PREIMO 2006 A - Matt.

Titolo nota 23/05/2006

$$P(x) = (x+1)^p (x-3)^q = x^n + a_1 x^{n-1} + a_2 x^{n-2}$$

$$a_1 = a_2$$

-1 è una rad molt p

3 è una rad molt q

$$-a_1 = -p + 3q$$

$$a_2 = -3pq + \frac{p(p-1)}{2} + 9\,\frac{q(q-1)}{2}$$

$$p - 3q = -3pq + \frac{p(p-1)}{2} + 9\frac{q(q-1)}{2}$$

$$p^2 - 6pq + 9q^2 = 3p + 3q$$

$$\underbrace{(p-3q)^2}_{9A^2} = 3(p+q)$$

$p = 3i$

$q = 3k$

$$\begin{cases} p+q = 3A^2 \\ p-3q = 3A \end{cases}$$

sottr:

$$4q = 3A^2 - 3A = 3A(A-1)$$

$$q = \frac{3A(A-1)}{4}$$

intero $\Leftrightarrow$ $A \equiv {0 \atop 1} \ (4)$

$P(x)$ $\quad K = 1, 2 \dots n+1$ $\quad P(k) = \frac{1}{k}$

$$Q(x) = P(x) \cdot x - 1$$

$$Q(x) = a_n (x-1)(x-2) \cdot (\dots) \cdot (x - \overset{1}{n} - 1)$$

$$P(n+2) = \frac{Q(x)+1}{n+2} = \frac{Q(n+2)+1}{n+2} = \frac{a_n (n+1)! + 1}{n+2}$$

$$Q(0) = -1$$

$$Q(x) = a_n \cdot (-1)(-2) \dots = a_n \cdot (-1)^{n+1} \cdot (n+1)!$$

$$a_n =$$

grado 2

$p(1) = 1 \qquad p(2) = 8 \qquad p(3) = 27$

$p(x) = ax^2 + bx + c \qquad x = 1, 2, 3$

$p(1) = 1 \qquad p(x) = 1 + (x-1)\, q(x)$

$p(2) = 8 \qquad 8 = p(2) = 1 + q(2) \Rightarrow q(2) = 7$

$p(x) = 1 + (x-1)\left[7 + (x-2)\, r(x)\right]$

$p(3) = 27 \qquad r(3) = \dots$

$$p(x) = A(x-1)(x-2) + B(x-1)(x-3) + C(x-2)(x-3)$$

$p(1) = 1$	0	0	$C(-1)(-2) = 1$
$p(2) = 8$	0	$B \cdot 1(-1) = 8$	0
$p(3) = 27$	$A \cdot 2 \cdot 1 = 27$	0	0

$x \equiv 1 \quad (p_1)$

$x \equiv 2 \quad (p_2)$

$x \equiv 5 \quad (p_3)$

$$x = A p_1 p_2 + B p_2 p_3 + C p_3 p_1$$

$1 =$	0	$B p_3 p_2$	0	(p_1)
	0	0	$\cdots$	(p_2)

$$\sum_{sym} x^2y = x^2(y+z)+y^2(x+z)+z^2(x+y) = x^2(1-x)+y^2(1-y)+z^2(1-z)$$

$x \le y \le z$ $\qquad$ $x+y \le \frac{2}{3}$ $\qquad$ $x+y > \frac{2}{3}$

$x+z > \frac{2}{3}$

$y+z > \frac{2}{3}$

$x+y+z > 1$

$$x+y \le \frac{2}{3}$$

$$3(x+y) \le 2$$

$$3xy(x+y) \le 2xy$$

$$2xy - 3xy(x+y) \ge 0$$

$$x^2+y^2-x^3-y^3+2xy-3xy(x+y) \ge x^2+y^2-x^3-y^3$$

$$(x+y)^2-(x+y)^3 \ge x^2-x^3+y^2-y^3$$

$$\sum_{sym} x^2y = x^2(1-x)+y^2(1-y)+z^2(1-z) \leq (x+y)^2(1-x-y)+z^2(1-z)=$$

$$(x+y)^2z+z^2(x+y) = z(x+y)(x+y+z) = z(x+y) \leq \left[\frac{(x+y)+z}{2}\right]^2 = \frac{1}{4}$$

$$\sum_{cyc} x^2y = x^2y+y^2z+z^2x = P(x,y,z)$$

$$P(x,y,z) \leq P(x+z,y,0)$$

$$P(x+z,y,0) - P(x,y,z) =$$

$$= (x+z)^2y - x^2y - y^2z - z^2x =$$

$$= \cancel{x^2 y} + 2xyz + z^2y - \cancel{x^2 y} - y^2 z - z^2 x =$$

$$= z\left(2xy + zy - y^2 - zx\right) =$$

$$= z\left(\underbrace{(x-y)(y-z)}_{\geq 0} + xy\right)$$

$\boxed{z = 0}$ sym $\Rightarrow$ $x \geq y \geq z \geq \ldots$ ---

cyc $\Rightarrow$ solo max o min

o etc. ....

