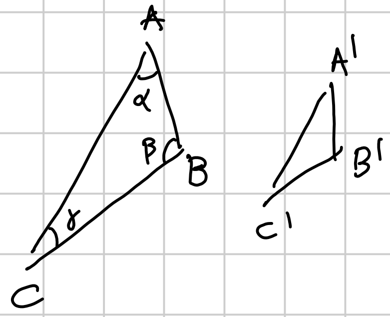
GEOMETRIA BASIC

Note Title 01/11/2019

Brevissimo recap

Similitudine



$\alpha = \alpha'$ e ayc.

$$\frac{AB}{A'B'} = \frac{BC}{B'C'} = \frac{CA}{C'A'}$$

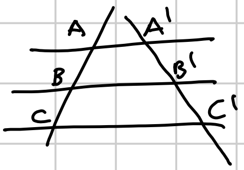
$$\alpha + \beta + \gamma = 180°$$

Ⓘ $\alpha = \alpha'$ $\beta = \beta'$

Ⓘⓘ $\alpha = \alpha'$ $\dfrac{AB}{A'B'} = \dfrac{AC}{A'C'}$

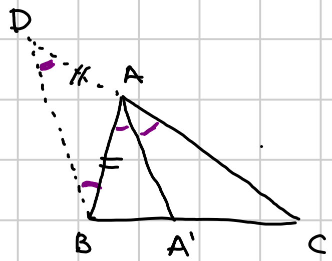
Ⓘⓘⓘ tutti i lati proporzionali

Talete



$$\frac{AB}{BC} = \frac{A'B'}{B'C'}$$

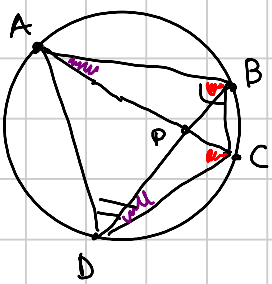
Thm della bisettrice



$$\Rightarrow \frac{BA'}{A'C} = \frac{AB}{AC}$$

$\overset{\triangle}{ABD}$ isoscele $\dfrac{AB}{AC} = \dfrac{AD}{AC} = \dfrac{A'B}{A'C}$

Ciclicità



ABCD conciclici

$\Leftrightarrow$ $A\hat{B}C = 180° - A\hat{D}C$

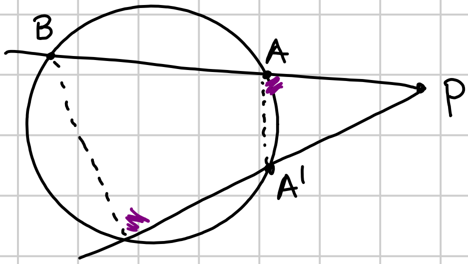
$\Leftrightarrow$ $B\hat{A}C = B\hat{D}C$

$\Leftrightarrow$ $P\overset{\triangle}{A}B \sim P\overset{\triangle}{D}C$

↳ simile

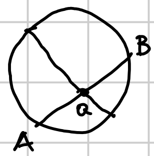
$\Leftrightarrow$ $PB \cdot PD = PA \cdot PC$

