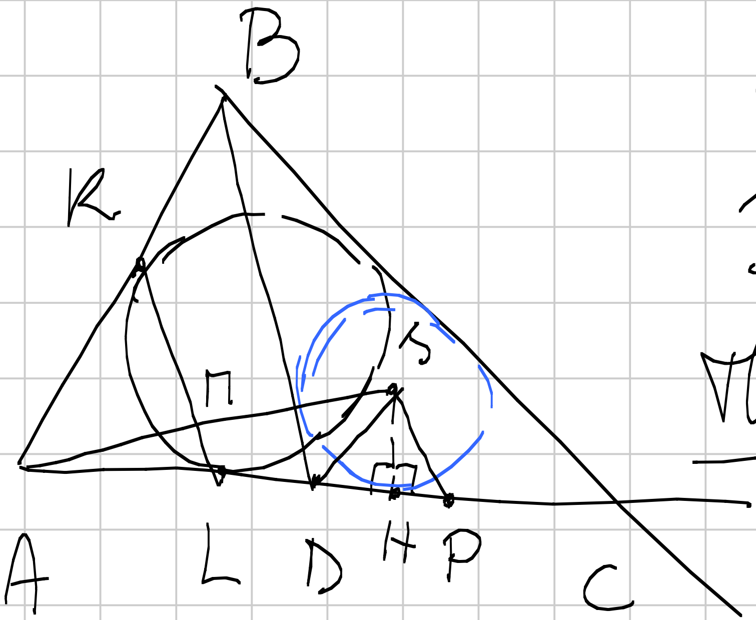
GEOMETRIA POMERIDIANA

Titolo nota 29/05/2007



$BD = BA$

S incentro di BDC

<u>Th</u>: $AM = MS$

$SP \parallel KL \Rightarrow Th \Leftrightarrow AL = LP$

$AL = \frac{b+c-a}{2}$

Vogliamo dim $\overset{\triangle}{DSP}$ isoscele.

$S\hat{P}A = K\hat{L}A = 90° - \frac{\alpha}{2}$

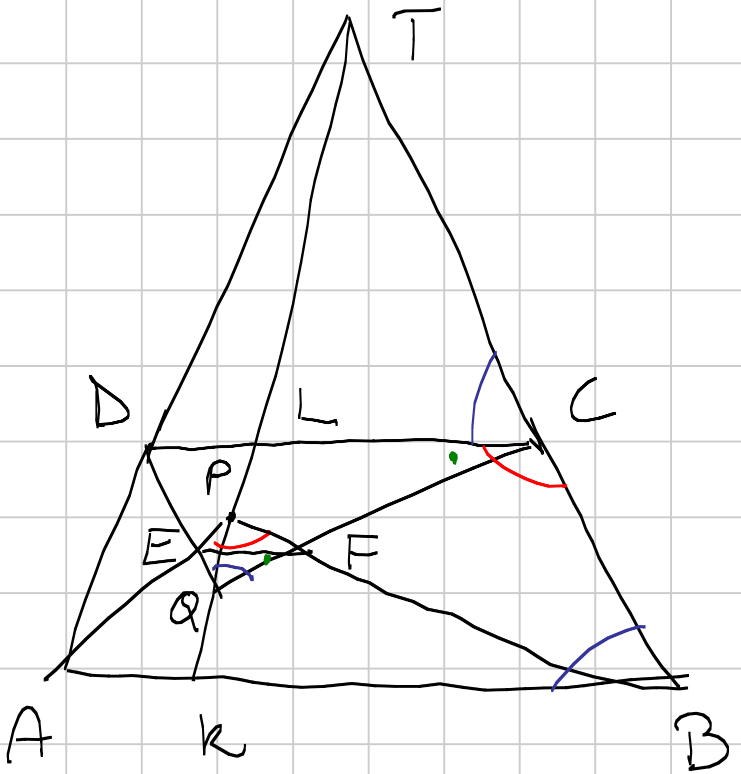
$$\alpha = B\hat{A}D = B\hat{D}A \Rightarrow B\hat{D}C = 180^\circ - \alpha$$

$$\Rightarrow B\hat{D}C = 90^\circ - \frac{\alpha}{2}$$

$$DH = \frac{DC + c - a}{2} \qquad DP = DC + c - a$$

$$AP = AD + DP = AC + c - a = b + c - a =$$

$$= 2\left(\frac{b + c - a}{2}\right) = 2AL$$



$$\frac{AK}{KB} = \frac{DL}{LC}$$

$$A\hat{P}B = B\hat{C}D$$

$$C\hat{Q}D = A\hat{B}C$$

Th: $PQBC$ conciclici

$EPFQ$ è ciclico.

