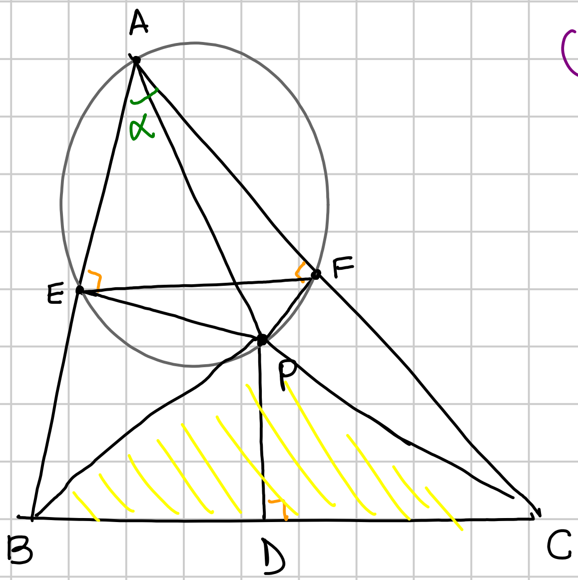
EGMO CAMP 2018 - Geometria

Note Title 1/22/2018

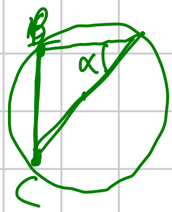
G1)



$(*) = \dfrac{EF \times PD}{AP} + cyc \overset{?}{=} cost$

AEPF ciclico
e AP è diametro

$EF = AP \cdot \sin\alpha$



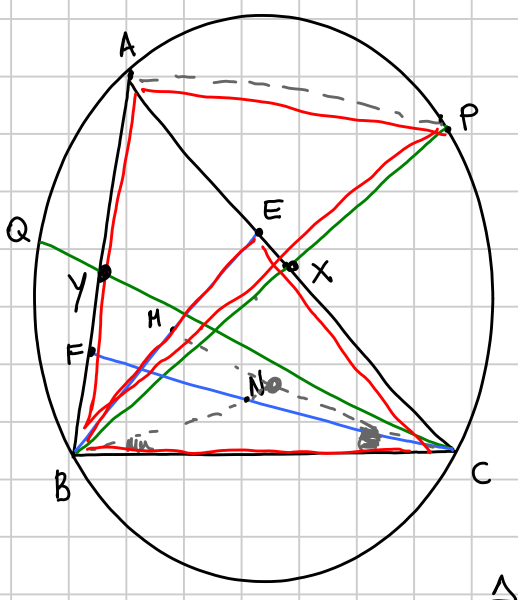
$BC \cdot \sin\alpha = 2R$

$(*) = PD \cdot \sin\alpha + cyc \overset{?}{=} cost$

Thm dei seni $\Rightarrow$ $\dfrac{BC}{\sin\alpha} = cost = k$

$(*) = \dfrac{PD \cdot BC}{k} + cyc = \dfrac{A_{BCP}}{k} + cyc = \dfrac{A_{ABC}}{k}$

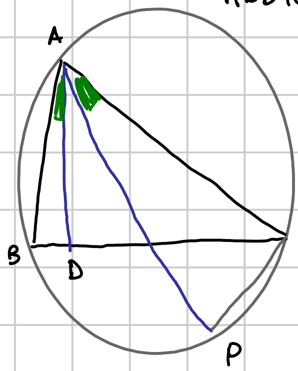
G4)



$\rightsquigarrow$ conti ???

Lemma

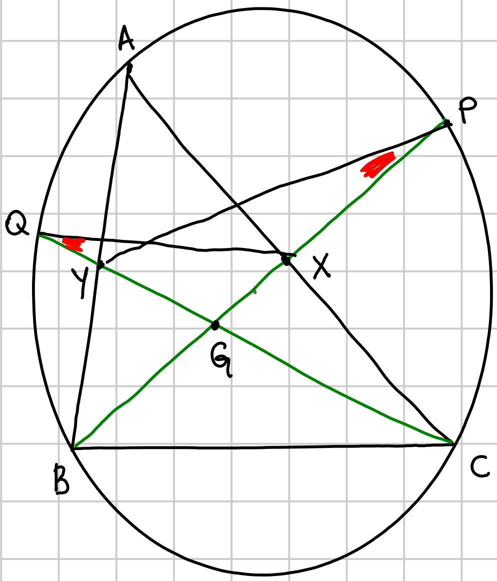
$\overset{\triangle}{ABD} \sim \overset{\triangle}{APC}$



x lemma $\rightsquigarrow$ $\overset{\triangle}{BEC} \sim \overset{\triangle}{BAP}$

$\widehat{HCB} = \widehat{YPB}$

x lemma $\rightsquigarrow$ $\overset{\triangle}{BCF} \sim \overset{\triangle}{QCA}$ $\Rightarrow$ $\widehat{NBC} = \widehat{XQC}$



