Stage di CAMPOBASSO - 2012

Titolo nota 24/02/2012

4 - Algebra

$$\frac{4}{x+y} = \frac{1}{x} + \frac{1}{y} \qquad x, y \in \mathbb{R}$$

$x \neq 0$, $y \neq 0$, $x+y \neq 0$.

$$4xy = y(x+y) + x(x+y)$$

$$4xy = xy + y^2 + x^2 + xy \longrightarrow x^2 + y^2 - 2xy = 0$$

$$(x-y)^2 = 0 \Rightarrow x = y$$

$\square^2 = 0 \iff$ la base è uguale a 0

$$ax^2 + bx + c = 0$$

$$x^2 = 0 \longrightarrow x = 0$$

$$(x-a)^2 = 0 \longrightarrow x = a$$

$$x^2 = c \begin{cases} \to x = \pm\sqrt{c} & c > 0 \\ \to x = 0 & c = 0 \\ \to \text{NO!} & c < 0 \end{cases}$$

$$(x-a)^2 = c \qquad c > 0$$

$$x-a=\pm\sqrt{c} \qquad x=a\pm\sqrt{c}$$

$\boxed{a>0}$

$$ax^2+bx+c=0$$

$$A^2x^2+2ABx+B^2-k=0$$

$$(Ax+B)^2-k=0$$

$$\boxed{A^2=a \quad 2AB=b}$$

$$B^2-k=c$$

$$A=\sqrt{a}$$

$$B=\frac{b}{2\sqrt{a}}$$

$$k=B^2-c=\frac{b^2}{4a}-c=\frac{b^2-4ac}{4a}$$

$$\left(\sqrt{a}\,x+\frac{b}{2\sqrt{a}}\right)^2=\frac{b^2-4ac}{4a}$$

$4a>0$ LO SO

$b^2-4ac>0$ LO CHIEDO

$\parallel$

Δ DISCRIMINANTE

$\boxed{\Delta>0}$

$$\left(\sqrt{a}\,x+\frac{b}{2\sqrt{a}}\right)^2=\frac{\Delta}{4a}$$

$$\sqrt{a}\,x+\frac{b}{2\sqrt{a}}=\pm\frac{\sqrt{\Delta}}{2\sqrt{a}}$$

$$x_{1,2}=\left(-\frac{b}{2\sqrt{a}}\pm\frac{\sqrt{\Delta}}{2\sqrt{a}}\right)\frac{1}{\sqrt{a}}=\frac{-b\pm\sqrt{\Delta}}{2a}$$

$\boxed{\Delta<0}$

NIENTE

$\boxed{\Delta=0}$

$$\left(\sqrt{a}\,x+\frac{b}{2\sqrt{a}}\right)^2=0 \longrightarrow x=-\frac{b}{2a}$$

Es: $x^2-3x+2=0$.

$$\Delta = b^2-4ac = 9-4\cdot 1\cdot 2 = 1$$

$$x_{1,2} = \frac{-b\pm\sqrt{\Delta}}{2a} = \frac{+3\pm\sqrt{1}}{2} = \begin{cases} 2 \\ 1 \end{cases}$$

Oss: Polinomio di 2° grado $p(x)$ t.c.
$p(x)=0$ abbia soluzioni $3, -14$

$$(x-3)(x+14) = x^2+11x-42$$

$$2(x-3)(x+14) = 2x^2+22x-84$$

Oss +: Un pol. di 2° grado $p(x)$ che abbia sol.
h, k

$$m(x-h)(x-k) =$$
$$= mx^2 - \underbrace{m(h+k)}_{\text{somma}}x + \underbrace{mhk}_{\text{prodotto}}$$

