Titolo nota 25/05/2006

Esiste un problema assegnato a 4 persone.

$1+3+3+3+3=13.$

Persone \ Probl	1	2	3	4	5	...							
1	x	x	x	x									
2	x				x	x	x						
3	x							x	x	x			
4	x										x	x	x
5		x			x			x			x		
6		x				x			x			x	
7			x		x	-		- -					
			x	x									

n_1 = numero delle persone che hanno risolto il 1° problema

n_2

$\vdots$

n_k

$$\binom{n_1}{2} + \binom{n_2}{2} + \dots + \binom{n_k}{2} \leq \binom{10}{2}$$

$$\sum_{i=1}^{k} \frac{n_i^2 - n_i}{\cancel{2}} \leq \frac{10 \cdot 9}{\cancel{2}}$$

$$\sum n_i^2 \geq \frac{\left(\sum n_i\right)^2}{k}$$

$$\frac{40^2}{k} - 40 \leq \sum n_i^2 - \sum n_i \leq 90$$

$$\frac{4 \cdot 40}{k} \leq 13 \qquad 13k \geq 160 \qquad k \geq 13$$

A	A	A	B	B	B	C	C	C	A
D	E	F	D	E	F	D	E	F	B
G	H	I	I	G	H	H	I	G	C
L	M	N	M	N	L	N	L	M	O

Spazio affine $\mathbb{R}^2, \mathbb{R}^3, \mathbb{R}^4, \dots$

retta in $\mathbb{R}^2$ $\rightarrow$ coso con eq.

$$ax + by = c$$

(occhio che molt. tutto per $\lambda \neq 0$ ottengo stessa retta) (è un $\Longleftrightarrow$)

piano in $\mathbb{R}^3$

$$a_1 x_1 + a_2 x_2 + \dots + a_n x_n = b$$

iperpiano in $\mathbb{R}^3$

Dati 2 p.ti diversi $\exists$ unica retta che li unisce

Date 2 rette distinte $\nearrow$ 0 p.ti (c cambia, ma a e b uguali)

$\searrow$ 1 p.to

$\mathbb{Z}_3$ = classi di congruenza mod 3

$(\mathbb{Z}_3)^2$ = piano affine su $\mathbb{Z}_3$

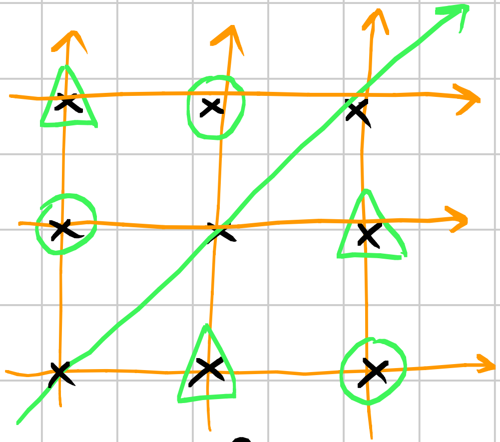
retta in $(\mathbb{Z}_3)^2$: insieme dei punti che soddisfano

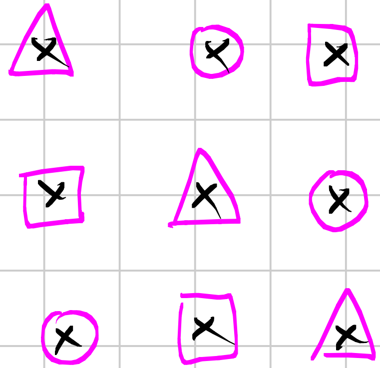
$$ax + by \equiv c \pmod 3$$

2 rette sono la stessa $\Leftrightarrow$ i coeff. si ottengono molt. per $\lambda \neq 0$ (3)