Potenza di P rispetto a Γ

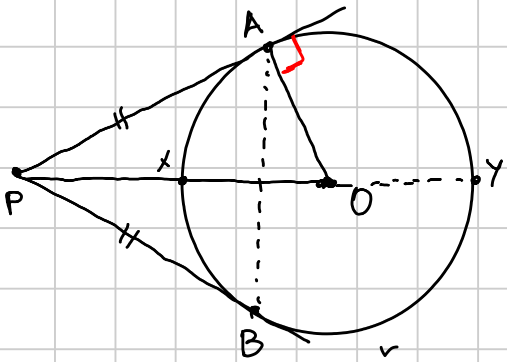


$Pow_\Gamma(P) := PA \cdot PB = PA' \cdot PB'$

$[\hat{PAA'} \sim \hat{PB'B}]$



$\boxed{Pow_\Gamma(Q) = -QA \cdot QB}$

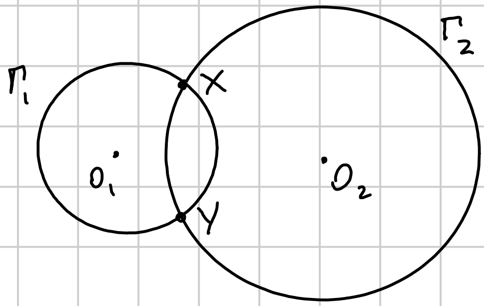


$Pow_\Gamma(P) = PA^2 = PB^2$

$Pow_\Gamma(P) = PX \cdot PY = (OP - OA)(OP + OA)$

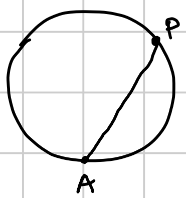
$= OP^2 - \underbrace{OA^2}_{= R_\Gamma}$

Asse radicale



Asse radicale = punti P tali che $Pow_{\Gamma_1}(P) = Pow_{\Gamma_2}(P)$

X, Y $\in$ asse radicale

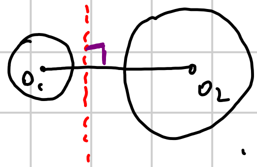


$\boxed{Pow_\Gamma P = 0}$

Fatto: Asse radicale è la retta passante per X, Y [se Γ_1, Γ_2 si intersecano]

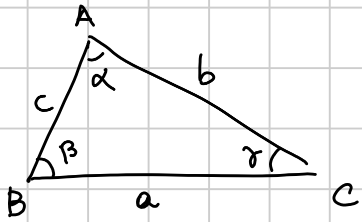
In generale è una retta ortogonale a O_1O_2

EX



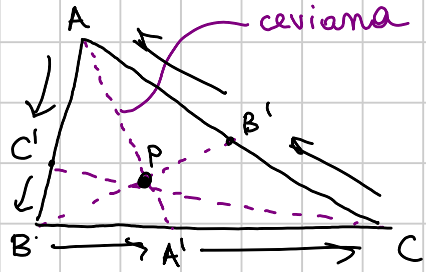
Triangolo

Notazione



$$\alpha+\beta+\gamma = 180^\circ$$

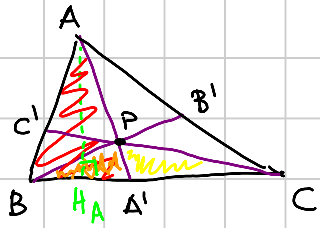
TRm di Ceva



AA', BB', CC' concorrono

$$\iff \frac{BA'}{A'C}\cdot\frac{CB'}{B'A}\cdot\frac{AC'}{C'B} = 1$$

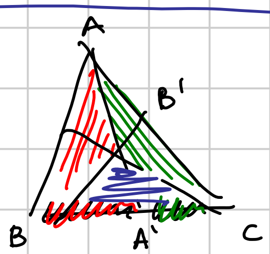
Dimostrazione $\Longrightarrow$



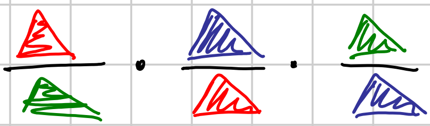
$$\frac{BA'}{A'C} = \frac{BA'\cdot AH_A}{A'C\cdot AH_A} = \frac{\cancel{2}\,\text{Area}(ABA')}{\cancel{2}\,\text{Area}(A'CA)} = \frac{\text{Area}(BA'P)}{\text{Area}(CA'P)} = \frac{\text{Area}(ABA') - \text{Area}(BA'P)}{\text{Area}(A'CA) - \text{Area}(CA'P)}$$

$$\boxed{\frac{x}{y} = \frac{u}{v} \implies \frac{x}{y} = \frac{u}{v} = \frac{x-u}{y-v} \qquad \frac{x}{y} = \frac{x-u}{y-v} \iff \cancel{xy} - xv = \cancel{xy} - yu \iff \frac{x}{y} = \frac{u}{v}}$$

