Titolo nota 23/05/2006

PREIMO A Pom.

$$(4+4+4)(a^2+b^2+c^2) \geq (a+2b+2c)^2$$

$$\text{RHS} \leq 12$$

$$\left(\frac{a}{m_a}+\frac{b}{m_b}+\frac{c}{m_c}\right)^2 \geq 12$$

$$\frac{a}{m_a} + \frac{b}{m_b} + \frac{c}{m_c} \geq 2\sqrt{3}$$

$$m_a \geq h_a = \frac{2A}{a}$$

$$a^2 + b^2 + c^2 \geq 4A\sqrt{3}$$

$\cancel{A2}$

$$\sum e$$

$$\sum e\, m_e^2$$

$$e\, m_e^2 = \frac{2(b^2+c^2)-e^2}{4} \cdot e = \frac{2(eb^2+ec^2)-e^3}{4}$$

$$\sum e\, m_e^2 = \frac{2\sum_{sym} a^2 b - \sum e^3}{4}$$

$$\sum e\, m_e^2 \underbrace{\sum \frac{e}{m_e} \sum \frac{e}{m_e}}_{LHS} \geq \left(\sum e\right)^3$$

$$LHS \geq \frac{\left(\sum e\right)^3}{\sum e\, m_e^2} \qquad \frac{\left(\sum e\right)^3}{\sum e\, m_e^2} \geq 4\, \frac{\cancel{\left(\sum e\right)^2}}{\sum e^2}$$

$$\sum \frac{a}{m_a} \geq 2\sqrt{3}$$

$$\frac{\cancel{4}\left(\sum e\right)}{2\sum_{sym} e^2 b - \sum e^3} \geq \frac{\cancel{4}}{\sum e^2}$$

$$\left(\sum e\right)\left(\sum e^2\right) \geq 2\sum_{sym} e^2 b - \sum e^3$$

$$\sum e^3 + \cancel{\sum_{sym} e^2 b} \geq \cancel{2}\sum_{sym} e^2 b - \sum e^3$$

$$\sum_{sym} e^3 \geq \sum_{sym} e^2 b$$

$$\frac{a}{a+b+c}\cdot\frac{1}{m_a}+\frac{b}{a+b+c}\cdot\frac{1}{m_b}+\frac{c}{a+b+c}\cdot\frac{1}{m_c}\geq 2\cdot\frac{1}{\sqrt{a^2+b^2+c^2}}$$

$$f(x)=\frac{1}{\sqrt{x}}$$

$$\sum_c \lambda_a\cdot f\left(\frac{1}{\cancel{2}4}\left(2b^2+2c^2-a^2\right)\right)\geq f\left(\sum \lambda_a\cdot\frac{1}{4}\cdot\left(2b^2+2c^2-a^2\right)\right)=$$

$$=\frac{1}{\sqrt{\dfrac{2\sum a^2b-\frac{\sum a^3}{2}}{\cancel{4}(a+b+c)}}}\overset{\nearrow H}{\geq}\frac{\cancel{2}}{\sqrt{a^2+b^2+c^2}}$$

$$(a+b+c)\frac{\sum a^2}{2}\overset{?\nearrow H}{\geq}2\sum a^2b-\frac{\sum a^2}{2}$$

$$\sum a^2b+\frac{\sum a^3}{2}\overset{\nearrow H}{\geq}2\sum a^2b-\frac{\sum a^3}{2}\rightarrow\sum a^3\geq\sum a^2b$$

$$m_a = \frac{1}{2}\sqrt{2(b^2+c^2)-a^2}$$

$$(a, b, c)$$

$$\left(\frac{\cancel{2}a}{\sqrt{2(b^2+c^2)-a^2}} + \cdots \right)^2 \geq \cancel{4} \frac{(a+b+c)^2}{a^2+b^2+c^2}$$

$$\left(\frac{1}{m_a}, \frac{1}{m_b}, \frac{1}{m_c} \right)$$

$$\geq \left(\frac{\cancel{(a+b+c)} \left(\frac{1}{\sqrt{2(b^2+c^2)-a^2}} + \cdots \right)}{3} \right)^2 \geq \frac{\cancel{(a+b+c)^2}}{a^2+b^2+c^2}$$

Hope

$$\left(\frac{\frac{1}{m_a} + \frac{1}{m_b} + \frac{1}{m_c}}{3} \right)^2$$

$$\geq \left(\left(\frac{3}{2(b^2+c^2)-a^2+2(a^2+b^2)-c^2+2(a^2+c^2)-b^2} \right)^{1/2} \right)^2 = \frac{1}{a^2+b^2+c^2}$$

ACÉ

$$\frac{1}{1+x_1} + \frac{1}{1+x_1+x_2} + \dots + \frac{1}{1+\dots+x_n} < \sqrt{\frac{1}{x_1} + \dots + \frac{1}{x_n}}$$

$$\left(\frac{1}{\sqrt{x_1}}, \frac{1}{\sqrt{x_2}}, \dots, \frac{1}{\sqrt{x_n}} \right)$$

$$\left(\frac{\sqrt{x_1}}{1+x_1}, \frac{\sqrt{x_2}}{1+x_1+x_2}, \dots, \frac{\sqrt{x_n}}{1+\dots+x_n} \right)$$

$$LHS^2 \leq RHS^2 \cdot \underbrace{\left(\frac{x_1}{(1+x_1)^2} + \frac{x_2}{(1+x_1+x_2)^2} + \dots \frac{x_n}{(1+\dots+x_n)^2} \right)}_{h}$$