$\Downarrow$

$$E\hat{P}Q = E\hat{F}Q$$

AD, BC, KL concorrono in T.

Supponiamo che $EF \parallel AB \parallel CD$

$\Downarrow$ $E\hat{F}Q = D\hat{C}Q \implies Q\hat{C}B =) - \cdot =$

$= Q\hat{P}B$

Devo dim $EF \parallel AB$

Scrivo Menelao per $\overset{\triangle}{AP\Gamma}$ e DQ

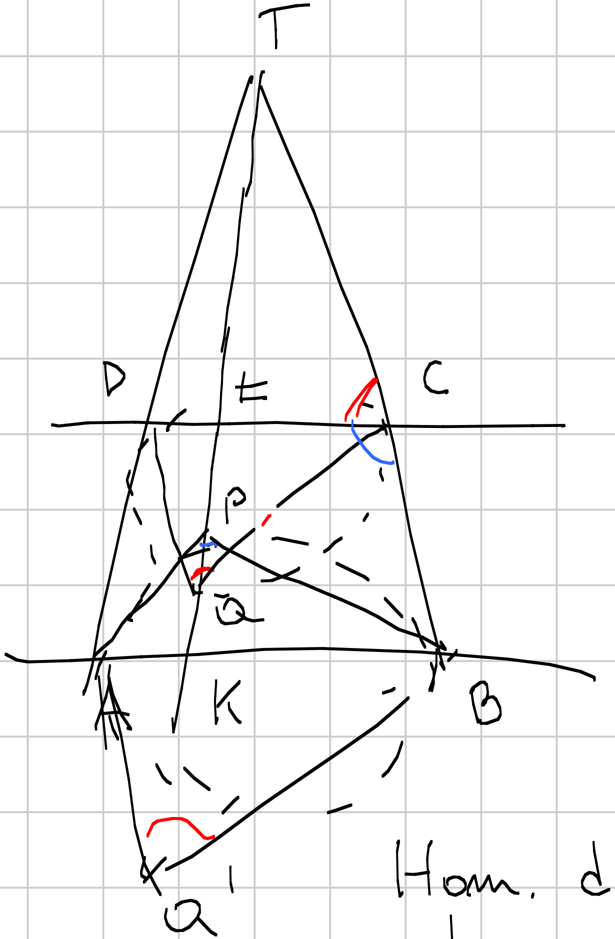
per $\overset{\triangle}{P\Gamma B}$ e QC

$$\cancel{\frac{\Gamma D}{DA}} \cdot \frac{AE}{EP} \cdot \cancel{\frac{PQ}{Q\Gamma}} = 1$$

$$\cancel{\frac{\Gamma C}{CB}} \cdot \frac{BF}{FP} \cdot \cancel{\frac{PQ}{\Gamma Q}} = 1$$

$$\Rightarrow \frac{AE}{EP} = \frac{BF}{FP}$$

$$\Rightarrow EF \parallel BA \quad \square$$



$\hat{TCD} = \hat{CQD}$

$\Longrightarrow$ retta TC é tangente alla circonferenza per DCQ

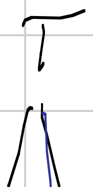
retta Tc é tangente alla circonferenza per APB

