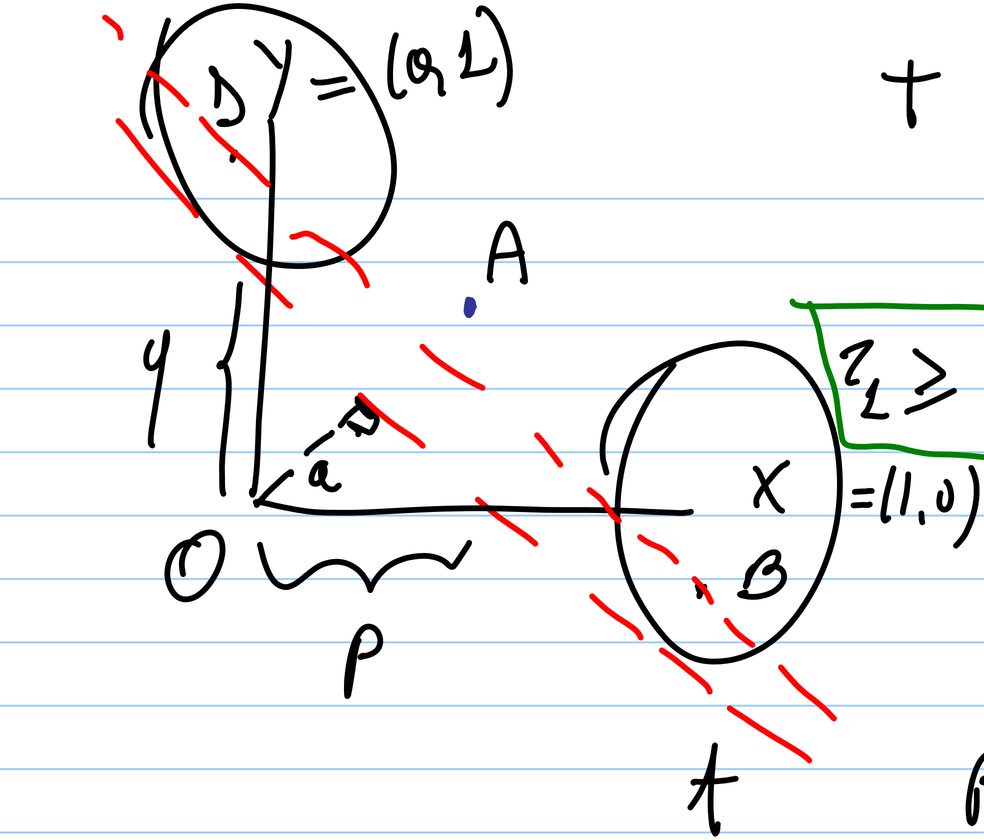


t.c. $r_1^2 + r_2^2 + r_3^2 + r_4^2 \leq \frac{1}{\pi}$

$\Rightarrow$ ABCD convesso

Oss: ABCD è convesso $\Rightarrow$ A non sta in $\overset{\triangle}{BCD}$

Se A sta in $\overset{\triangle}{BCD}$



t tg. com. alle cf. di centri Y, X

$$\boxed{r_1 \geq d(O,A) \geq d(O,t) = a}$$

$t: \; qx + py = pq$

$$\frac{pq}{\sqrt{p^2+q^2}} = a$$

$$r_2 = d(t, X) = \frac{q - pq}{\sqrt{p^2+q^2}} \qquad r_1 = d(t, Y) = \frac{p - pq}{\sqrt{p^2+q^2}}$$

$$r_2^2 + r_4^2 + r_1^2 \geqslant r_2^2 + r_4^2 + a^2 =$$

$$= \frac{q^2}{p^2+q^2} + \frac{p^2q^2}{p^2+q^2} - \frac{2pq}{p^2+q^2}q + \frac{p^2}{p^2+q^2} + \frac{p^2q^2}{p^2+q^2} -$$