$\rightarrow$ $x + y = 1$ $y = 1 - x$

$$P(y, 1-x) = x^2(1-x) = 4\left(\frac{x}{2}\cdot\frac{x}{2}\cdot(1-x)\right) \leq$$

$$\leq \left(\frac{\frac{x}{2} + \frac{x}{2} + (1-x)}{3}\right)^3 4 = \frac{4}{27}$$

$$\frac{x}{2} = 1-x \quad \Rightarrow \quad \boxed{x = \frac{2}{3} \quad y = \frac{1}{3} \quad z = 0}$$

$$P(x+z, y, 0) - P(x, y, z) = xz(2y - x - z) \geq 0$$

$$y \geq x \geq z$$

UNSMOOTHING!

STRAIGHTFORWARD...

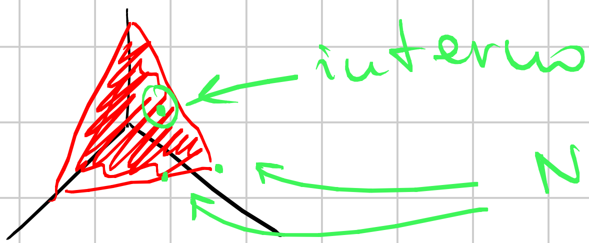
$$\text{MAX}\left\{ p(x,y,z) \;:\; \begin{matrix} x\geq 0,\, y\geq 0,\, z\geq 0 \\ \underbrace{x+y+z}=1 \end{matrix} \right\}$$

$$g(x,y,z) = x+y+z-1 = 0$$

Nei p.ti di max si ha

$$\nabla p = \lambda \nabla g \qquad \boxed{x=y=z=\frac{1}{3}}$$

SE I PUNTI DI MAX sono all' "interno" dell' insieme considerato



CHIUSO + LIMITATO

⇓

esistenza

$$P(x,y,z) = \frac{1}{x} + y + z$$

NON è definita su tutto $\triangle$

e continua

$H_p \quad a+b+c \geq \frac{1}{a}+\frac{1}{b}+\frac{1}{c} \qquad a,b,c \in \mathbb{R}^+$

$T_s : \quad a+b+c \geq \frac{3}{abc}$

$\sigma_1 = \frac{a+b+c}{3} \qquad \sigma_2 = \frac{ab+bc+ca}{3} \qquad \sigma_3 = abc$

$H_p : \quad \sigma_1 \geq \frac{\sigma_2}{\sigma_3} \qquad T_s : \quad \sigma_1 \geq \frac{1}{\sigma_3}$

$\sigma_2^2 \geq \sigma_1 \cdot \sigma_3$

$\cancel{\sigma_1}\,\sigma_2^2 \geq \cancel{\sigma_1}\,\cancel{\sigma_3}\,\frac{\cancel{\sigma_2}}{\cancel{\sigma_3}} \qquad \cancel{\sigma_2}\,\sigma_1 \geq \frac{\cancel{\sigma_2}}{\sigma_3}$

$\sigma_2 \geq 1 \qquad \sigma_1 \geq \frac{1}{\sigma_3}$ q.e.d.

$\frac{1}{2} \quad \frac{1}{2} \quad \frac{15}{4}$

Hp $\quad a+b+c \geq \frac{1}{a}+\frac{1}{b}+\frac{1}{c} = \frac{ab+bc+ca}{abc}$

1

Th $\quad a+b+c \geq \frac{3}{abc} \quad -3$

$1 \geq \frac{ab+bc+ca}{abc(a+b+c)} \quad -2$

Hope

$(a+b+c)\cdot\left(\frac{ab+bc+ca}{abc(a+b+c)}\right)^2 \geq \frac{3}{abc}$

$$\cancel{(a+b+c)} \cdot \frac{(ab+bc+ca)^2}{abc(a+b+c)^{\cancel{2}}} \geq 3$$

$$\frac{1}{2}\sum_{sym} a^2b^2 \geq \frac{1}{2}\sum_{sym} a^2bc$$

che é vera per Bunching!

Tesi $\quad abc(a+b+c) \geq 3$

$$A \geq B$$

$$A \geq C \geq B$$

$$A \geq C$$
$$B \geq C$$

NO

Hp $\quad \boxed{3\left(\frac{abc(a+b+c)}{ab+bc+ca}\right)^2} \geq 1 \cdot 3$

$$abc(a+b+c) \geq 3\left(\frac{abc(a+b+c)}{ab+bc+ca}\right)^2$$

grado 6   4   grado $\cancel{2}$ 4

$$a+b+c \geq \frac{1}{a}+\frac{1}{b}+\frac{1}{c} \qquad MA \qquad abc<1$$

$abc = 1$

$a=1 \quad b=1 \quad c=1$

$a=n \quad b=\frac{1}{n} \quad c=1$

$$1+n+\frac{1}{n} \geq 1+n+\frac{1}{n}$$

$$a=n^2 \quad b=\frac{1}{n} \quad c=\frac{1}{n} \quad abc=1$$

$$n^2 + \frac{1}{n} + \frac{1}{n} \geq n + n + \frac{1}{n^2}$$ VERA ABBONDANTE

$$a = n\sqrt{n} \qquad b = \frac{1}{n} \qquad c = \frac{1}{n}$$

$$n\sqrt{n} + \frac{1}{n} + \frac{1}{n} \geq n + n + \frac{1}{n\sqrt{n}}$$ OK

$$abc = \frac{1}{\sqrt{n}}$$