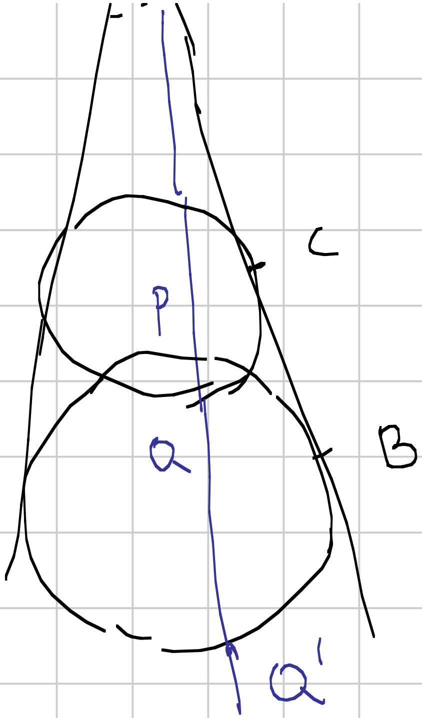
Hom. da T che manda $C \to B$

$D \to A$

$Q \to Q'$

circ. DCQ $\longrightarrow$ circ. ABQ'





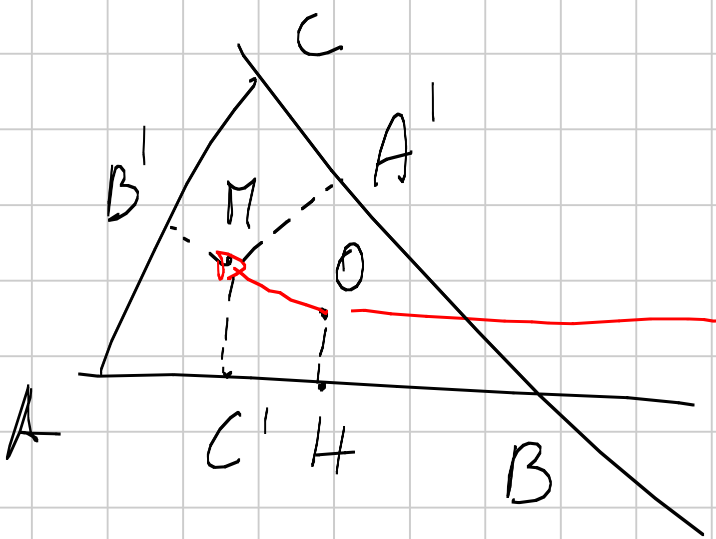
Circ. DCQ $\to$ ABP

CB PQ ciclico

$TC \cdot TB = TP \cdot TQ$

$$\frac{TC}{TQ} = \frac{TB}{TQ'}$$

$TP \cdot TQ' = TB^2$



$$f(M) = AC' + BA' + CB'$$

$$\text{Th}: \ f(M) = f(N) \iff MN \parallel IO$$

$$M \neq N$$

Metto l'origine in O.

$$C'H = \vec{M} \cdot \frac{(\vec{B} - \vec{A})}{c} \qquad AC' = \frac{c}{2} + \vec{M} \cdot \frac{(\vec{B} - \vec{A})}{c}$$

$$BA' = \frac{a}{2} + \vec{M} \cdot \frac{(\vec{C} - \vec{B})}{a}$$

$$\vec{v} = \frac{\vec{B} - \vec{A}}{c} + \frac{\vec{C} - \vec{B}}{a} + \frac{\vec{A} - \vec{C}}{b}$$

$$CB' = \frac{b}{2} + \vec{M} \cdot \frac{(\vec{A} - \vec{C})}{b}$$

$$f(M) = \frac{a+b+c}{2} + \vec{M} \cdot \vec{v}$$

$$f(M) = f(N) \iff \vec{M} \cdot \vec{v} = \vec{N} \cdot \vec{v}$$

$$\iff (\vec{M} - \vec{N}) \cdot \vec{v} = 0 \iff \vec{M} - \vec{N} \perp \vec{v}$$