$\widehat{XQC} \overset{?}{=} \widehat{YPB}$

Avrei finito se XYQP fosse ciclico

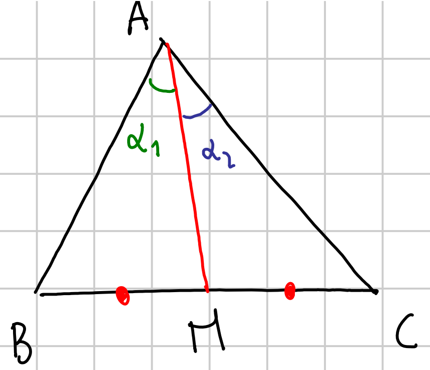
So BCPQ è ciclico

$\Rightarrow$ $\underset{\substack{\wr \\ GX}}{GB} \cdot GP = \underset{\substack{\wr \\ GY}}{GC} \cdot GQ$ $*$

$GX = \frac{1}{2} GB$ x es.

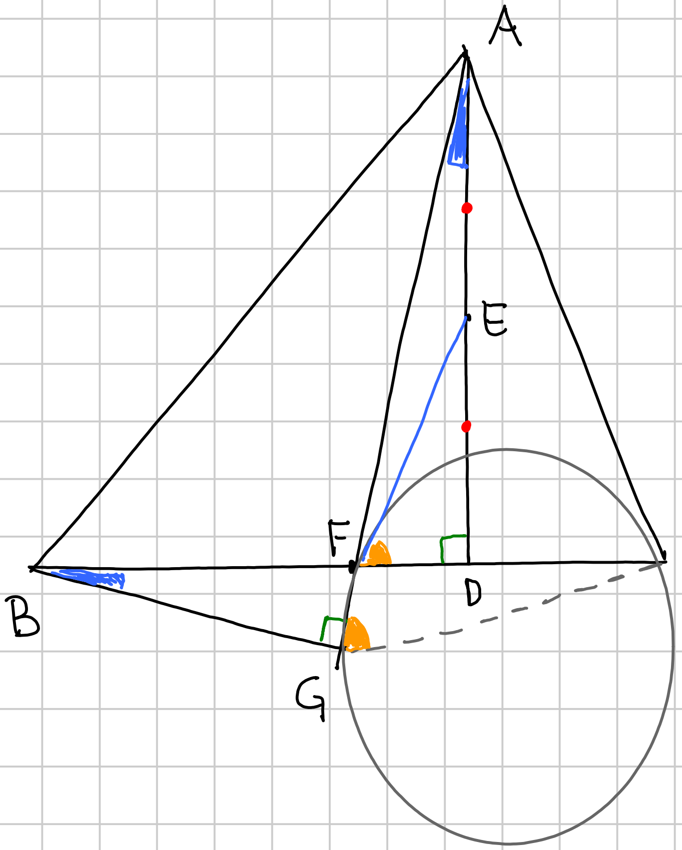
$*$ $\Rightarrow$ $GX \cdot GP = GY \cdot GQ$ $\Rightarrow$ XYQP ciclico

G2)



MEDIANA:

$$\frac{AB}{AC} = \frac{\sin(\alpha_2)}{\sin(\alpha_1)} \quad \text{se AM mediana}$$



Sol 1: trigonometria

$E\hat{F}D = u$

$F\hat{G}C = z$

- $\overset{\triangle}{AFD}$: EF e' mediana

$\rightarrow$ (A) $\frac{FD}{AF} = \frac{\sin(AFE)}{\sin(u)} = \frac{\sin(AFD-u)}{\sin(u)}$

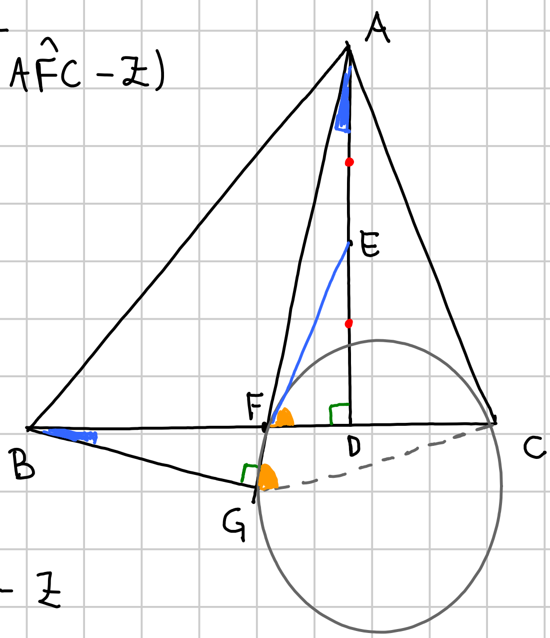
- $\overset{\triangle}{BGC}$: FG e' mediana

