CESENATICO - FAKE EDITION 4

Titolo nota 05/04/2007

1) $(p+1)^q$

- $q = 2 \rightarrow$ ok $\boxed{(p,2)\ \checkmark}$
- $q \neq 2$ $(p+1)^q$ è un quadrato $\Leftrightarrow$ $p+1$ è un quadrato

$\longrightarrow p+1 = r_1^{\alpha_1} r_2^{\alpha_2} \dots r_k^{\alpha_k}$

$(p+1)^q = r_1^{q\alpha_1} \dots\dots r_k^{q\alpha_k} \Leftarrow$ $\left.\begin{array}{l} q\alpha_i \text{ pari } \forall i \\ q \text{ disp.} \end{array}\right\} \Rightarrow \alpha_i$ pari $\forall i$

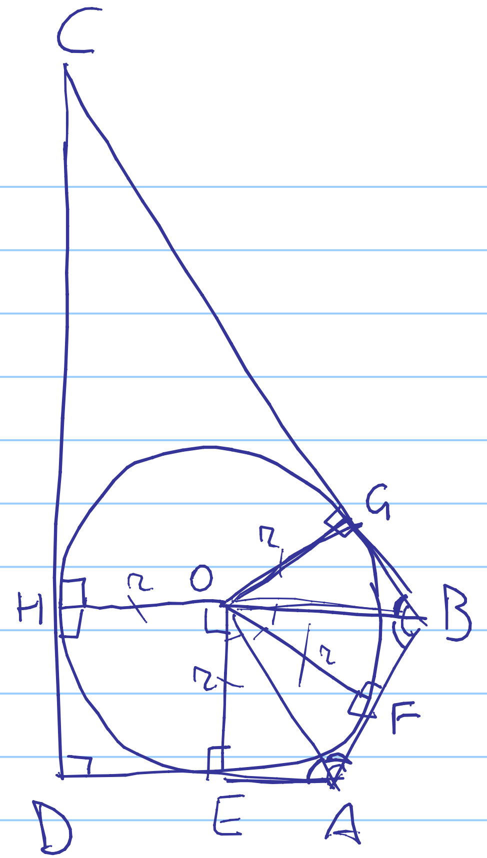
$p+1 = k^2$

$p = k^2 - 1 = \underbrace{(k-1)}\underbrace{(k+1)} \Rightarrow k-1 = 1 \longrightarrow k = 2$

$p = 3$ $\boxed{(3,q)\ \checkmark}$ $(p+1)^q$ è un quadrato

$= 4^q = 2^{2q} = (2^q)^2$

2)



$BC = 1$

<u>th</u> AD?

$D\hat{A}B = A\hat{B}C = 120°$

$C\hat{D}A = 90°$

$OEDH$ è un quadrato

$\overset{\triangle}{OBG}$, $\overset{\triangle}{OBF}$, $\overset{\triangle}{OFA}$, $\overset{\triangle}{OEA}$ sono congruenti

$H\hat{O}B = 90° + 90° = 180°$

$\Rightarrow H, O, B$ sono allineati

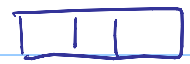
HCB rettangolo $90°, 60°, 30°$

$$\left.\begin{aligned} \Rightarrow HB &= \frac{1}{2} CB = \frac{1}{2} \\ \| \\ OB + OH &= \frac{2}{\sqrt{3}} r + r \end{aligned}\right\} \Rightarrow \frac{1}{2} = r\left(1 + \frac{2}{\sqrt{3}}\right)$$

