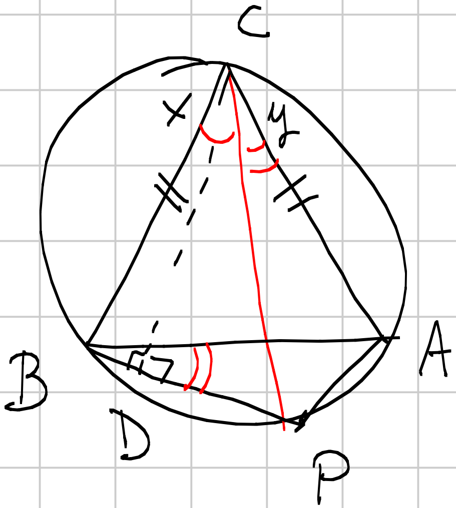
PREMIO 2008 – GEOMETRIA – MATTINA

Titolo nota 20/05/2008

(1)



$$PA + PB = 2PD$$

$$PA = 2R \sin y$$

$$PB = 2R \sin x$$

$$C\hat{B}P = \frac{\pi}{2} - \frac{x+y}{2} + y = \frac{\pi}{2} + \frac{y}{2} - \frac{x}{2}$$

$$C\hat{P}B = \frac{\pi}{2} - \frac{y}{2} - \frac{x}{2}$$

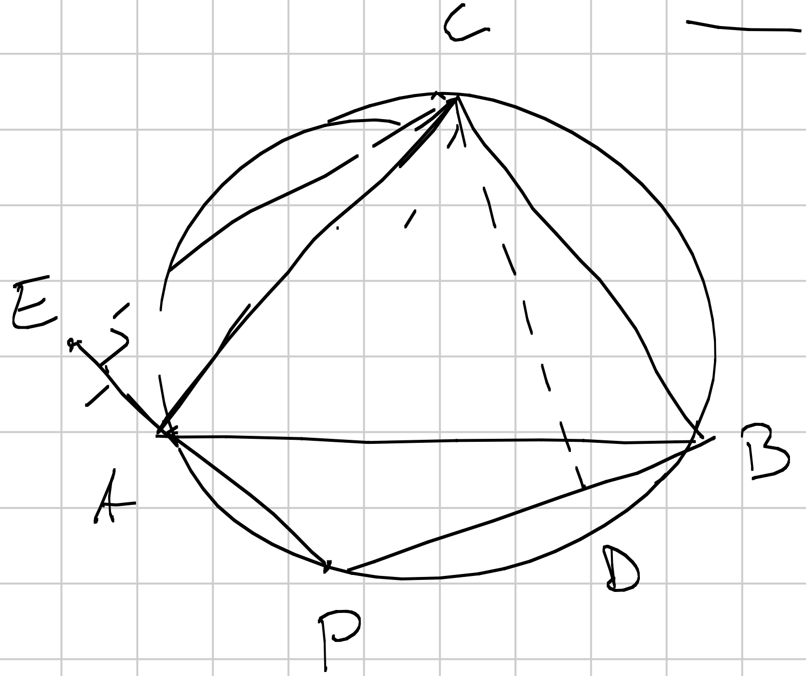
$$PD = CP \cdot \cos(C\hat{P}B) =$$

$$= 2R \sin(C\hat{B}P) \cos(C\hat{P}B) =$$

$$= 2R \cos\left(\frac{x}{2} - \frac{y}{2}\right) \sin\left(\frac{x}{2} + \frac{y}{2}\right) =$$

$$= 2R\left(\cos\frac{x}{2}\cos\frac{y}{2} + \sin\frac{x}{2}\sin\frac{y}{2}\right)\left(\sin\frac{x}{2}\cos\frac{y}{2} + \sin\frac{y}{2}\cos\frac{x}{2}\right) =$$