$\rightarrow$ $\frac{GC}{BG} = \frac{\sin(90°)}{\sin(z)} = \frac{1}{\sin(z)}$

$$\frac{GC}{GB} = \frac{\sin(G\hat{B}C)}{\sin(G\hat{C}B)} = \frac{FD}{AF} \cdot \frac{1}{\sin(A\hat{F}C - z)}$$

- $G\hat{B}C = G\hat{A}D = F\hat{A}D$

$\sin(F\hat{A}D) = \frac{FD}{AF}$



- $A\hat{F}C = z + G\hat{C}B \rightarrow G\hat{C}B = A\hat{F}C - z$

$$\frac{FD}{AF} = \frac{\sin(AFE)}{\sin(m)} = \frac{\sin(AFD - u)}{\sin(u)} \qquad (A)$$

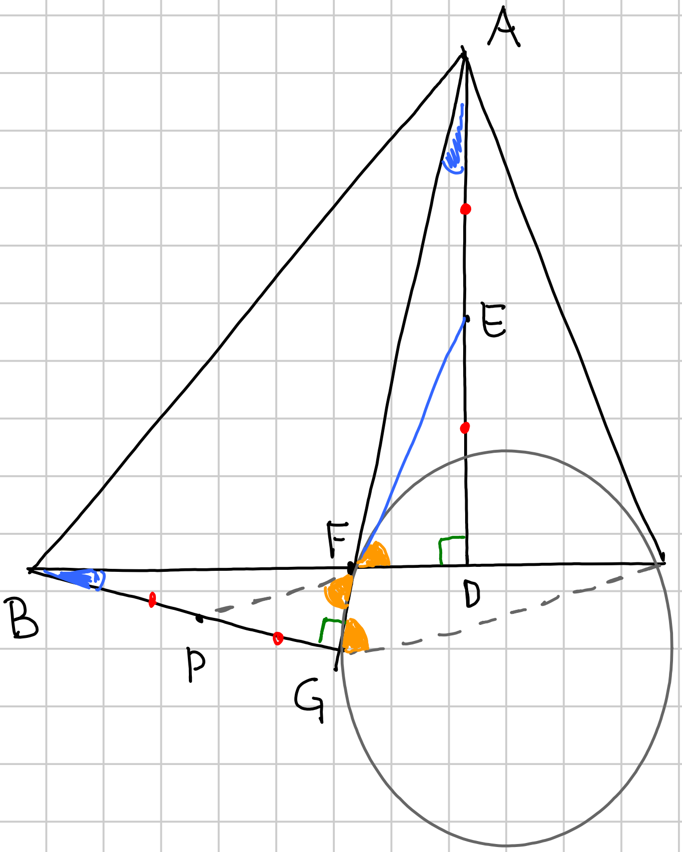
$$\frac{GC}{BG} = \frac{\sin(90^\circ)}{\sin(z)} = \frac{1}{\sin(z)}$$

$$\frac{GC}{GB} = \frac{\sin(G\hat{B}C)}{\sin(G\hat{C}B)} = \frac{FD}{AF} \cdot \frac{1}{\sin(A\hat{F}C - z)}$$

$$\frac{1}{\sin(z)} = \frac{GC}{BG} = \frac{FD}{AF} \cdot \frac{1}{\sin(A\hat{F}C - z)}$$

$$\Rightarrow \quad \frac{\sin(A\hat{F}C - z)}{\sin(z)} = \frac{FD}{AF} \overset{(A)}{=} \frac{\sin(A\hat{F}D - m)}{\sin(m)}$$

$$f(x) = \frac{\sin(\alpha - x)}{\sin(x)} \qquad x \in \left(0, \frac{\pi}{2}\right)$$