$$= \frac{\text{Area}(ABP)}{\text{Area}(ACP)}$$

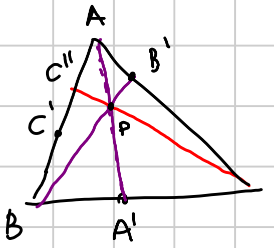


$$\frac{BA'}{A'C}\cdot\frac{CB'}{B'A}\cdot\frac{AC'}{C'B} =$$



$$= 1$$

$\Longleftarrow$

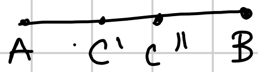


$$\boxed{\frac{BA'}{A'C}\cdot\frac{CB'}{B'A}\cdot\frac{AC'}{C'B}=1}$$

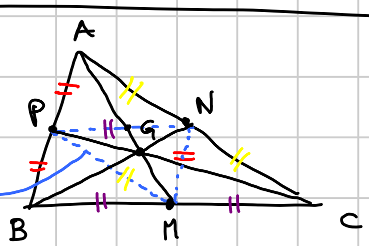
AA', BB', CC'' concorrono

$$\Rightarrow \boxed{\frac{BA'}{A'C}\cdot\frac{CB'}{B'A}\cdot\frac{AC''}{C''B}=1}$$

$$\Longrightarrow \frac{AC'}{C'B}=\frac{AC''}{C''B} \Rightarrow C'=C''$$



Baricentro



triangolo mediale

G baricentro

$\frac{BM}{MC}=1$

(esiste ovviamente per Ceva)

$PN \parallel BC$ (Talete)

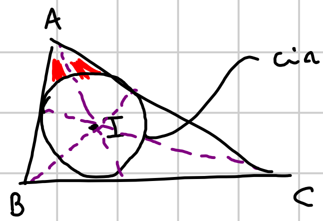
$\overset{\triangle}{MNP} = \overset{\triangle}{APN} \sim \overset{\triangle}{ABC}$ (con rapporto di similitudine 2)

$\Rightarrow AG = 2GM$

__Fatto__ G è il baricentro anche del triangolo mediale

$\frac{AG}{GM}$ = rapporto di similitudine $=2$

Incentro

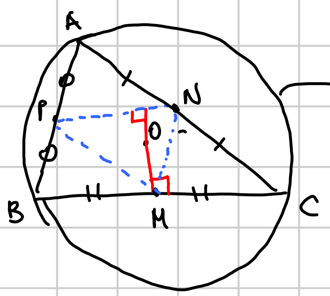


circonferenza inscritta

Il centro è l'incentro I

AI, BI, CI sono bisettrici.

Circocentro

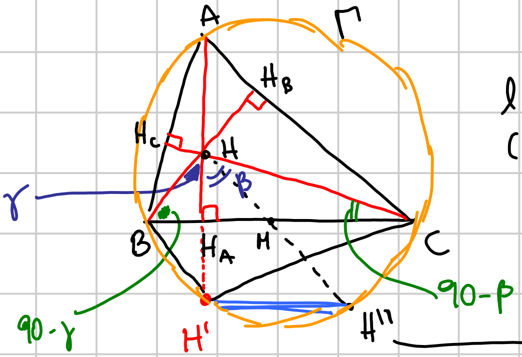


circonferenza circoscritta

il centro è il circocentro O

O è l'ortocentro del triangolo mediale $\overset{\triangle}{MNP}$

Ortocentro



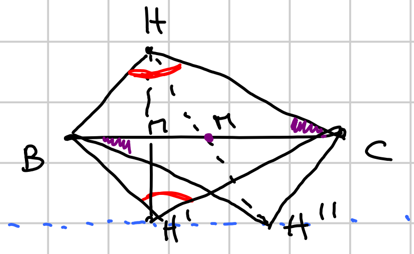
l' incontro delle altezze (dim che esiste) è l' ortocentro H