$$= R(\sin x + \sin y)$$



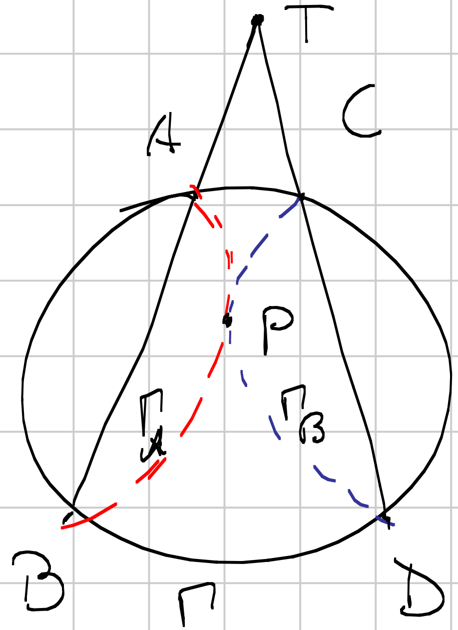
$EA = BD \qquad PE = PD$

$PA = PE - EA$

$PB = PD + DB$

$2PD$

2



α β

P

AB = asse radicale di Γ e Γ_α

CD = asse radicale di Γ e Γ_β

$\Downarrow$

T centro radicale.

$$pow_{\Gamma_\alpha}(T) = TA \cdot TB = TP^2$$

$$\parallel$$

$$pow_{\Gamma_\beta}(T) = TC \cdot TD = TP^2$$

$$\parallel$$

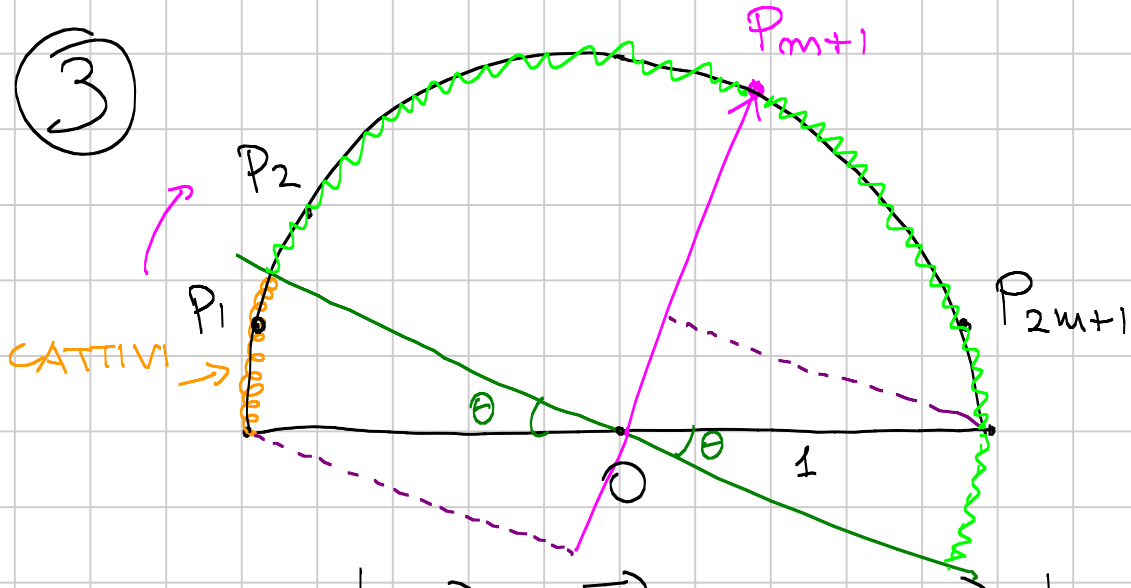
$$pow_{\Gamma}(T)$$

$$\left\{ P \text{ t.c. } TP = \sqrt{pow_\Gamma(T)} \right\} = \omega$$

e meno le intersezioni con le rette AB e CD.

$A, B, P,$ $\quad C, D, P$

③



$n = 2m+1$

n dispari serve (altrimenti li piglio a 2 a 2 simm. asse verticale)

<u>Tesi</u> $\left| \vec{OP_1} + \vec{OP_2} + \dots + \vec{OP_n} \right| \geq 1$ $\vec{S}$ = vettore somma