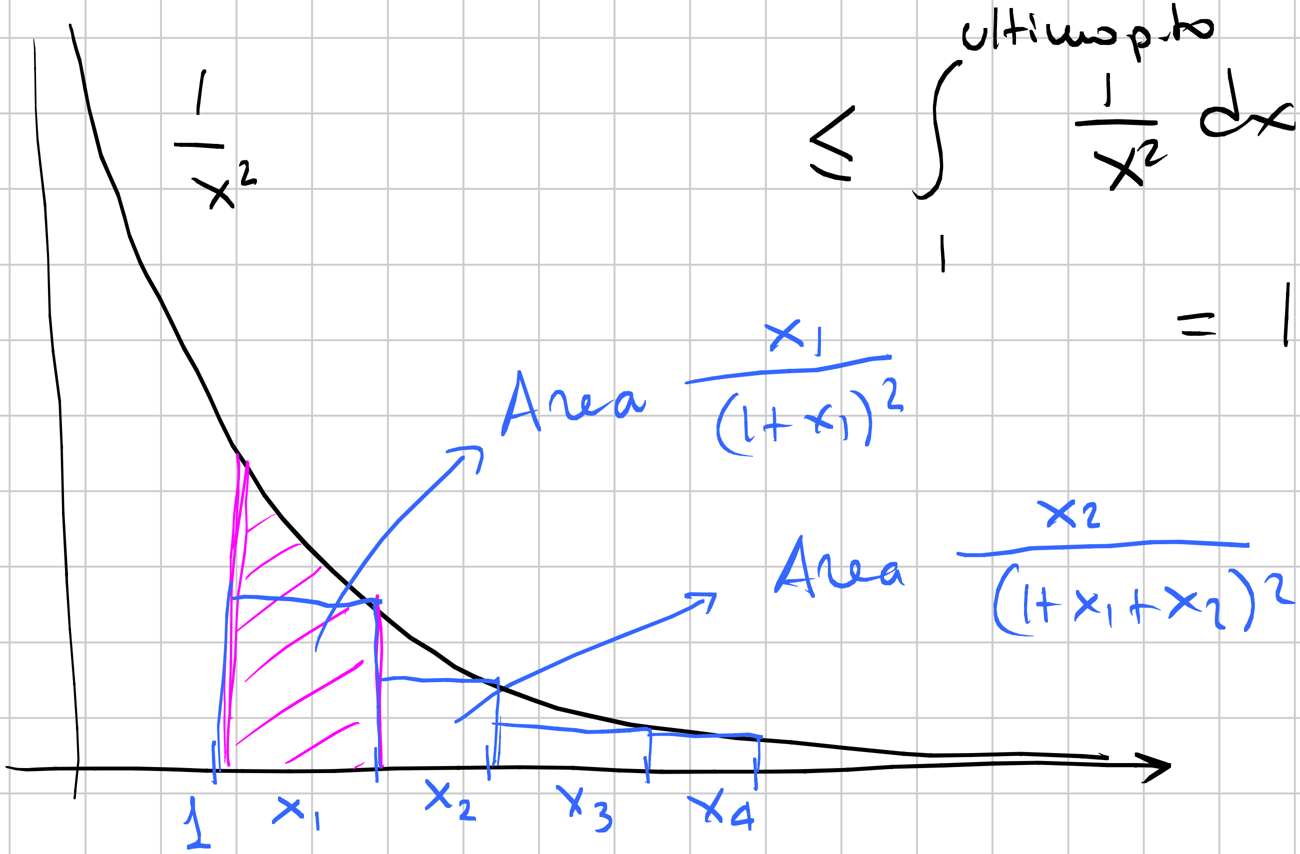
$$\frac{x_1}{(1+x_1)^2} < \frac{x_1}{1(1+x_1)} = 1 - \frac{1}{1+x_1}$$

$$\frac{x_2}{(1+x_1+x_2)^2} < \frac{x_2}{(1+x_1)(1+x_1+x_2)} = \frac{1}{1+x_1} - \frac{1}{1+x_1+x_2}$$

$$h < 1 - \frac{1}{1+x_1+\dots+x_n}$$

$$\frac{x_1}{(1+x_1)^2}+\frac{x_2}{(1+x_1+x_2)^2}+\dots\frac{x_n}{(1+\dots+x_n)^2}\Bigg) = \text{somma aree dei rett.}$$

$$\le \int_1^{\text{ultimo p.to}} \frac{1}{x^2}\,dx = \left[-\frac{1}{x}\right]_1^{\dots} = 1-\frac{1}{1+x_1+\dots+x_n}$$



