Stage di CAMPOBASSO - 2012

Titolo nota 23/02/2012

COMBINATORIA

PRINCIPIO DI INDUZIONE:

P

$P(n)$ n SODDISFA P

Se $P(0)$ CASO BANALE

$P(n) \Rightarrow P(n+1)$ PASSO INDUTTIVO

Allora $P(n)$ $\forall n$

$$\sum_{i=0}^{n} i = \frac{n(n+1)}{2}$$

PRINCIPIO DEI CASSETTI:

n oggetti

m cassetti

$n > m$ $\exists$ un cassetto con almeno due oggetti.

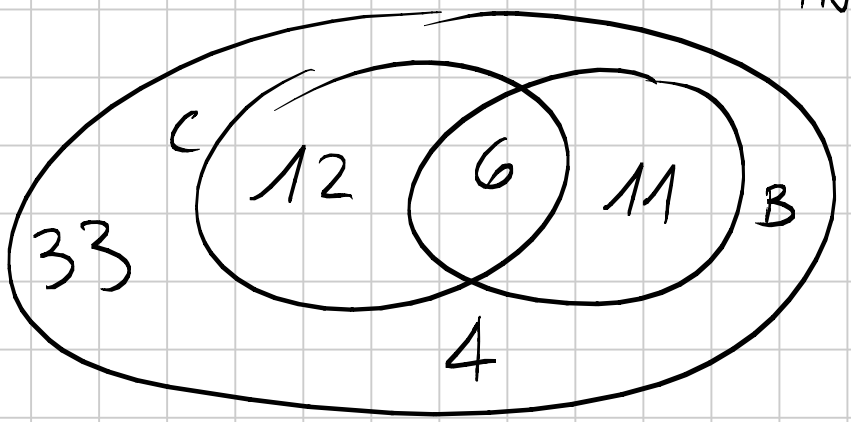
ES: Classe 33 alunni

18 Calcio

17 Basket

4 Nulla

? QUANTI PRATICANO BASKET CALCIO INSIEME

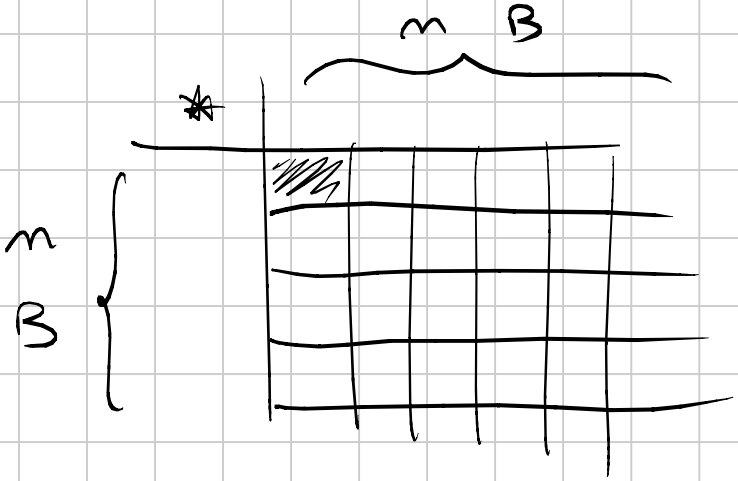


29 $B \cup C$ $18+17=35$

Quante operazioni? 4 NO!

B $B \times B \longrightarrow B$

$\#B$ $|B| = n$



n^{n^2}

Quanti modi per la stessa operazione

$n!$

$n!$ $R: (n!)^2$

$n!$ n fattoriale $n \cdot (n-1) \cdot (n-2) \cdot \ldots \cdot 1$

$|B| = 5$

$|A| = 2$

$A = \{0, 1\}$

$f: B \longrightarrow A$ $\quad 2^5$

a	b	c	d	e	
0	1	0	0	1	$= \{b, e\}$
2	2	.	.	2	

$|B| = m$

$|A| = m$

$\# f: B \to A : m^m$

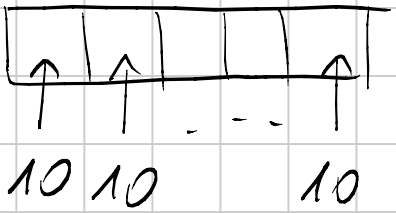
$\mathcal{P}(B)$

$|\mathcal{P}(B)| = 2^{|B|}$

es: PIN 5 cifre

cifre = 10

0, 1, ..., 9



R: 10^5

SCOPO: Quanti sottoinsiemi di k elementi da un insieme di n elementi

TORNEO: 10 SQUADRE

4 PREMI diversi

O	A	B	L
10	9	8	7

Generale $n \cdot (n-1) \cdot \ldots \cdot (n-k+1)$

es: Numeri di 3 cifre (diverse tra loro)