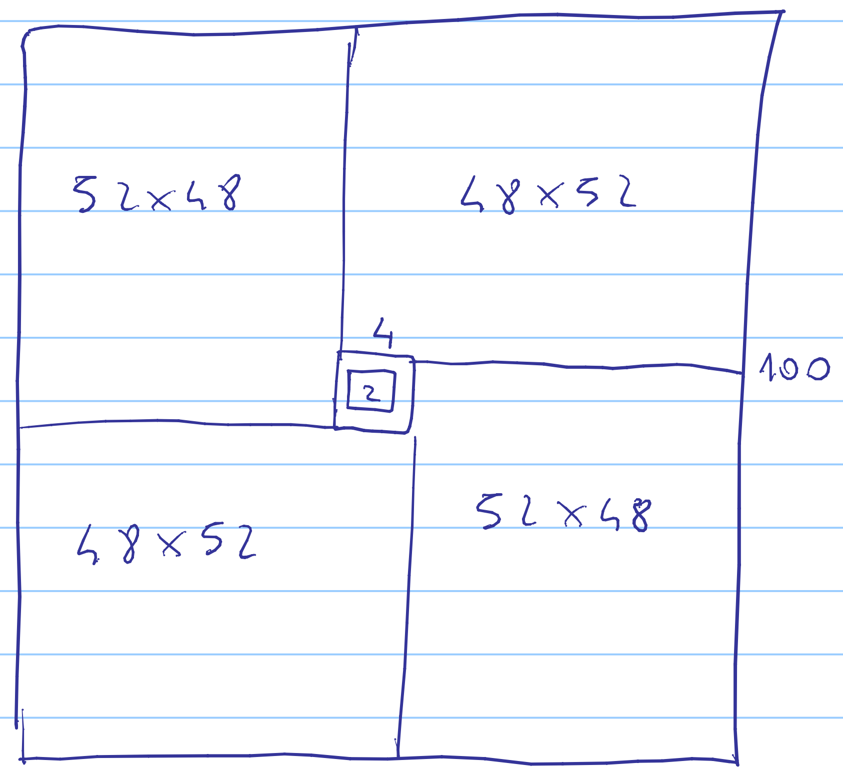
$$\Rightarrow r = \frac{\sqrt{3}}{2(\sqrt{3}+2)}$$

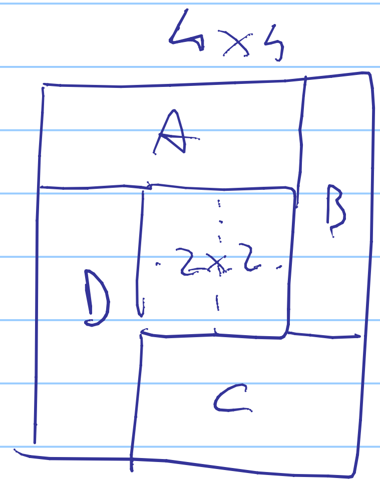
$$AD = ED + EA = 2 + \frac{2}{\sqrt{3}} = 2\left(1 + \frac{1}{\sqrt{3}}\right) =$$

$$= \ldots = \frac{\sqrt{3} - 1}{2}$$

3) 100 × 100  3 × 1 (o 1 × 3)