$H' \in \Gamma$, $H'' \in \Gamma$

↪ simmetrico di H rispetto a BC ($HH_A = H'H_A$)

→ simmetrico di H rispetto a M [$HM = MH''$]

$$B\hat{H'}C = B\hat{H}C = \beta + \gamma = 180° - \alpha$$

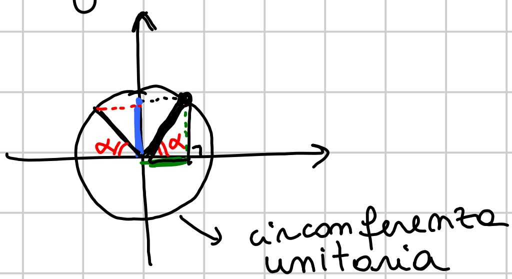


$H'H'' \parallel BC$

BH''CH è un parallelogramma
[(B e C sono simmetrici rispetto ad M)
HC è parallela a BH'']

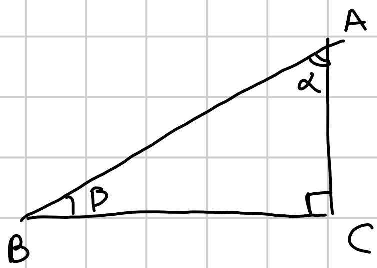
→ BC e HH'' si incontrano nel pto medio di entrambi

Trigonometria



sin α =

cos α =



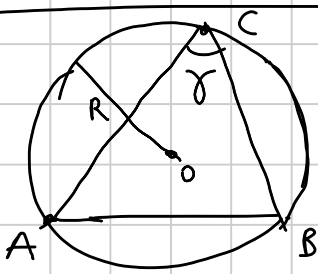
$$AC = AB \cdot \cos\alpha = AB \cdot \sin\beta$$

$\beta = 90° - \alpha$ ⇝ $\sin(90° - \alpha) = \cos\alpha$

$\cos(90° - \alpha) = \sin\alpha$

$$\sin(180° - \alpha) = \sin\alpha$$

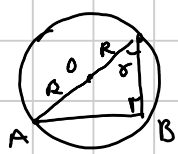
$$\cos(180° - \alpha) = -\cos\alpha$$



$$\boxed{AB = 2R \sin\gamma}$$

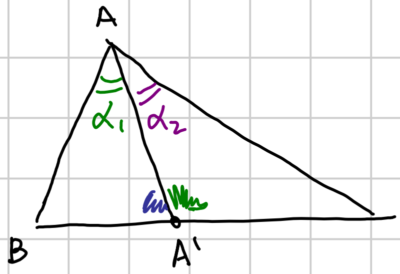
$$\frac{AB}{\sin\gamma} = 2R$$

$$\frac{BC}{\sin\alpha} = 2R$$



Thm dei seni

$$\frac{AB}{\sin\gamma} = \frac{BC}{\sin\alpha} = \frac{CA}{\sin\beta}$$

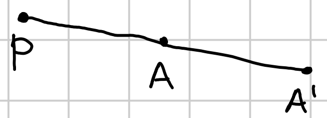


$$\frac{\sin\alpha_1}{\sin\alpha_2}$$

$$\begin{cases} \dfrac{BA'}{\sin\alpha_1} = \dfrac{AB}{\sin A\hat{A'}B} \\[2ex] \dfrac{A'C}{\sin\alpha_2} = \dfrac{AC}{\sin A\hat{A'}C} \end{cases} \Rightarrow \frac{\sin\alpha_1}{\sin\alpha_2} = \frac{BA'}{A'C} \cdot \frac{AC}{AB}$$

Omotetia

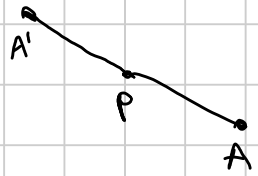
(trasformazione del piano)
di centro P e fattore λ