$$- \frac{2pq}{p^2+q^2}p + a^2 =$$

$$\frac{p^2+q^2}{p^2+q^2} + a^2 - 2a \cdot \frac{q}{\sqrt{p^2+q^2}} + a^2 - 2a\frac{p}{\sqrt{p^2+q^2}} + a^2 =$$

$$= 1 + 3a^2 - 2a\frac{p+q}{\sqrt{p^2+q^2}} \geq$$

$$\frac{p+q}{2} \leq \sqrt{\frac{p^2+q^2}{2}} \qquad (p, q \geq 0)$$

$$\frac{p+q}{\sqrt{p^2+q^2}} \leq \sqrt{2}$$

$$6a - 2\sqrt{2} = 0$$

$$a = \frac{2\sqrt{2}}{6} = \frac{\sqrt{2}}{3}$$

$$\geq 1 + 3a^2 - 2a\sqrt{2} \geq$$

$$\geq 1 + \frac{3 \cdot 2}{9} - \frac{2 \cdot \sqrt{2} \cdot \sqrt{2}}{3} =$$

$$= 1 + \frac{2}{3} - \frac{4}{3} = 1 - \frac{2}{3} = \frac{1}{3}$$

$$r_1^2 + r_2^2 + r_4^2 \geq \frac{1}{3} \gneq \frac{1}{\pi}$$

Se A sta in $\overset{\triangle}{BCD}$, allora

$$r_1^2 + r_2^2 + r_4^2 > \frac{1}{\pi} \Rightarrow$$ <u>Assurdo.</u>

2

$$m^{2006} + m + 1$$

$$2006 = 3 \cdot k + 2$$

$$x^m - 1 = 0 \qquad x^3 - 1 = 0$$

$$x^3 = 1$$

$$x^{3k} = 1 \Rightarrow x^{3k+2} = x^2$$

$\omega \in \mathbb{C}$ t.c. $\omega^2 + \omega + 1 = 0 \Rightarrow (\omega^3 - 1) = 0$

$$\Longrightarrow \underbrace{\omega^{2006} + \omega + 1 = \omega^2}_{\times \text{ le cose dette sopra}} + \omega + 1 = 0$$

Tutte le radici di x^2+x+1 (complesse)

sono radici di $x^{2006}+x+1$

$\Longrightarrow$ (poiché x^2+x+1 non ha radici doppie)

$$x^2+x+1 \mid x^{2006}+x+1$$

$$x^{2006}+x+1 = (x^2+x+1)\cdot q(x)$$

Se $q(x)=1 \Rightarrow x^{2006}\cancel{+x}\cancel{+1} = x^2\cancel{+x}\cancel{+1}$

$$x^{2004}=1$$

3 Se $\forall x > y \quad f(x) > f(y)$

esiste al più 1 valore t t.c. $f(t) = 0$.