<u>Tesi vera</u> $\vec{S} \cdot \vec{OP_{m+1}} \geq 1$ così avrei finito, perché

$$1 \leq \vec{S} \cdot \vec{OP_{m+1}} \underset{\text{C.S.}}{\leq} |\vec{S}| \cdot \underbrace{|\vec{OP_{m+1}}|}_{=1}$$

$$\vec{u} \cdot \vec{v} = |\vec{u}| \cdot |\vec{v}| \cdot \cos\varphi$$

$$\vec{u} \cdot \vec{u} = |\vec{u}|^2$$

$$\vec{S}\cdot\vec{OP}_{m+1} = (\ldots)\cdot\vec{OP}_{m+1}$$

$$= \vec{OP}_1\cdot\vec{OP}_{m+1} + \ldots + \underbrace{\vec{OP}_{m+1}\cdot\vec{OP}_{m+1}}_{1} \underbrace{+ \ldots + \vec{OP}_{2m+1}\cdot\vec{OP}_{m+1}}_{\text{parte dx}}$$

i cattivi stanno qui

$$\vec{OP}_k\cdot\vec{OP}_{m+1} = \underbrace{|\vec{OP}_k|}_{=1}\cdot\underbrace{|\vec{OP}_{m+1}|}_{=1}\cdot\cos\varphi_k = \cos\varphi_k \geq \cos(90^\circ-\theta)$$

$k > m+1$

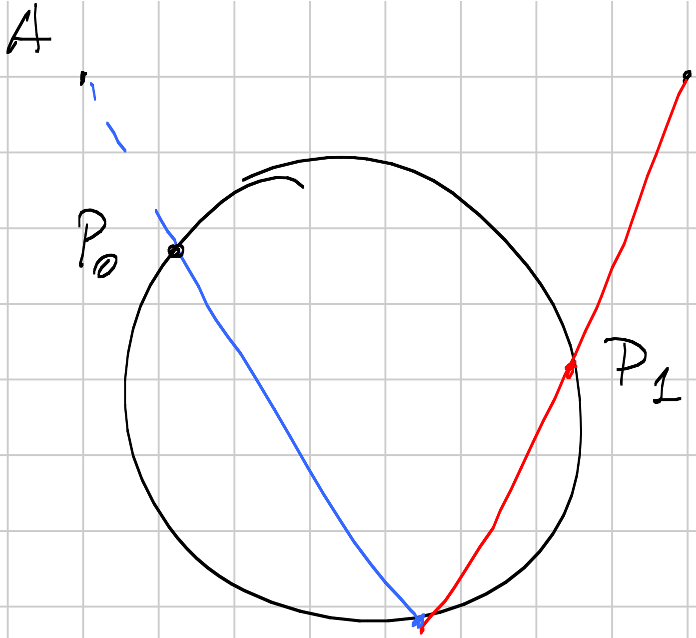
perchè $\varphi_k \leq 90^\circ-\theta$

$$\left.\begin{array}{l}\text{Parte dx} \geq m\cos(90^\circ-\theta)\\ \text{Parte sx} \geq -m\cos(90^\circ-\theta)\end{array}\right\}\ \text{sommando si ha la tesi}$$

$$\vec{OP}_k\cdot\vec{OP}_{m+1} = 1\cdot1\cdot\cos\varphi_k \geq -\cos(90^\circ-\theta)$$

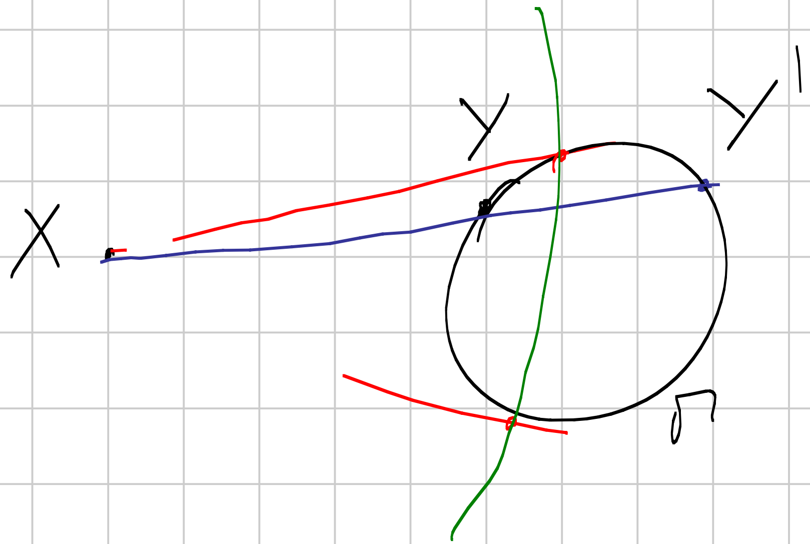
$k < m+1$

(4)