$$\vec{I} = \frac{a\vec{A} + b\vec{B} + c\vec{C}}{a+b+c}$$

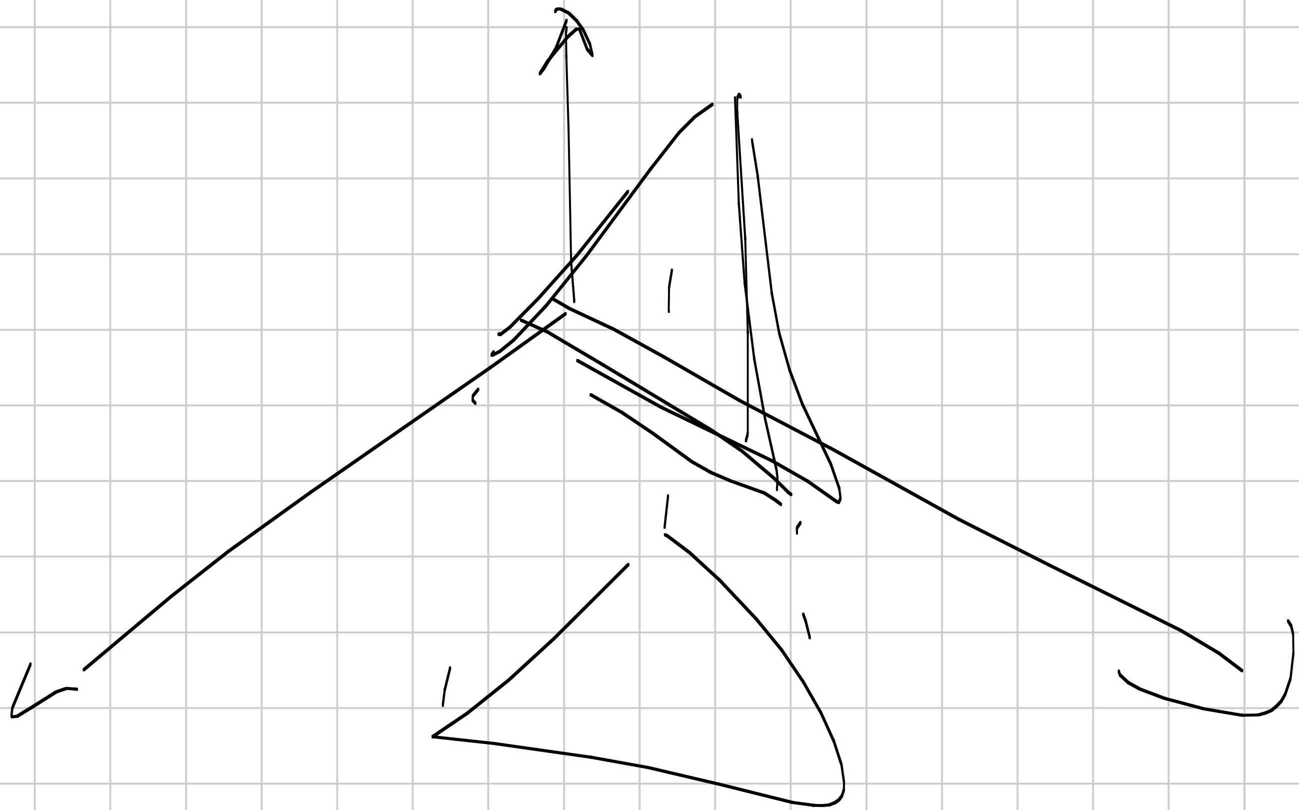
$$\vec{A} \cdot \vec{A} = \vec{B} \cdot \vec{B} = \vec{C} \cdot \vec{C} = R^2$$

$$c^2 = (\vec{A} - \vec{B}) \cdot (\vec{A} - \vec{B}) = 2R^2 - 2\vec{A} \cdot \vec{B}$$

$$(a\vec{A} + b\vec{B} + c\vec{C}) \cdot \left(\frac{\vec{B} - \vec{A}}{c} + \frac{\vec{C} - \vec{B}}{a} + \frac{\vec{A} - \vec{C}}{b} \right) =$$