9 10 10

~~0~~ 1 2 ←

numeri a 3 cifre non necessariamente distinte $10^3 - 10^2 = 900$

3 2 1

3 cifre imposte

1, 2, 3

$3! = 3 \cdot 2 \cdot 1 = 6$

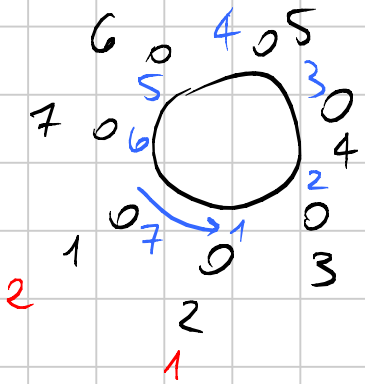
lettera — destinatario (BUSTA)

POSTINO DISTRATTO n lettere
n buste

R: $n!$

PERMUTAZIONI

7 persone



$$\frac{7!}{}$$

$6!$

GENERALE

n $\bigcirc$ $(n-1)!$ $\frac{}{n!}$

ANAGRAMMI

MOLISE $6!$

MATEMATICA $10!$ NO!

AMTEAMTICA
AMTEAMTICA

$$\frac{10!}{2 \cdot 2 \cdot 3!} = 151200$$

$10!/2! \cdot 2! 3!$

5 CARCERATI
2 CONDANNE A MORTE (TIZIO E CAIO)

5 × tizio ↑ caio
4 × caio ↓ tizio

$$\frac{5 \cdot 4}{2} = 10$$

$$\frac{5 \cdot 4}{2} = \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 3 \cdot 2 \cdot 1} = \frac{5!}{2! \, 3!}$$

GENERALE n pers. k cond.

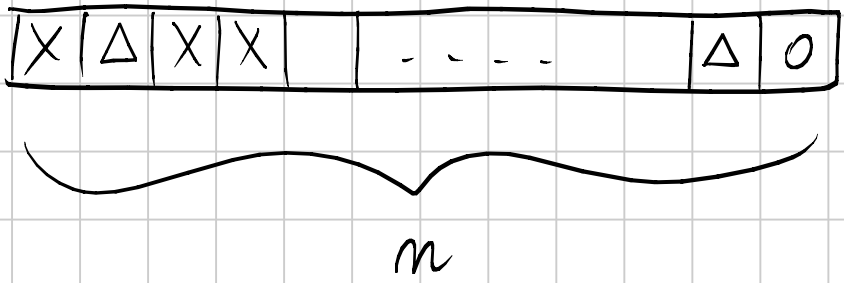
$$\frac{n!}{k!(n-k)!} = \binom{n}{k}$$

COEFFICIENTE BINOMIALE

$$\frac{n \cdot (n-1) \cdot \ldots \cdot (n-k+1)}{k!} = \frac{n!}{(n-k)!} \cdot \frac{1}{k!}$$

Δ	$a\,\Delta$
O	$b\,O$
X	$c\,X$

$a+b+c=n$



$$\frac{n!}{a!\,b!\,c!} = \binom{n}{a}\cdot\binom{n-a}{b} = \frac{n!}{a!(n-a)!}\cdot\frac{(n-a)!}{b!\,c!}$$