Es: $x^2-5x+4=0$ $\qquad$ $x^2-Sx+P=0$

$x_{1,2}=1,4$

Es: $x^2+x-1=0$ $\qquad \alpha, \beta \qquad \alpha+\beta=-1$

$$\Delta = 1-4(1)(-1)=5 \qquad \alpha\beta=-1$$

$$\alpha^2+\beta^2=(\alpha+\beta)^2-2\cdot\alpha\beta = S^2-2P =$$
$$=(-1)^2-2(-1)=1+2=3$$

$$\frac{1}{\alpha}+\frac{1}{\beta}=\underbrace{\frac{S}{P}}_{=1}=\frac{\alpha+\beta}{\alpha\beta}=\frac{\cancel{\alpha}}{\cancel{\alpha}\beta}+\frac{\cancel{\beta}}{\alpha\cancel{\beta}}=\frac{1}{\beta}+\frac{1}{\alpha}$$

$$\alpha^4+\beta^4$$

$$(\alpha+\beta)^4=\alpha^4+\beta^4+4\alpha^3\beta+\overbrace{6\alpha^2\beta^2}^{-6P^2}+4\alpha\beta^3$$

$$4\alpha\beta(\alpha^2+\beta^2)$$
$$-4P(S^2-2P)$$

$$S^4-6P^2-4PS^2+8P^2=S^4-4PS^2+2P^2$$

$$\alpha^4+\beta^4=1+4+2=7.$$

$$\alpha^3\beta+\beta^3\alpha=P(S^2-2P)$$

$$\frac{1}{\alpha^4\beta} + \frac{1}{\alpha^3\beta^2} + \frac{1}{\alpha^2\beta^3} + \frac{1}{\alpha\beta^4} = \text{esercizio.}$$

<u>Teo di Ruffini</u>

t. noto

$$p(x) = \underbrace{a_n}_{} x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$$

polinomio $a_n \neq 0$ di grado n

Coeff. DIRETTORE

$c \in \mathbb{R}$ si dice RADICE di $p(x)$ se $p(c) = 0$.

$p(x)$ di grado n

$q(x)$ di grado $m < n$

$$\Rightarrow p(x) = q(x) \cdot A(x) + R(x)$$

quoziente resto

$$\deg R(x) < \deg q(x).$$

Caso particolare: se divido per $q(x) = ax + b$

$$\Rightarrow R(x) = r \in \mathbb{R}.$$

<u>Teo</u>: $c \in \mathbb{R}$ è una radice di $p(x)$

$$\Updownarrow$$

$(x-c)$ divide $p(x)$

<u>Teo</u>: Il resto di $p(x)$ diviso per $x-c$ è $p(c)$.

Esercizio: Non esiste nessun polinomio monico di grado 3, con 3 radici intere distinte che valga 3 in due interi distinti.

$$P(x) = x^3 + \square x^2 + \nabla x + \text{\&}$$

$a \in \mathbb{Z}$ t.c. $P(a) = 0 \Rightarrow P(x) = (x-a) P_1(x)$

$b \in \mathbb{Z}$ t.c. $P(b) = 0 \to (b-a) P_1(b) = 0 \to P_1(b) = 0$

$\to P_1(x) = (x-b) P_2(x)$.

$c \in \mathbb{Z}$ t.c. $P(c) = 0 \to (c-a) P_1(c) = 0 \to P_1(c) = 0$

$\Rightarrow (c-b) P_2(c) = 0 \Rightarrow P_2(c) = 0$

$P_2(x) = (x-c) P_3(x) = k(x-c)$

$$P(x) = \cancel{k}(x-a)(x-b)(x-c)$$

$P(m) = P(n) = 3 \quad m \neq n \quad m, n \in \mathbb{Z}$

$1, -1, -3$

$(m-a)(m-b)(m-c) = 3$

$(n-a)(n-b)(n-c) = 3$

$1, -1, -3 \to m$

$\left.\begin{matrix} -3, 1, -1 \\ -1, -3, 1 \end{matrix}\right\} \to n$

$m-a = 1 \quad m-b = -1 \quad m-c = -3$

$n-a = -3 \quad n-b = 1 \quad n-c = -1$

p(x) deg 3 con 3 radici α, β, γ monico

$$(x-\alpha)(x-\beta)(x-\gamma) = x^3 - \underbrace{(\alpha+\beta+\gamma)}_{S} x^2 + \underbrace{(\alpha\beta+\beta\gamma+\gamma\alpha)}_{Q} x - \underbrace{\alpha\beta\gamma}_{P}$$

$$\alpha^2+\beta^2+\gamma^2 = S^2 - 2Q$$

$$\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma} = \frac{Q}{P}$$

$$\alpha^3+\beta^3+\gamma^3 = S^3 - 3QS + 3P$$

$$(\alpha+\beta+\gamma)^3 = (\alpha^2+\beta^2+\gamma^2+2\alpha\beta+2\beta\gamma+2\alpha\gamma)(\alpha+\beta+\gamma)$$