Anche negli affini finiti per 2 p.ti distinti passa un'unica retta

Date 2 rette $\rightsquigarrow$ vedi $\mathbb{R}^2$

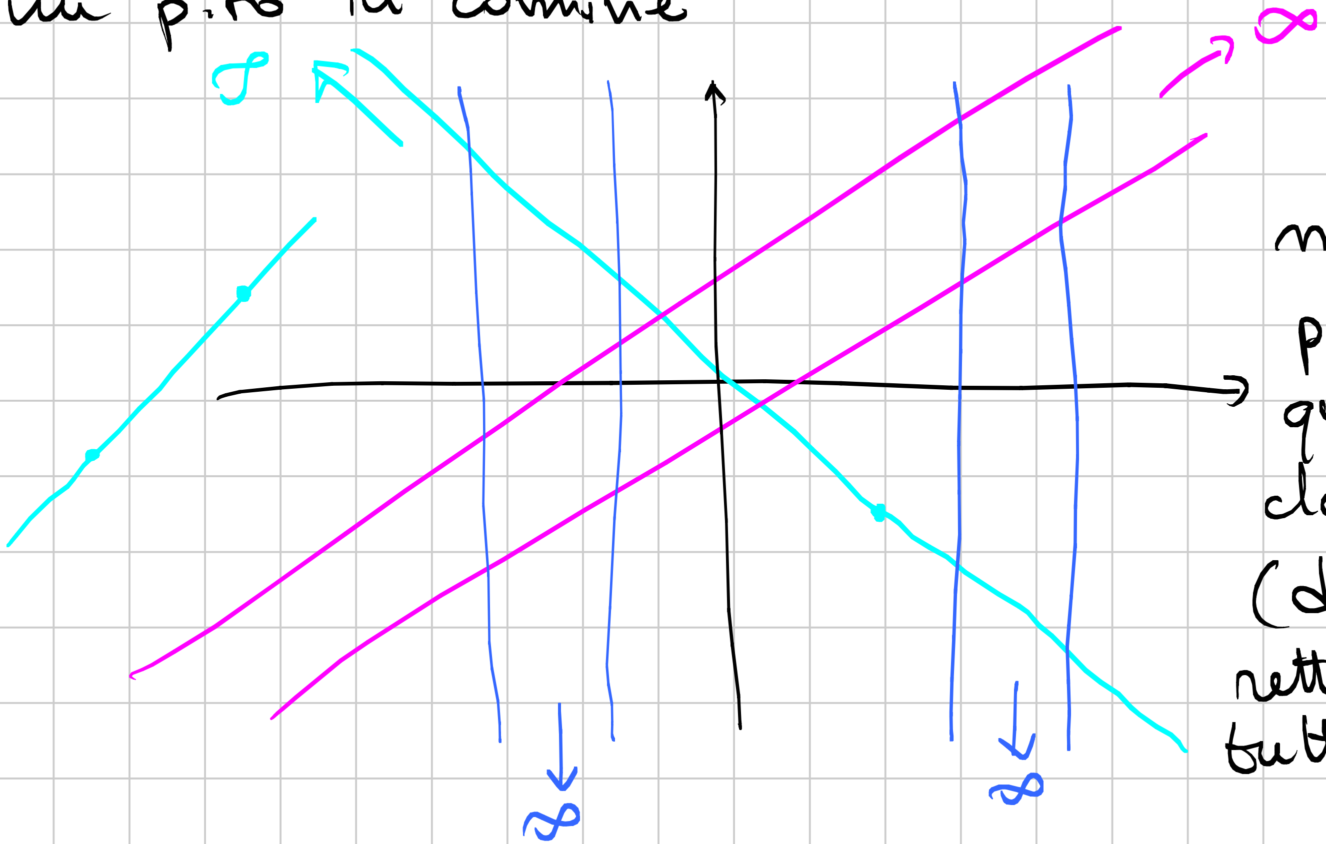




Quante classi? $4 = 3 + 1$

$\mathbb{RP}^2$ = piano proiettivo reale

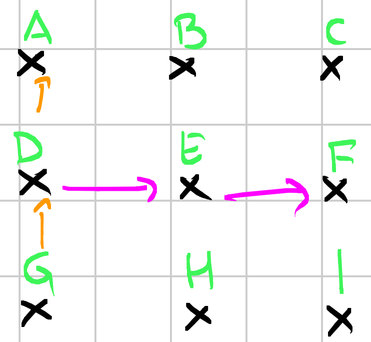
Seccato perchè 2 rette non hanno PER FORZA un p.to in comune

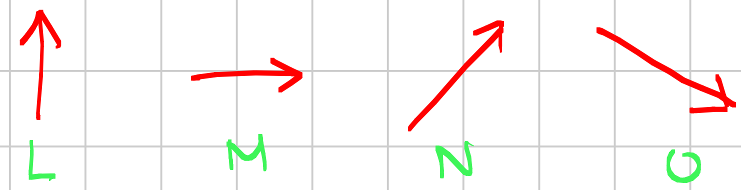


Morale: si mettono tanti p.ti all'infinito quanti sono le classi di parall. (direzioni)

retta all'∞ con tutti i p.ti all'∞.