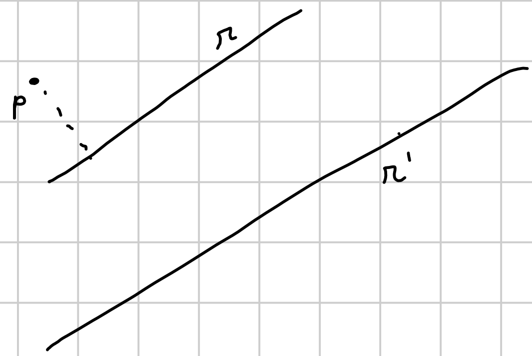


$A \longmapsto A'$ t.c. P, A, A' sono allineati

$$\frac{PA'}{PA} = \lambda$$

$\lambda < 0$



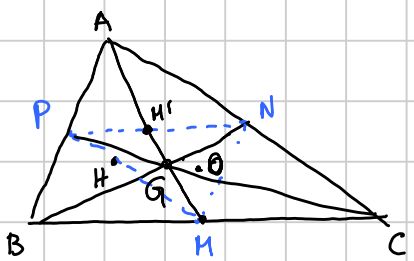


retta $\longmapsto$ retta parallela

gli angoli si conservano

le lunghezze NON si conservano

$\overset{\triangle}{ABC} \longmapsto \overset{\triangle}{A'B'C'} \sim \overset{\triangle}{ABC}$



Facciamo un'omotetia di centro G e fattore -2

$M \longmapsto A$

$N \longmapsto B$

$P \longmapsto C$

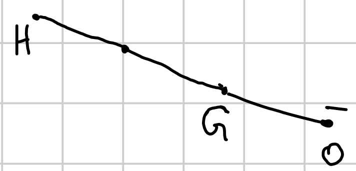
$O \longmapsto H$

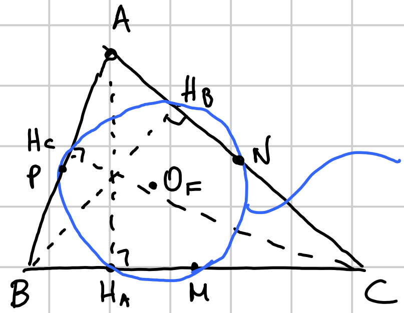
circocentro di $\overset{\triangle}{ABC}$

↳ ortocentro di $\overset{\triangle}{MNP}$

$\Longrightarrow$ O, H, G allineati $\longrightarrow$ retta di Eulero

$$2OG = GH$$





circonferenza di Feuerbach

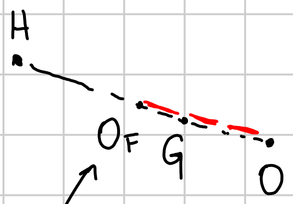
$H_A H_B H_C M N P$ sono conciclici

O_F centro di Feuerbach

$O_F \xmapsto{\text{omotetia di prima}} O$ circocentro di $\hat{ABC}$

$\Longrightarrow$ O, G, O_F sono allineati

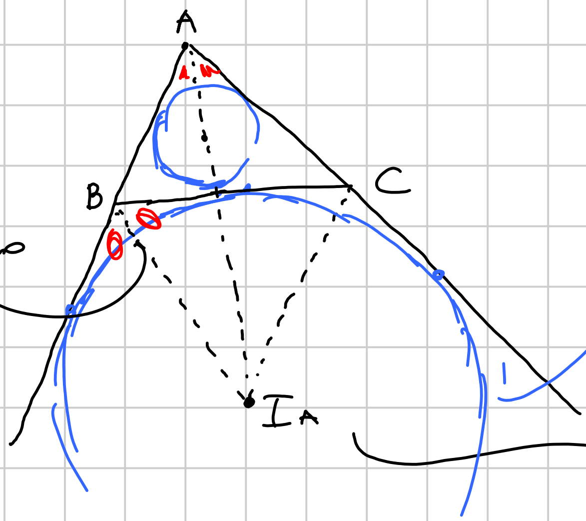
$H, G, O, O_F \in$ retta di Eulero.