$p(0) = a$

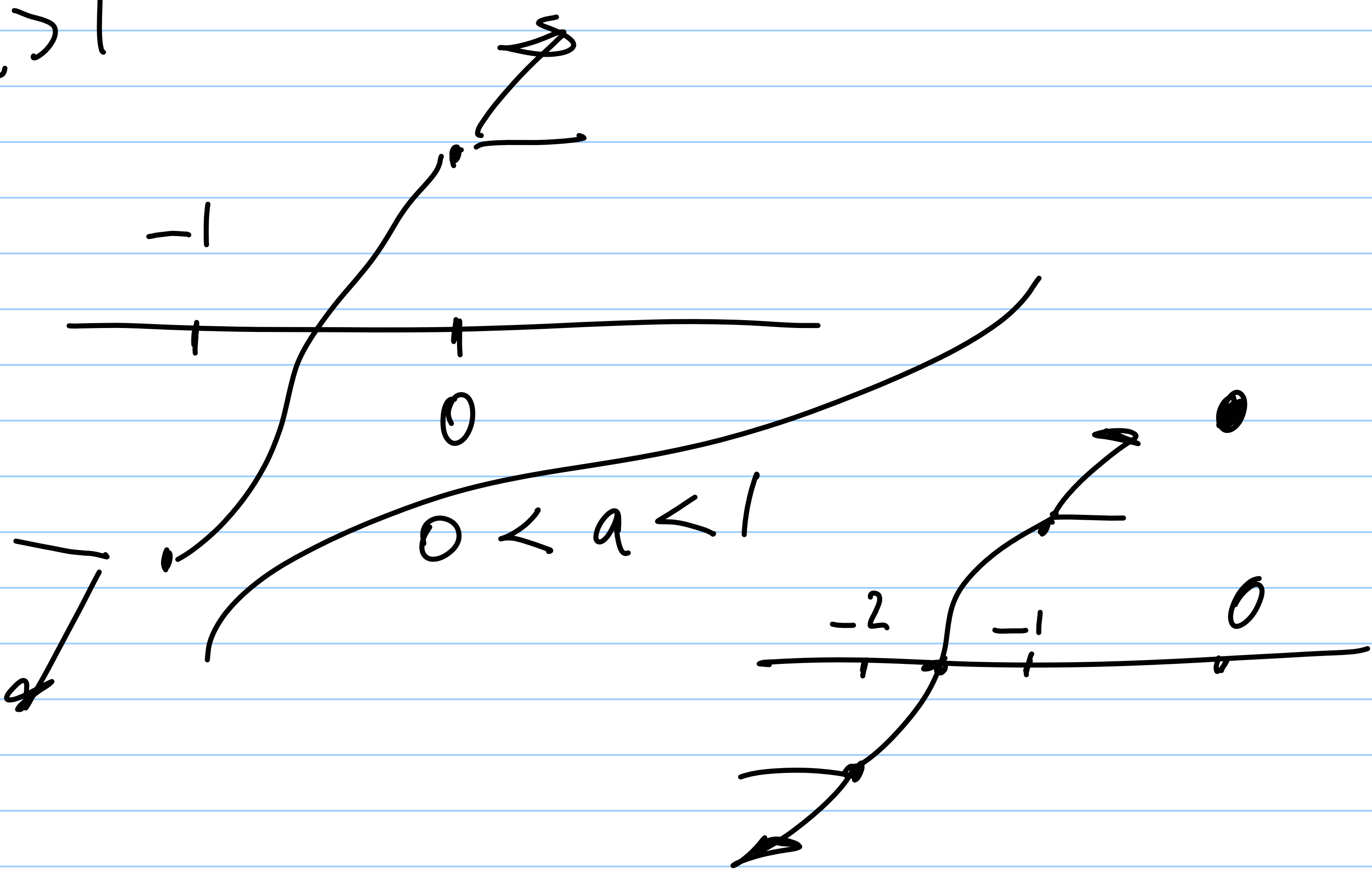
$p(-1) = a^2(1-a)$ se $a > 1$ $p(-1) < 0$

se $a = 1$ $p(-1) = 0$

se $a < 1$ $p(-1) > 0$

$p(-2) = -8a^3 + 4a^2 - 2a + a = -a(8a^2 - 4a + a) < 0$

$a > 1$



$0 < a < 1$

$$p'(x) = 3a^3x^2 + 2a^2x + a$$

$$\Delta = 4a^4 - 12a^4 = -8a^4 < 0$$

$p'(0) = a > 0 \Rightarrow p$ è crescente.

$$p(x) = a^3x^3 + a^2x^2 + ax + a =$$

$$= \underbrace{\left(ax + \frac{1}{3}\right)^3}_{\text{crescente}} + \underbrace{\left(\frac{2}{3}ax + a - \frac{1}{27}\right)}_{\text{crescente}} \Rightarrow$$

$a > b \Rightarrow a^3 > b^3$

$a^3x^3 + a^2x^2 + ax + a$

$\Downarrow$

$(ax + c)^3 - \frac{1}{3}ax - \frac{1}{27} + ax + a = (\ )^3 + \frac{2}{3}ax + \ldots$

$a^3x^3 + \underbrace{3c}_{1} a^2x^2 + 3c^2ax + c^3$

$\frac{1}{3}ax + \frac{1}{27}$

$c = \frac{1}{3}$

$p(x) \qquad x = ty + u$