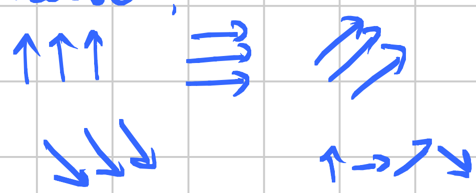
$\mathbb{Z}_3\mathbb{P}^2 = \mathbb{P}^2(\mathbb{Z}_3) \to$ 13 p.ti





1. G D A L
2. H E B L
3.
4. D E F M

Quante sono le rette?



$(p+1)p+1$
$= p^2+p+1$

$$\mathbb{Z}_5\mathbb{P}^2 = \underset{\text{affini}}{25} + \underset{\text{coeff. ang.}}{5} + \underset{\text{dir. vert.}}{1}$$

$$\binom{n}{k}\binom{n-k}{k}\cdot 4^k \cdot 4^{n-2k} = \frac{n!\,4^{n-k}}{k!\,k!\,(n-2k)!}$$

$$\sum$$

$$4^n\left(\frac{x}{4}+\frac{1}{x}+1\right)^n = 4^n\left(\frac{x^2+4+4x}{4x}\right)^n =$$

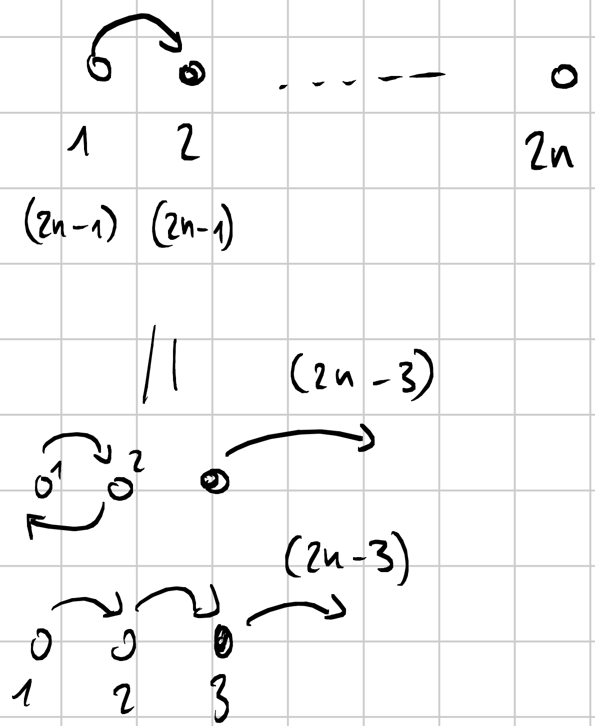
$$\frac{\cancel{4^n}\,(x+2)^{2n}}{\cancel{4^n}\,x^n} = \binom{2n}{n}2^n$$

$$(x+y)^n = \sum_{k=0}^{n}\binom{n}{k}x^k y^{n-k} = \sum_{k=0}^{n}\frac{n!}{k!(n-k)!}x^k y^{n-k}$$

$$(x+y+z)^n = \sum_{i+j+k=n}\frac{n!}{i!\,j!\,k!}x^i y^j z^k \qquad \frac{n!}{i!\,j!\,k!} = \binom{n}{i}\binom{n-i}{j}$$