a) quadrato 2×2 centrale

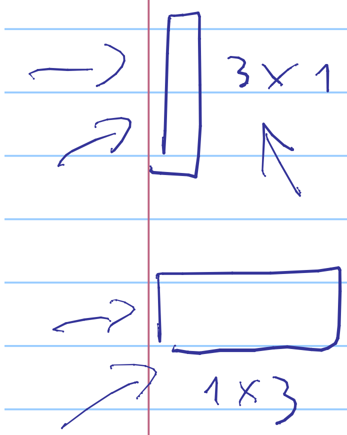


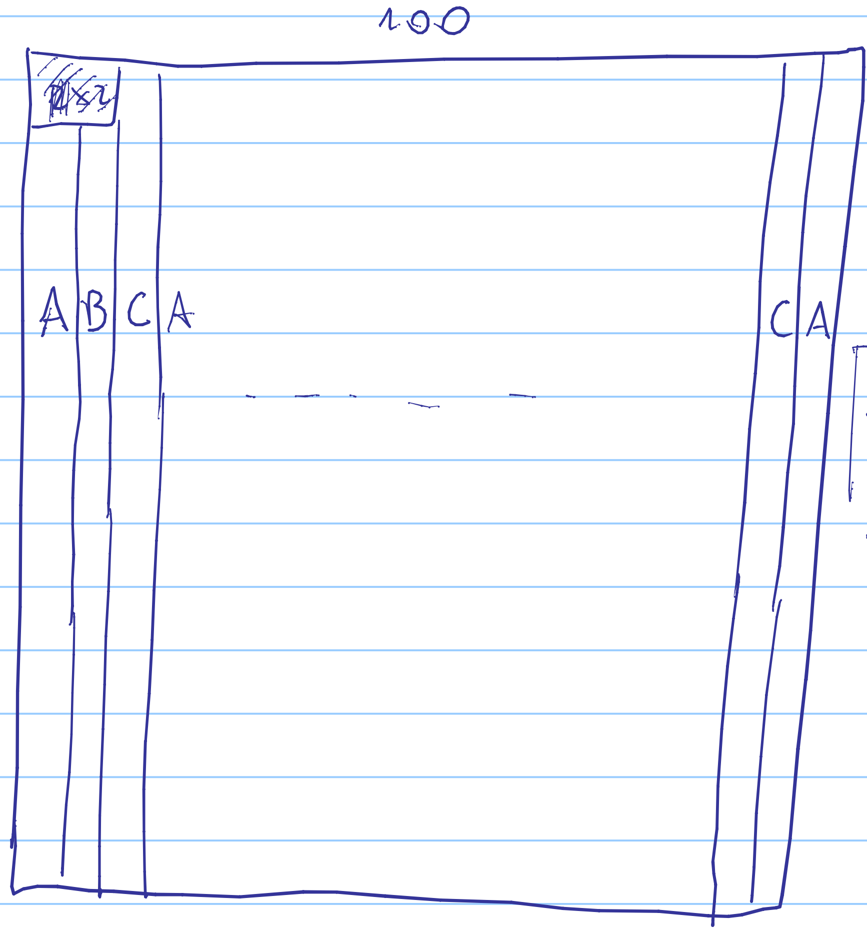


b) quadrato 2x2 in un angolo

Non si può!

1	2	3	1	2	3
2	3	1	2	3	1
3	1	2	3	1	2





A : 3400 − 2 caselle

B : 3300 − 2

C : 3300

2

$\boxed{\#C - \#B}$

$+1 \quad -1 \rightarrow 0$

$A \rightarrow 0$

$B \rightarrow -3$

$C \rightarrow +3$

4) $r_1, \ldots, r_m \in \mathbb{Q}$ $\quad r_i > 0$

$r_1 + \ldots + r_m = 1$

$f(n) = n - [r_1 n] - [r_2 n] - \ldots - [r_m n]$

$f : \mathbb{N} \to \mathbb{Z}$ $\quad \min f, \max f$?

$x \in \mathbb{R}$ $\quad \boxed{0 \leq x - [x] < 1}$

$f(n) = 1 \cdot n - [r_1 n] - \ldots - [r_m n]$

$= (r_1 + \ldots + r_m) n - [r_1 n] - \ldots [r_m n]$

$= \sum_{i=1}^{m} (r_i n - [r_i n])$

$f(n) \geq \sum_{i=1}^{m} 0 = 0$ $\quad f(0) = 0$ $\Longrightarrow$ $\boxed{\min f = 0}$

$\min f \geq 0$

$f(n) \geq -7$

- $\max f$

$f(n) = \sum_{i=1}^{m} \left(r_i n - [r_i n] \right) < \sum_{i=1}^{m} 1 = m$ $\qquad (x - [x]) =$

$= (x + m - [x+m])$

$\forall n \in \mathbb{N} \quad f(n) < m \,, \; f(n) \in \mathbb{Z}$

$\boxed{f(n) \leq m-1}$ $\cancel{\Rightarrow} \max f = m-1$

$r_i(n + b_1 \cdots b_m)$

$\shortparallel$

$r_i n +$ numeri int.