$$= \ldots = 0 \implies IO \perp \vec{v} \implies \text{ok.}$$

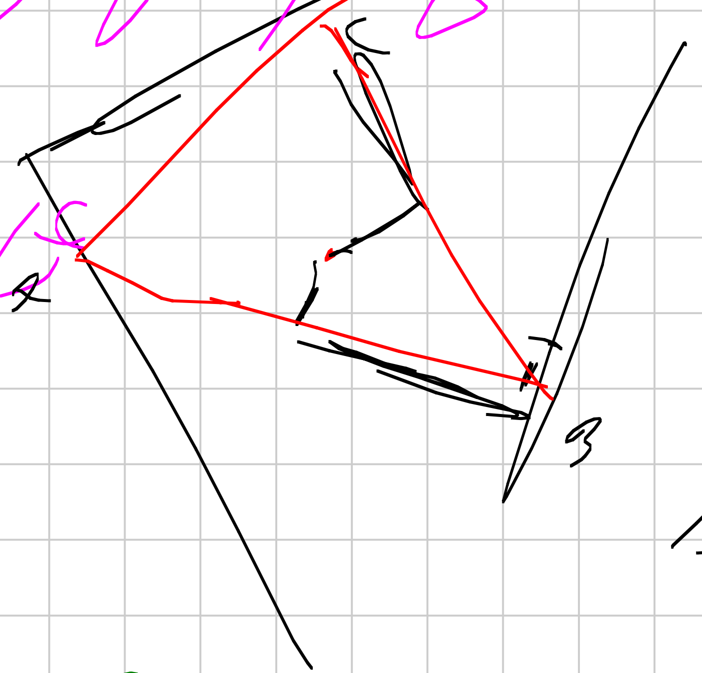
Oppure si verifica che $f(0) = f(I)$

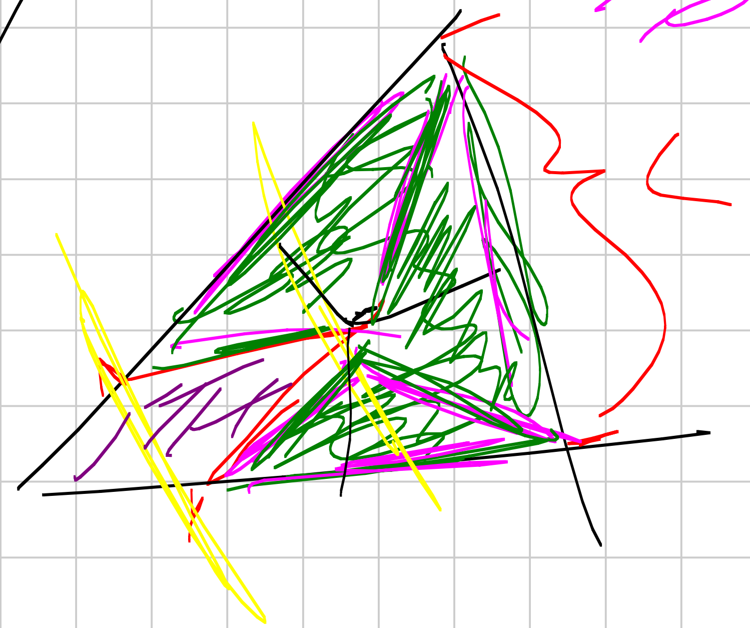


TANTO LO DIC CI

SCRIVETO IL AZO

IL NOME

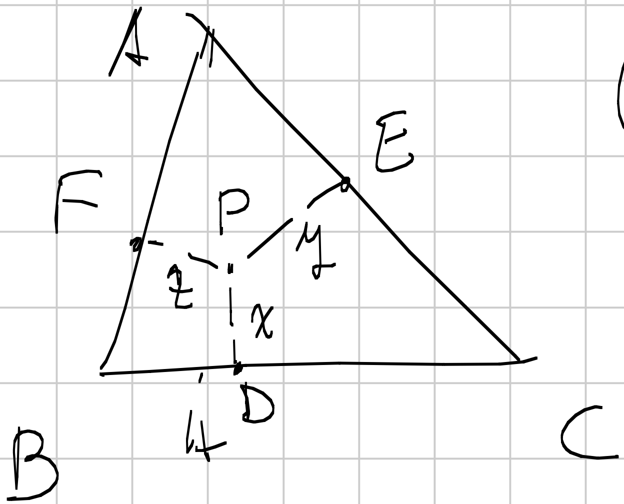




$$\frac{(\angle \alpha)\cdot L}{2} = \vartheta$$

$$\vartheta = \pi - \alpha$$

Coordinate Trilineari



$(x : y : z)$

$[x, y, z]$

Se $AH = 2$

$(3 : 0 : 0)$

$(1 : 0 : 0)$

$(1000 : 0 : 0)$

$PD \cdot BC + PE \cdot AC + PF \cdot AB = 2S$