Δ O

TRINOMIALE

DIVISORI DI UN INTERO POSITIVO

$$2012 = 2^2 \cdot 503$$

quanti divisori positivi 2012

$$2^a \cdot 503^b$$

vincoli su a, b: $a = 0, 1, 2$
$b = 0, 1$

3 per a
2 " b $\quad 3 \cdot 2$ divisori 2012

GENERALE $\quad n = p_1^{a_1} \cdot p_2^{a_2} \cdot \ldots \cdot p_k^{a_k}$

divisori positivi di $n = (a_1+1)(a_2+1)\ldots(a_k+1)$

superenalotto

$\binom{90}{6} \sim 720$ milioni

es: fare 4 / 5

$\binom{n}{k}$

TR. TARTAGLIA

0 1
1 1 1
2 1 2 1
3 1 3 3 1
4 1 4 6 4 1
0 1 2 3 4

$\binom{4}{2} = \frac{4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 2} = 6$

FORMULA RICORSIVA

col k col k+1

riga n

a b

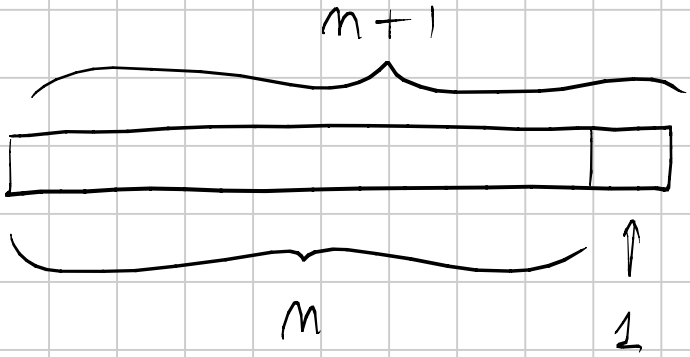
riga n+1

c

col k+1

$c = a + b$

$$\binom{n+1}{k+1} = \binom{n}{k} + \binom{n}{k+1}$$



$$\binom{n}{k+1} \qquad \binom{n}{k}$$

$$\binom{n}{k} = \binom{n}{n-k}$$

SIMMETRIA

$$(x+y)^n = (x+y)(x+y)\cdot\ldots\cdot(x+y)$$

$$a_1 x^1 y^1 + \ldots + k\, x^5 y^{15} + \ldots$$

$n = 20$

k $\quad \binom{20}{5} = \binom{20}{15}$

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k}$$

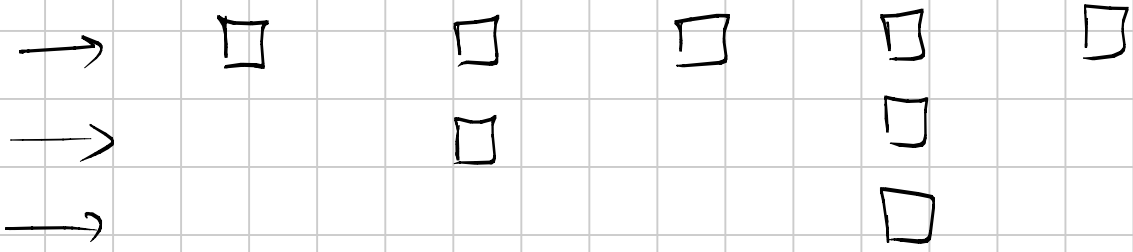
DIVISIBILITA'

$2012!$

max potenza di 2 che divide $2012!$

$2^{10} = 1024$ NO!

$2012! = 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdot 8 \cdot 9 \cdot 10 \cdot 11$



$\frac{2012}{2} = 1006$

$\frac{2012}{4} = 503$

$\left[\frac{2012}{8}\right] = 251$

↓ PARTE INTERA

$$R: \left[\frac{2012}{2}\right] + \left[\frac{2012}{4}\right] + \ldots + \left[\frac{2012}{1024}\right]$$

PROBABILITA'

es: Prob. lanciando 3 dadi che la somma ≤ 5

$1+1+1 = 3$

$\vdots$

$6+6+6 = 18$

16 POSSIBILI

3
4
5

3 FAVOREVOLI

$3/16$ NO! 20%

□ □ □ 6^3

$6 \cdot 6 \cdot 6$

$\frac{10}{6^3} = \frac{5}{108}$

3 $1+1+1$ 1

4 $2+1+1$ 3

5 $(2+2+1)(1+1+3)$ 6

$\sim 5\%$