Sol 2: sintetica

$BP = PG \qquad BF = FC$

$PF \parallel GC$

$ABGD$: $B\hat{G}A = B\hat{D}A$

$B\overset{\triangle}{G}F \sim A\overset{\triangle}{D}F \qquad (1)$

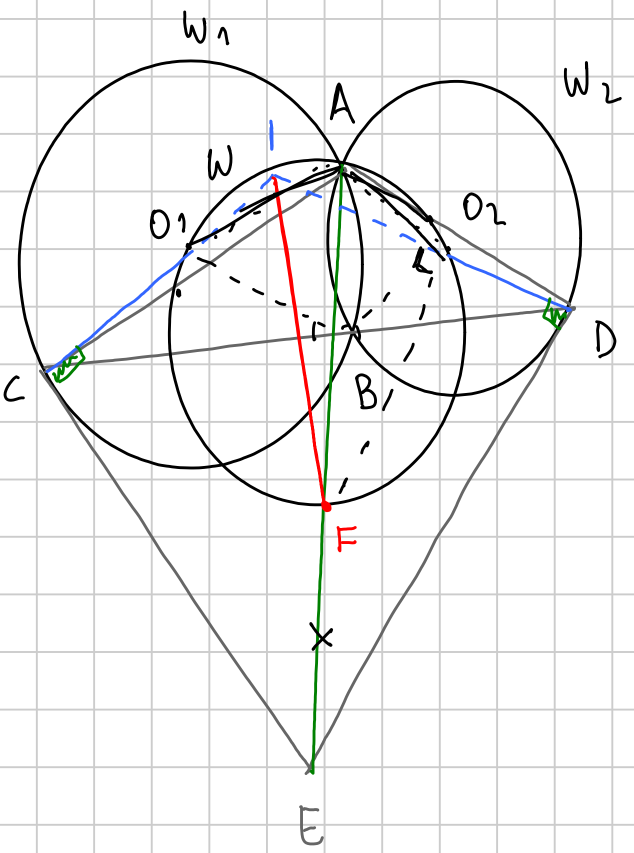
$\Downarrow$

P pt. medio di BG (2)
E pt. medio di AD

↳ $(1)+(2)$: $P\overset{\triangle}{G}F \sim E\overset{\triangle}{D}F$

$$\left.\begin{array}{l} \rightarrow \quad P\hat{F}G = E\hat{F}D \\ \qquad\quad \shortparallel \\ \qquad F\hat{G}C \end{array}\right\} \quad EF \text{ tg in } E \text{ a } (FGC)$$

G3.)



EF = diam di ω

$I = CO_1 \cap DO_2$

1) $I \in \omega$?

ICED è ciclico, $\Gamma = (ICED)$

- $C\hat{E}I = C\hat{D}I = B\hat{D}O_2 = O_2\hat{B}D$
- $D\hat{E}I = D\hat{C}I = B\hat{C}O_1 = O_1\hat{B}C$

$\to C\hat{E}D = O_2\hat{B}D + O_1\hat{B}C =$

$= 180° - O_1\hat{B}O_2$

$= 180° - O_1\hat{A}O_2$

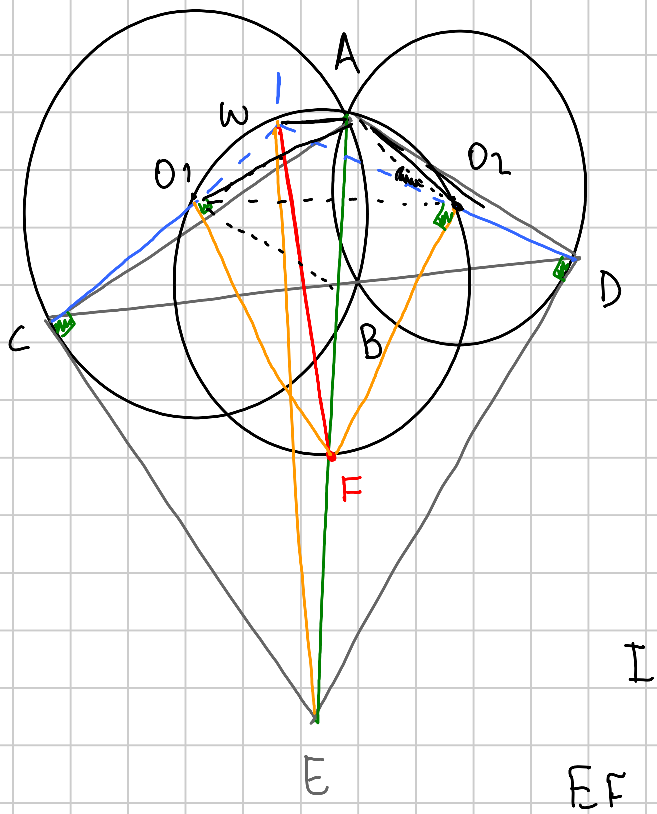
$\to A \in \Gamma$

verifica

$\to C\hat{I}D = C\hat{A}D = O_1\hat{A}O_2$ ($\simeq$)

$\quad \| \qquad\qquad\qquad\qquad \Big\}\ I \in \omega$

$O_1\hat{I}O_2$



$\begin{cases} A\hat{F}D = A\hat{C}D = A\hat{C}B \\ A\hat{F}O_2 = A\hat{O}_1O_2 = \dfrac{A\hat{O}_1B}{2} \end{cases}$

$\to O_2F \parallel ED$

$\to I\hat{O}_2F = I\hat{D}F = 90°$

IF è diam di ω

$EF = IF \iff \overset{\triangle}{IFE}$ isoscele

$I\hat{F}A = A\hat{O}_2I = 2A\hat{D}O_2 = 2A\hat{D}I = 2A\hat{E}I$

$\overset{\|}{I\hat{F}E} + A\hat{E}I$