ortocentro di $\hat{ABC}$

circocentro di $\hat{MNP}$

circocentro di $\hat{ABC}$.
ortocentro di $\hat{MNP}$

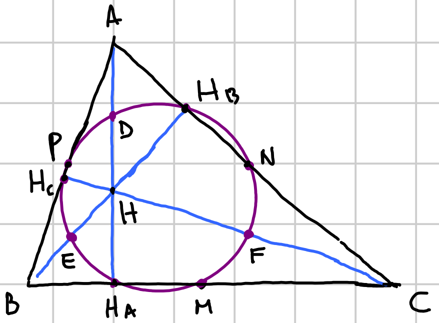


angolo esterno

circonferenza exscritta

excentro relativo ad A.

G15
Bonus

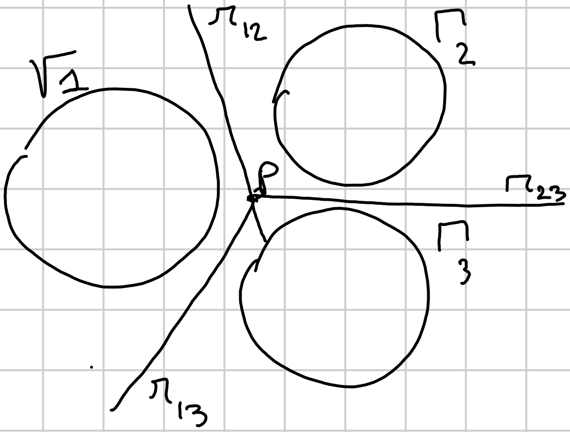


D, E, F punti medi di AH, BH, CH

G14 M ē il punto medio BC (non di AB)

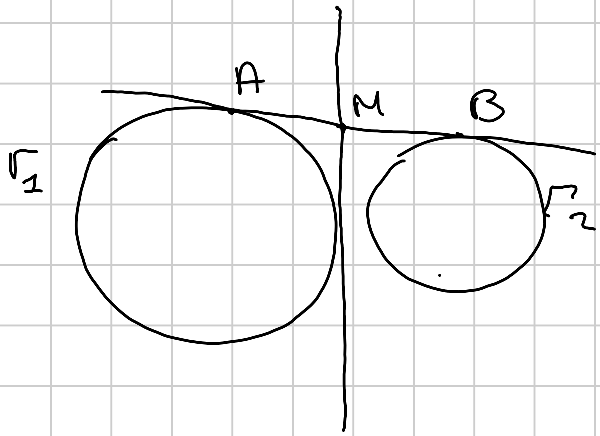
ESERCIZI

G2.



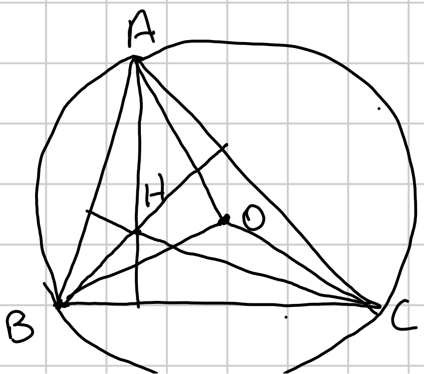
$$Pow_{\Gamma_1}(P) = Pow_{\Gamma_2}(P)$$
$$Pow_{\Gamma_2}(P) = Pow_{\Gamma_3}(P)$$
$$\Rightarrow Pow_{\Gamma_1}(P) = Pow_{\Gamma_3}(P)$$

G3.