$f(n) = m-1$

$r_i = \dfrac{a_i}{b_i} \qquad a_i, b_i \in \mathbb{N}, \quad MCD(a_i, b_i) = 1$

$\boxed{n_0 = b_1 b_2 \cdots b_m - 1}$ $\qquad \boxed{f(n) \text{ è per. di per. } b_1 \cdots b_m}$

$f(n_0) = \sum_{i=1}^{m} \left(\frac{a_i}{b_i}(b_1 b_2 \cdots b_m - 1) - \left[\frac{a_i}{b_i}(b_1 b_2 \cdots b_m - 1) \right] \right) =$

$$= \sum_{i=1}^{m} \left(\underbrace{a_i b_1 b_2 \cdots \cancel{b_i} \cdots b_m - \frac{a_i}{b_i} - (a_i b_1 b_2 \cdots \cancel{b_i} \cdots b_m - 1)}_{K-1 \leq K - r_i < K} \right)$$

$$= \sum_{i=1}^{m} \left(1 - \frac{a_i}{b_i} \right) = m - 1$$

$$\Longrightarrow \max f = m - 1$$

$r_1 = \cdots = r_{m-1} = 0 \quad , r_m = 1$

$f(n) = 0$

5) p primi t.c. $\dfrac{2^{p-1}-1}{p}$ è un quadrato perfetto.

- $p=2$ NO! $\frac{1}{2}$
- p primo dispari

$$\frac{2^{p-1}-1}{p} \in \mathbb{N} \ ?$$

<u>Piccolo teorema di Fermat</u>

Versione 1: $\left.\begin{array}{l} p \text{ primo} \\ p \nmid a \ (\text{cioè } a \not\equiv 0 \ (p)) \end{array}\right\} \Longrightarrow a^{p-1} \equiv 1 \ (p)$ (cioè $\underline{p \mid a^{p-1}-1}$)

Versione 2: p primo $\Longrightarrow a^p \equiv a \ (p)$

$\boxed{p=3}$ è soluzione

- p primo ≥ 5 $\quad$ p dispari

$$\underbrace{\left(2^{\frac{p-1}{2}}-1\right)}_{A}\underbrace{\left(2^{\frac{p-1}{2}}+1\right)}_{B} = p\,m^2$$

A, B sono dispari consecutivi $\Rightarrow MCD(A,B) = 1$

p divide uno fra A e B, l'altro è un quadrato

$p \mid A$ $\qquad B = q_1^{\alpha_1} \cdots q_k^{\alpha_k}$ $\qquad q_i$ primi distinti

$q_i \neq p$

α_i dispari

$$q_i^{\alpha_i} \mid AB = \underline{p}\, m^2 \Rightarrow q_i^{\alpha_i} \mid m^2 = q_i^{\beta_i} \cdots$$

q_{α} $\quad \alpha_i$

β_i pari

disp $\alpha_i < \beta_i$ pari

p divide uno tra A e B, l'altro (tra A e B) è un quadrato perfetto

$A = \boxed{2^{\frac{p-1}{2}}} - 1$ disp

$B = 2^{\frac{p-1}{2}} + 1$ disp

OSS K disp $\Rightarrow K^2 \equiv 1 \ (4)$

$K = 2h+1 \Rightarrow K^2 = \underline{4h^2} + \underline{4h} + 1 \equiv 1 \ (4)$

$\Rightarrow m \equiv 3 \ (4)$ allora m non è un quadrato perfetto

A, B disp, ~~uno dei~~ due è un quadrato

$A \equiv -1 \equiv 3 \ (4)$

$B \equiv 1 \ (4)$

$\Rightarrow$ B è un quadrato perfetto

$2^{\frac{p-1}{2}}+1$ è " "

$$2^{\frac{p-1}{2}} \cancel{+1} = m^2-1 = (m+1)(m-1) = 4 \cdot 2 = 8 = 2^3$$

$\Rightarrow m+1 = 2^{\alpha}$

$m-1 = 2^{\beta}$ $\qquad 2^{\alpha}-2^{\beta} = m+1-(m-1) = 2$

$\Rightarrow 2^{\alpha} = 4$ e $2^{\beta} = 2 \Rightarrow \ldots$

$\Rightarrow \frac{p-1}{2} = 3 \Rightarrow \boxed{p = 7}$ ✓

$$\frac{2^{7-1}-1}{7} = 9 = 3^2 \qquad \square$$