(a, b, m, n) $\quad m > n > 1$

$$x^n + ax + b \mid x^m + ax + b$$

$a = b = 0$

$$x^n \mid x^m$$

$(0, 0, m, n)$

$a = 0 \quad b \neq 0$

$$x^n + b \mid x^m + b$$

$$x^n + b \mid x^m \cancel{+b} - x^n \cancel{-b}$$

$$x^n + b \mid (x^{m-n} - 1)\cancel{x^n}$$

$$(x^n + b)\,q(x) = x^{m-n} - 1$$

$$w^n + b = 0$$

$$w^n = -b$$

$$e^{i\theta} = -b\,e^{i}$$

$$b^2 = 1$$

$$b = 1 \qquad b = -1$$

$$b = 1$$

$$x^n + 1 \mid x^{m-n} - 1$$

$$m - n = 2kn$$

$x^n + 1 \mid x^{2k} - 1$

$m = (2k+1)n$

$w^{2n} = 1 \quad w^n \neq 1$

$(0, 1, n, (2k+1)n)$

$b = -1$

$x^n - 1 \mid x^{m-n} - 1$

$m - n = kn$

$m = (k+1)n$

$(0, -1, n, kn)$

$k > 1$

$b = 0$

$$x^n + ax \mid x^m - x^n$$

$$x^n + ax \mid x^n \left(x^{m-n} - 1\right)$$

$$x^{n-1} + a \mid \cancel{x^{n-1}} \left(x^{m-n} - 1\right)$$

$$(1, 0, n, (n-1)k + n)$$

$$(-1, 0, n, (n-1)k + n)$$

$a \neq 0, \quad b \neq 0$

$$x^n + ax + b \mid x^{m-n} - 1$$

$$w = \cos\theta + i \operatorname{sen}\theta$$

$$\cos n\theta + i \operatorname{sen} n\theta + a\cos\theta + ia\operatorname{sen}\theta + b = 0$$

$$\operatorname{sen} n\theta + a\operatorname{sen}\theta = 0$$

$$\cos n\theta + a\cos\theta + b = 0$$

$$\operatorname{sen}^2 n\theta = a^2 \operatorname{sen}^2\theta$$

$$\cos^2 n\theta = a^2\cos^2\theta + 2ab\cos\theta + b^2$$

$$1 = a^2 + 2ab\cos\theta + b^2$$

$$\cos\theta = \frac{1-a^2-b^2}{2ab} \in \mathbb{Q} + a, b \in \mathbb{Z} + b = \pm 1$$

$$\Downarrow$$

$$\cos\theta = \left\{ \frac{1}{2}, -\frac{1}{2}, 1, -1 \right\}$$

$x^n + ax + b$

$x^n - 2x + 1$

$x^2 - 2x + 1 \mid x^{n-2} - 1$

$x^n + 2x + 1$

$x^2 + 2x + 1 \mid x^{n-2} - 1$ NO

NO

$\underbrace{x^2+x+1} \mid \underbrace{x^n + x + 1}$ $\qquad n = 2$

$x^2 + x + 1 \mid x^{m-2} - 1$ $\qquad 3 \mid m-2$

$x^2 - x + 1 \mid x^n - x + 1$ $\qquad x^2 - x + 1 \mid x^{m-2} - 1$

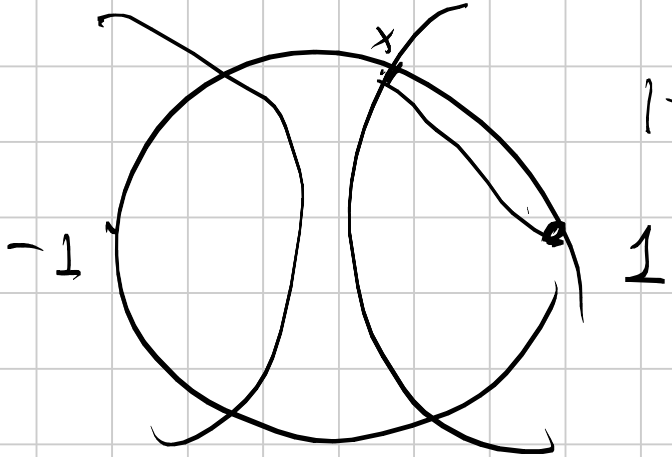
$n = 2$ $\qquad 6 \mid m-2$

$x^n + ax + b \mid x^{m-n} - 1$ $\qquad b \neq 0$

$b = \pm 1$

$|a| = |ax| \leq |x^n| + |b| = 2$

$x^n \pm 1$ $\qquad x^n \pm x \pm 1$



$|x^n| = |\pm x \pm 1|$

$x^n + ax + b$ tutte le sue radici sono radici di 1.

$z_1, \dots, z_n$ Supponiamo $n > 2$

$z_1 + \dots + z_n = \pm$ coeff. di $x^{n-1} = 0$

$$\pm \frac{a}{b} = \frac{1}{z_1} + \dots + \frac{1}{z_n} = \bar{z}_1 + \dots + \bar{z}_n = \overline{z_1 + \dots + z_n} = 0$$

$\Downarrow$

Fatto gen.: se $|z| = 1$, allora $\frac{1}{z} = \bar{z}$ $\quad a = 0$