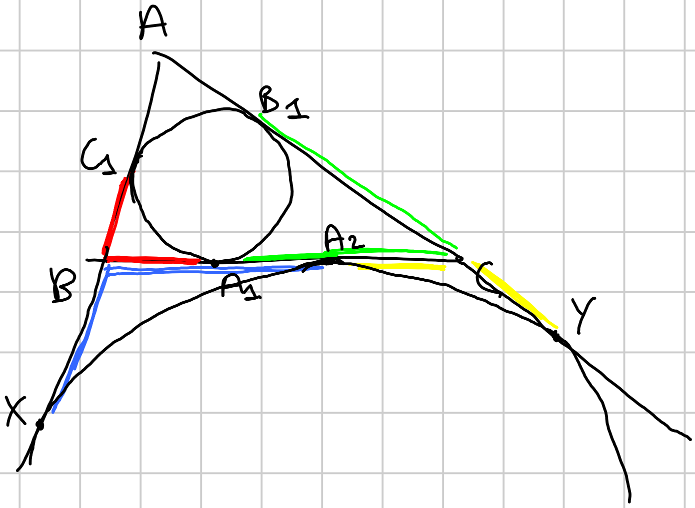
$$\underbrace{Pow_{\Gamma_1}(M)}_{\underset{AM \cdot AM}{\parallel}} = \underbrace{Pow_{\Gamma_2}(M)}_{\underset{MB \cdot MB}{\parallel}}$$
$$AM^2 = MB^2 \Rightarrow AM = MB$$

G12.



$$B\hat{A}H = 90° - \beta$$
$$A\hat{O}C = 2\beta$$
$$O\hat{A}C = \frac{180° - 2\beta}{2} = 90° - \beta$$
$$\Rightarrow B\hat{A}H = O\hat{A}C$$

G6.



$AX = AY$

$AC_1 = AB_1$

$AX - AC_1 = AY - AB_1$

$XC_1 = B_1Y$

$\|$

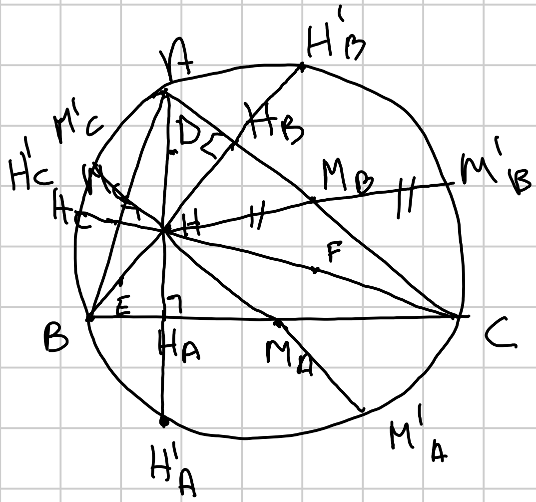
$XB + BC_1 = B_1C + CY$

$BA_2 + BA_1 = A_1C + A_2C$

$BA_1 + \cancel{A_1A_2} + BA_1 = \cancel{A_1A_2} + A_2C + A_2C$

$\cancel{2}BA_1 = \cancel{2}A_2C$

G15.



Omotetia di centro H e fattore 2

$H_A \rightarrow H'_A$

$M_A \rightarrow M'_A$ e cyc

$\Rightarrow$ ($H'_A M'_A H'_B M'_B H'_C M'_C$ sono ciclici

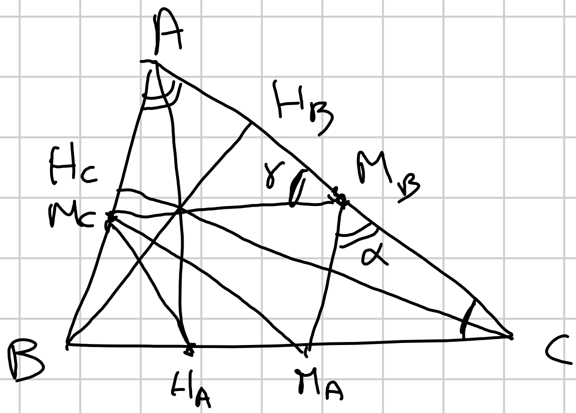
$\Rightarrow H_A M_A H_B M_B H_C M_C$ sono ciclici