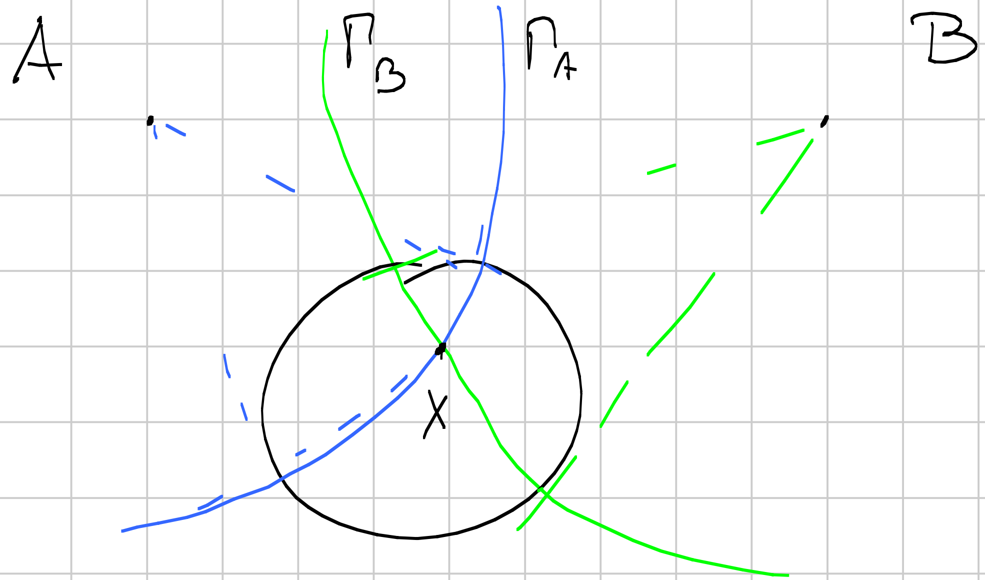
$P_0, P_1, \ldots$

$\exists\, k > 0$ t.c. $P_k = P_0$



$Y \longrightarrow Y'$

inv. risp. alla sf. con centro in X e ortogonale a Γ



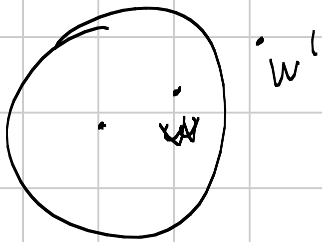
$P_0 \longrightarrow P_1$

$inv_{\Gamma_B}(inv_{\Gamma_A}(P_0)) = P_1$

Richiami

$\mathbb{C}$

$|z| = 1$



$$w' = \frac{1}{\overline{w}}$$