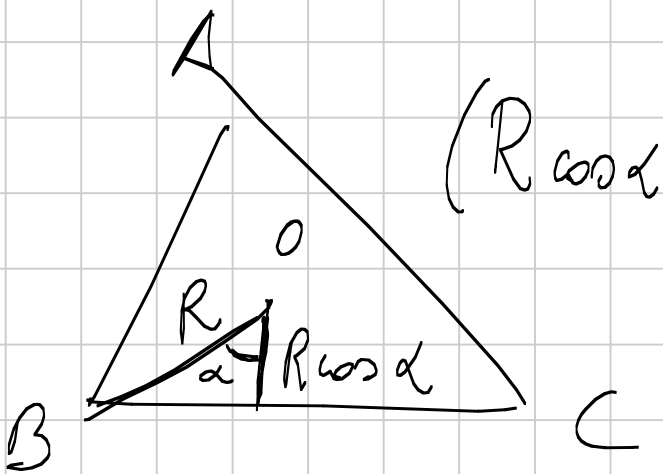
$ax + by + cz = 2S$

Incentro : $(r : r : r) \simeq (1 : 1 : 1)$

I vertici : $(1 : 0 : 0)$, $(0 : 1 : 0)$, $(0 : 0 : 1)$

I piedi delle bisettrici : $(1 : 1 : 0)$, $(0 : 1 : 1)$, $(1 : 0 : 1)$

Circocentro :



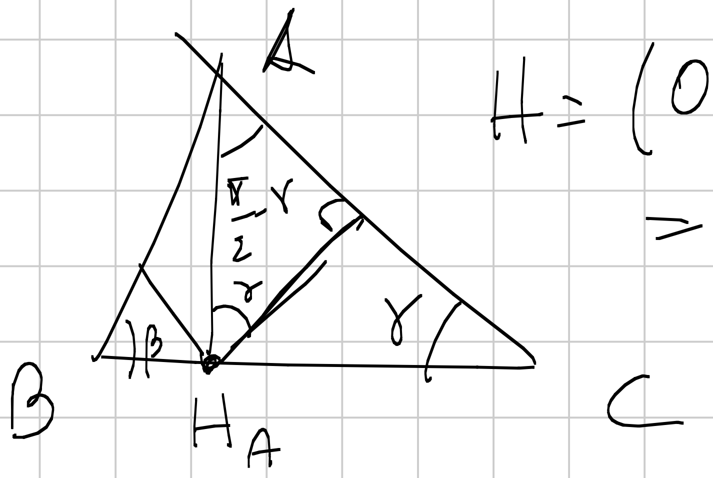
$(R\cos\alpha : R\cos\beta : R\cos\gamma)$
$\wr\wr$
$(\cos\alpha : \cos\beta : \cos\gamma)$

Baricentro : $\left(\frac{1}{a}, \frac{1}{b}, \frac{1}{c}\right)$

$$\| $$

$$(bc, ac, ab)$$

I piedi delle altezze :