$D \rightarrow A$

$E \rightarrow B$

$F \rightarrow C$

$D, E, F \in$ circ di Feuerbach



$\odot M_A M_B M_C$

$M_C \hat{M}_B M_A = \beta$

$M_C \hat{H}_A M_A = M_C \hat{H}_A A + \underbrace{A \hat{H}_A C}_{} = $ 90°

$= M_C \hat{H}_A A + 90°$

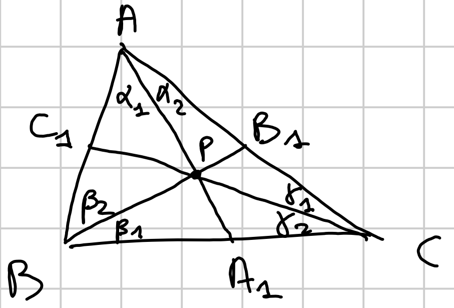
$= M_C \hat{A} H_A + 90°$

$= 90° - \beta + 90° =$

$= 180° - \beta$

$H_A \in \odot M_A M_B M_C$

G10.



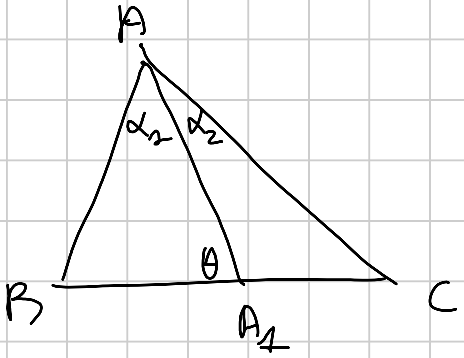
AA_1, BB_1, CC_1 concorrono $\iff \dfrac{BA_1}{A_1C} \cdot \dfrac{CB_1}{B_2A} \cdot \dfrac{AC_1}{C_1B} = 1$ (*)

☆ $\boxed{\dfrac{BA_1}{A_1C} = \dfrac{AB \cdot \sin\alpha_1}{AC \cdot \sin\alpha_2}}$ $\qquad \dfrac{CB_1}{B_1A} = \dfrac{BC \cdot \sin\beta_1}{AB \cdot \sin\beta_2}$

$\dfrac{AC_1}{C_1B} = \dfrac{AC \cdot \sin\gamma_1}{BC \cdot \sin\gamma_2}$

(*) $\iff \dfrac{\cancel{AB} \cdot \sin\alpha_1}{\cancel{AC} \sin\alpha_2} \cdot \dfrac{\cancel{BC} \cdot \sin\beta_1}{\cancel{AB} \cdot \sin\beta_2} \cdot \dfrac{\cancel{AC} \cdot \sin\gamma_1}{\cancel{BC} \cdot \sin\gamma_2} = 1$

$\iff \dfrac{\sin\alpha_1 \cdot \sin\beta_1 \cdot \sin\gamma_1}{\sin\alpha_2 \cdot \sin\beta_2 \cdot \sin\gamma_2} = 1$



$\dfrac{BA_1}{\sin\alpha_1} = \dfrac{AB}{\sin\theta} \Rightarrow \sin\theta = \dfrac{AB}{BA_1} \cdot \sin\alpha_1$

$\|$

$\dfrac{A_1C}{\sin\alpha_2} = \dfrac{AC}{\underbrace{\boxed{\sin 180° - \theta}}_{\sin\theta}} \Rightarrow \sin\theta = \dfrac{AC}{A_1C} \cdot \sin\alpha_2$

$\dfrac{AB}{BA_1} \cdot \sin\alpha_1 = \dfrac{AC}{A_1C} \cdot \sin\alpha_2$

$\Longrightarrow \boxed{\dfrac{BA_1}{A_1C} = \dfrac{AB \cdot \sin\alpha_1}{AC \cdot \sin\alpha_2}}$ ☆