$z_0, R \qquad w' = z_0 + \frac{R^2}{\overline{w} - \overline{z_0}}$$

$z\bar{z} = 1$

$$\mathrm{pow}_\Gamma(A) = a\bar{a} - 1$$

$$\mathrm{pow}_\Gamma(B) = b\bar{b} - 1$$

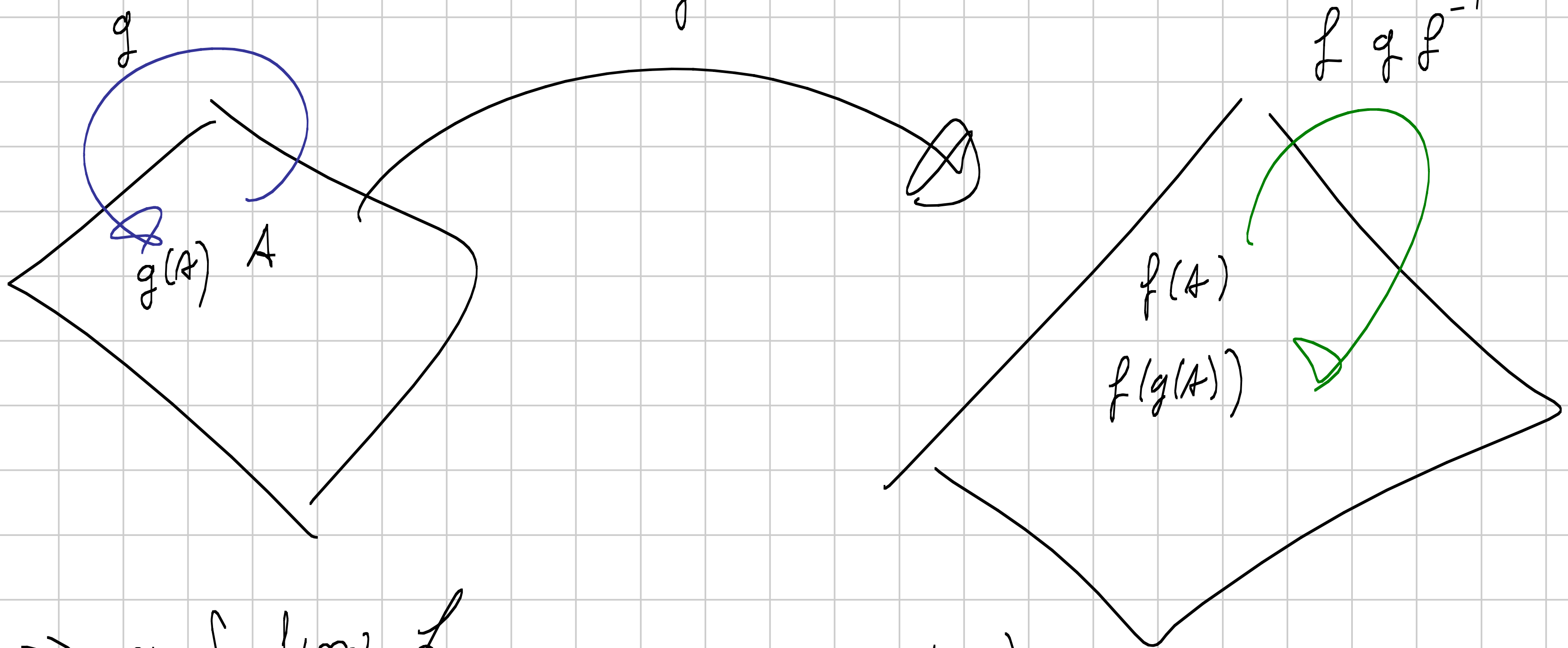
$$\mathrm{inv}_{\Gamma_A} : w \longrightarrow a + \frac{a\bar{a} - 1}{\bar{w} - \bar{a}} = v$$

$$\mathrm{inv}_{\Gamma_B} : v \longrightarrow b + \frac{b\bar{b} - 1}{\bar{v} - \bar{b}}$$