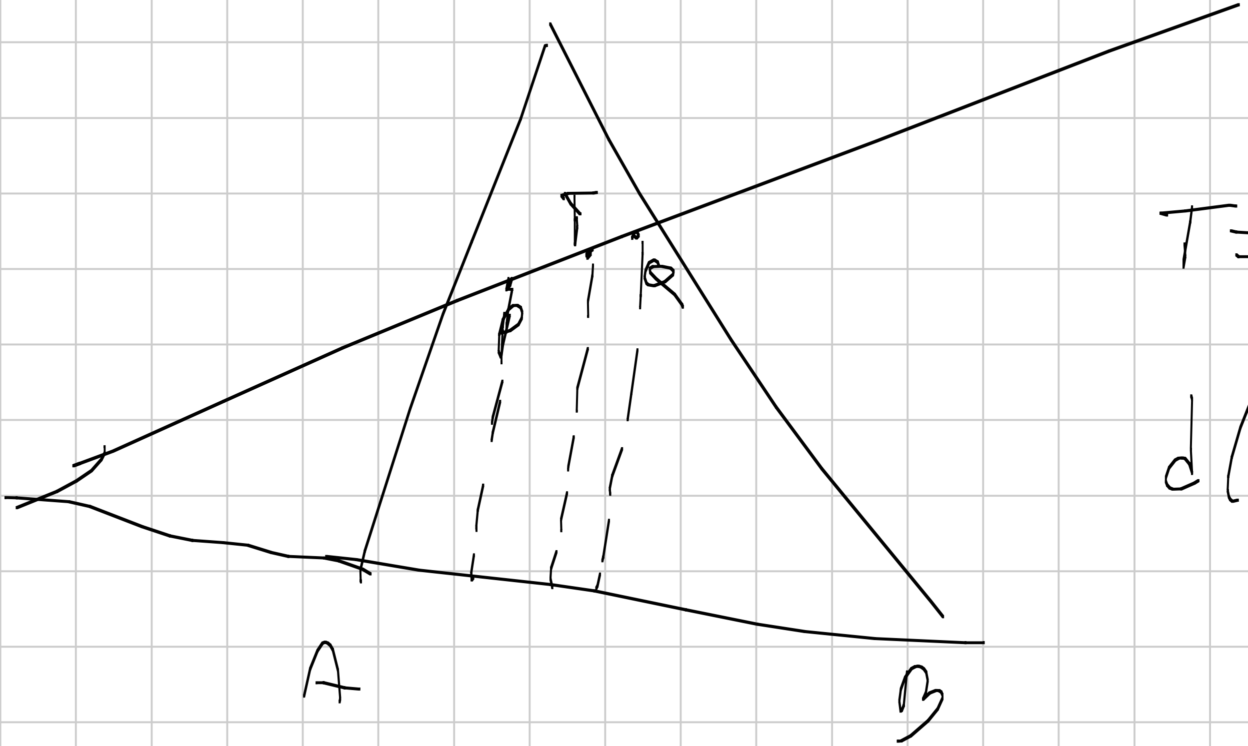


$H = (0 : AH \cdot \cos\gamma : AH \cos\beta) =$

$= (0 : \cos\gamma : \cos\beta)$

$H_B = (\cos\gamma : 0 : \cos\alpha)$ $\qquad$ $H_C = (\cos\beta : \cos\alpha : 0)$

$\{(x:y:z) \mid px + qy + rz = 0\}$ è una retta.



$$T = \lambda P + (1-\lambda) Q$$

$$d(T, AB) = \square \cdot d(P, AB) + \triangle \cdot d(Q, AB)$$

retta per $(1:1:0)$ e $(1:0:1)$

$px+qy+rz=0$

$x-y-z=0$

$$\begin{cases} p+q=0 \\ p+r=0 \end{cases} \qquad \begin{aligned} p&=-q \\ p&=-r \end{aligned}$$

$$\boxed{\cos\alpha - \cos\beta - \cos\gamma = 0} \iff O \in \text{piedi bisettrici}$$

retta per $(\cos\beta : \cos\alpha : 0)$ $(\cos\gamma : 0 : \cos\alpha)$

$p\cos\beta + q\cos\alpha = 0 \qquad p\cos\gamma + r\cos\alpha = 0$

$p = -\cos\alpha \quad q = \cos\beta \quad r = \cos\gamma$

$-\cos\alpha\, x + \cos\beta\, y + \cos\gamma\, z = 0$

$\boxed{-\cos\alpha + \cos\beta + \cos\gamma = 0} \iff I \in$ piedi altezze

$P = (x ; y ; z)$

$Q =$ con. isog. di $P = \left(\frac{1}{x} ; \frac{1}{y} ; \frac{1}{z}\right)$

$ax + by + cz = 0$