$$= \alpha^3+\beta^3+\gamma^3+3\alpha^2\beta+3\alpha\beta^2+2\alpha\beta\gamma+3\beta^2\gamma+3\beta\gamma^2+$$

$$+2\alpha\beta\gamma+3\alpha^2\gamma+3\alpha\gamma^2+2\alpha\beta\gamma =$$

$$= \alpha^3+\beta^3+\gamma^3+3\alpha^2\beta+3\alpha\beta^2+3\beta^2\gamma+3\beta\gamma^2+3\alpha^2\gamma+$$

$$+3\alpha\gamma^2+6\alpha\beta\gamma$$

$$x^4 - Sx^3 + Dx^2 - Tx + P$$

$$P \rightarrow \alpha\beta\gamma\delta$$

$$T \rightarrow \alpha\beta\gamma+\alpha\beta\delta+\alpha\gamma\delta+\beta\gamma\delta$$

$$D \rightarrow \alpha\beta+\beta\gamma+\gamma\delta+\alpha\gamma+\alpha\delta+\beta\delta$$

Formule di Viete

Relazioni Radici - coefficienti

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p(x) coeff. interi, p(2009) = 2009

Quante soluzioni intere distinte può avere al max l'eq. p(x) = 2000

$$q(x) = p(x) - 2000 \qquad b_1, b_2, \dots, b_m \in \mathbb{Z}$$

$$q(x) = (x-b_1)(x-b_2)\cdot\ldots\cdot(x-b_m)\cdot r(x)$$

$$\boxed{p(x) = r(x)(x-b_1)(x-b_2)\cdot\ldots\cdot(x-b_m) + 2000}$$

$$p(2009) = r(2009)(2009-b_1)\cdot\ldots\cdot(2009-b_m) + 2000 =$$

$$= 2009$$

$$r(2009)\cdot\underbrace{(2009-b_1)\cdot\ldots\cdot(2009-b_m)} = 9$$

$3, -3, 1, -1.$

$$(2009-b_1)(2009-b_2)(2009-b_3)(2009-b_4)$$

$-3 \qquad -1 \qquad 1 \qquad 3$

$b_1 = 2012 \qquad b_2 = 2010 \qquad b_3 = 2008 \qquad b_4 = 2006$

$$p(x) = (x-2012)(x-2010)(x-2008)(x-2006) + 2000$$

$$\uparrow$$

$r(x)$

$$a^2 - b^2 = (a+b)(a-b) \qquad a-b \text{ divide } a^n - b^n$$

$$a^3 - b^3 = (a-b)(a^2+ab+b^2) \qquad \boxed{n \text{ pari}} \quad n = 2k$$

$$a^{2k} - b^{2k} = (a^k - b^k)(a^k + b^k)$$

$\boxed{\text{n dispari}}$ $\quad p(x) = x^n - 1 \qquad p(1) = 0$

$$\Rightarrow p(x) = (x-1)\,q(x)$$

$$x = \frac{a}{b} \qquad \underset{\parallel}{p\left(\frac{a}{b}\right)} = \left(\frac{a}{b} - 1\right)\left(q\left(\frac{a}{b}\right)\right)$$

$$\frac{a^n}{b^n} - 1 = \left(\frac{a}{b} - 1\right) q\left(\frac{a}{b}\right)$$

$$a^n - b^n = (a-b)\,\overbrace{b^{n-1} q\left(\frac{a}{b}\right)}^{\text{senza denominatore}}$$

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p pol. a coeff. interi, $a, b \in \mathbb{Z}$

$\Rightarrow a-b$ divide $p(a) - p(b)$

$$p(x) = x^3 + 7x^2 + 2x - 5$$

$$p(a) - p(b) = \underbrace{a^3 - b^3} + 7\underbrace{(a^2 - b^2)} + 2\underbrace{(a-b)}$$

$p(x)$ a coeff. interi t.c. $p(2) = 3 \quad p(4) = 2$

$a = 4 \quad b = 2 \quad \underset{\substack{\parallel \\ 2}}{4-2}$ divide $p(4) - p(2) = 1$

$p(a)=0$ $\quad$ $p(n)=3$

$p(m)=3$

$a-m$ divide 3

$a-n$ divide 3