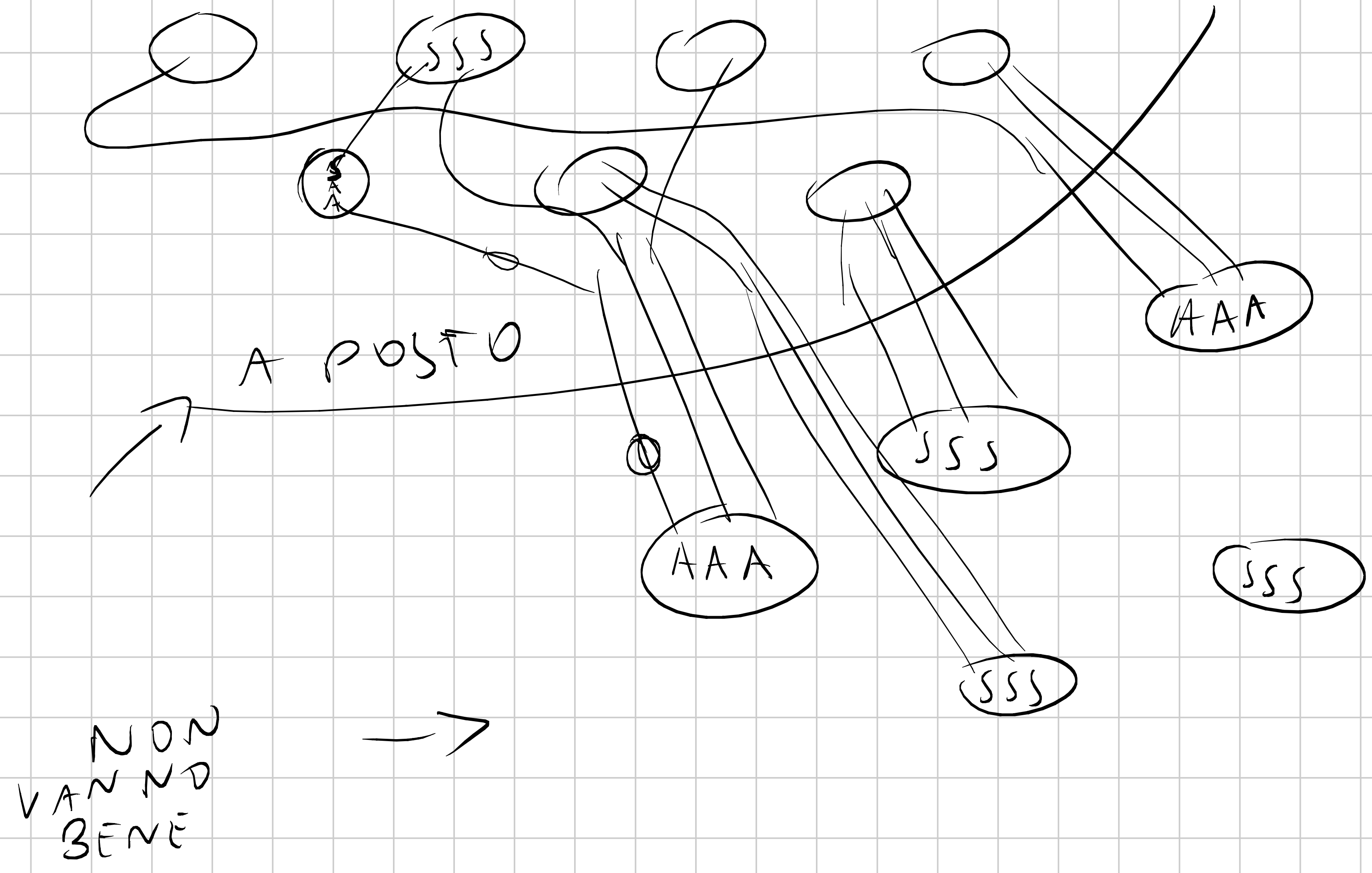
$$\begin{pmatrix} 1 & 2 & \cdots & 2n \\ k & j & \cdots & w \end{pmatrix}$$

$$\begin{array}{cc} \left(\begin{array}{c} 1 \\ 9 \\ \vdots \\ n \end{array}\right) & \begin{array}{c} A \\ B \\ \\ A \end{array} \end{array}$$



$$\begin{array}{cc} \left(\begin{array}{c} n \\ \vdots \\ m \end{array}\right) & \begin{array}{c} A \\ B \\ \vdots \\ B \end{array} \end{array} \qquad \begin{array}{cc} \left(\begin{array}{c} j \\ w \end{array}\right) & \begin{array}{c} A \\ \vdots \\ B \end{array} \end{array} \qquad \begin{array}{cc} \left(\begin{array}{c} \\ \\ \end{array}\right) & \begin{array}{c} A \\ \vdots \\ B \end{array} \end{array}$$

$$(2n-1)^2(2n-3)^2 \cdots 3^2 \cdot 1$$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