$$b + \frac{b\bar{b} - 1}{\bar{a} + \frac{a\bar{a} - 1}{w - a} - \bar{b}}$$

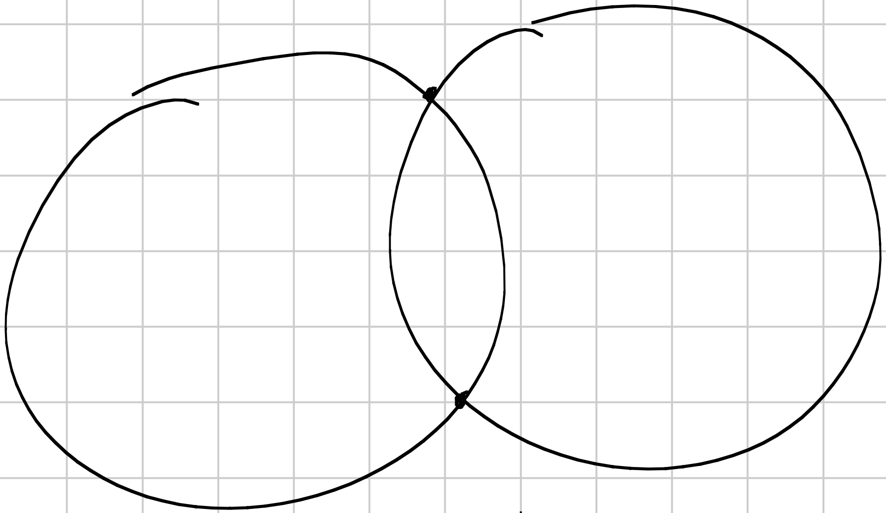
$P \longrightarrow Q$

$$h = f g f^{-1} \quad (f_* g)$$



$f(\mathcal{L})$

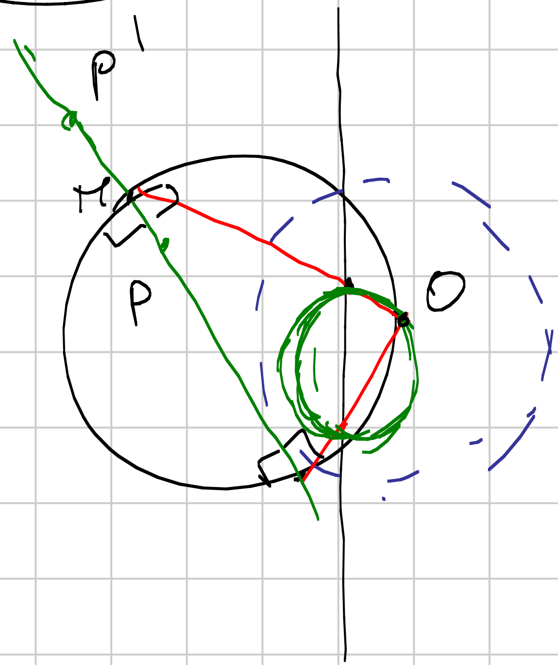
$g \Rightarrow$ punct. fix din $\mathcal{L}$



P, P' inversi risp a Γ

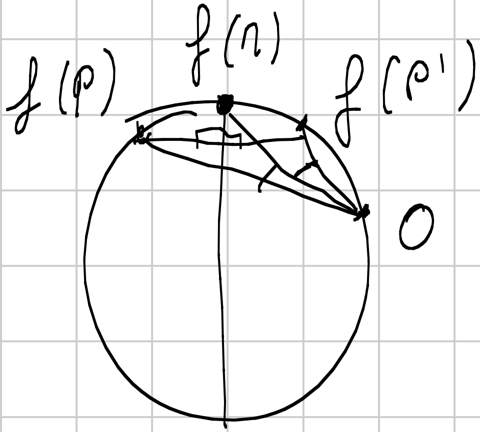
$\Downarrow$

$f(P), f(P')$ simm. risp. a $f(\Gamma)$



$\Pi\hat{O}P$

$\Pi\hat{O}P'$



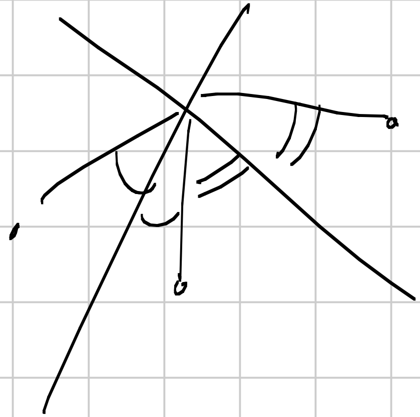
$$P_1 = \mathrm{inv}_{\Gamma_B}\left(\mathrm{inv}_{\Gamma_A}(P_0)\right)$$

$$f : \text{inv. di centro } X \in \Gamma_A \cap \Gamma_B$$

$$f \circ \mathrm{inv}_{\Gamma_B} \circ \mathrm{inv}_{\Gamma_A} \circ f^{-1} = \underbrace{f \circ \mathrm{inv}_{\Gamma_B} \circ f^{-1}} \circ \underbrace{f \circ \mathrm{inv}_{\Gamma_A} \circ f^{-1}} =$$

$$= \mathrm{sym}_{f(\Gamma_B)} \circ \mathrm{sym}_{f(\Gamma_A)} = \mathrm{rot}_T^{2\alpha}$$

α = angolo tra Γ_A, Γ_B



$P_0 \longrightarrow P_1$

$P_0 \longrightarrow P_k = \sigma_\Gamma^{2k\alpha}(P_0)$

$2k\alpha = 2m\pi$

— * —

fgf^{-1}

g inversione risp. Γ

f inv. di centro $P \in \Gamma$

fgf^{-1} simmetria risp a $f(\Gamma)$