PRE-IMO 2008 **GEOMETRIA** POMERIGGIO

Titolo nota 20/05/2008

ES. 5

HX

Lemma: $AN \parallel XM$

$\Delta X = MN$

$HX = \dfrac{HM}{\operatorname{tg} \delta}$

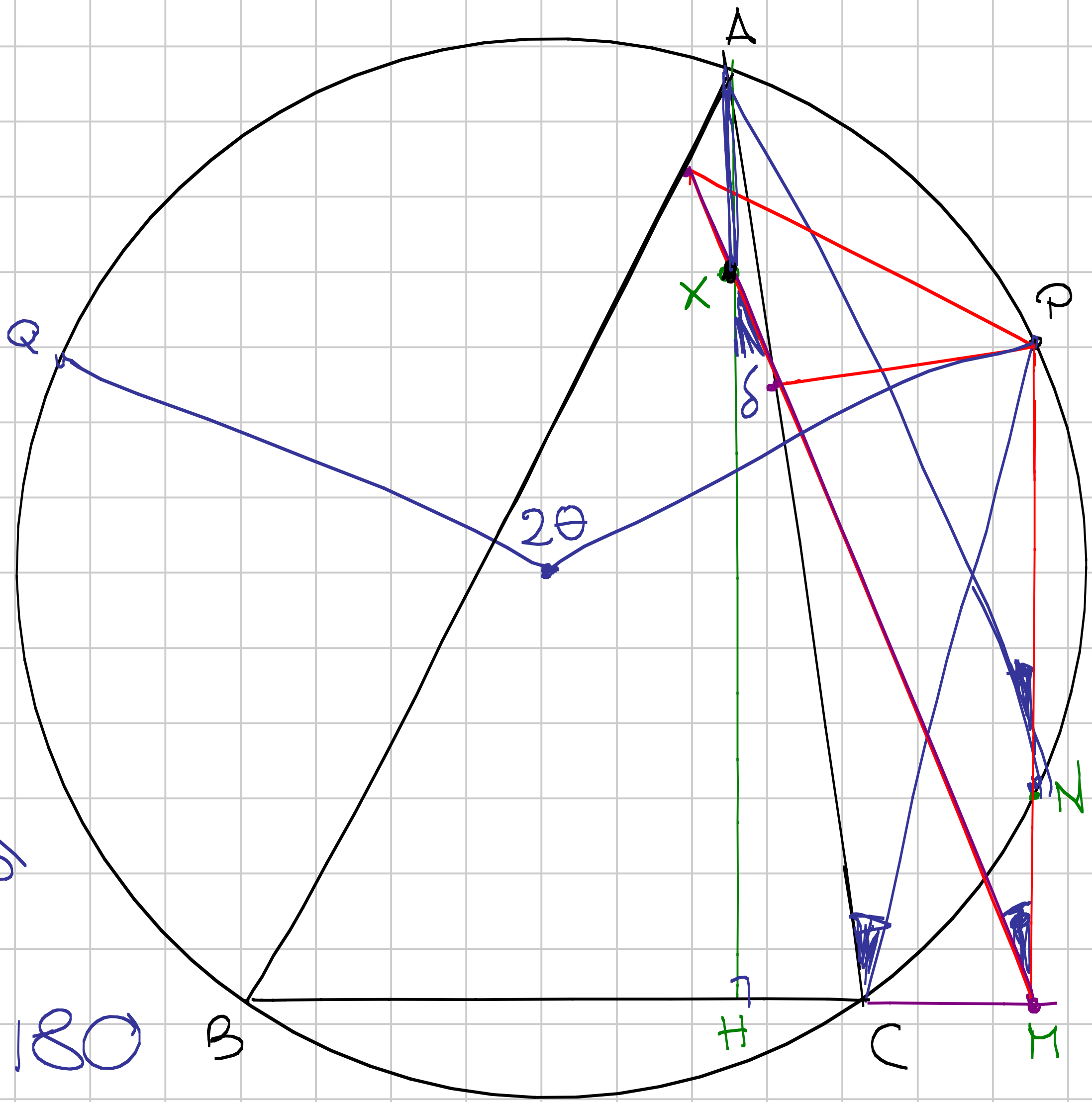
HM

δ

$\delta = \overset{\frown}{AP}$

$\overset{\frown}{CP} = \beta - \overset{\frown}{AP}$

$\alpha + \overset{\frown}{CP} + \theta + \overset{\frown}{CP} = 180$



$$\overset{\frown}{CP} = \frac{180-\alpha-\theta}{2} = 90-\frac{\alpha}{2}-\frac{\theta}{2}$$

$$\frac{\alpha+\beta+\gamma}{2} = 90$$

$$\overset{\frown}{CP} = \beta - \overset{\frown}{AP} = \frac{\theta+\beta-\gamma}{2}$$

$$CM = \frac{2R\sin\theta - a}{2}$$

$$HX =$$

$$HM = \frac{2R\sin\theta - a}{2} + b\cos\gamma$$

$$\beta-\gamma = X$$

$$\beta+\gamma = y$$

ES. 6

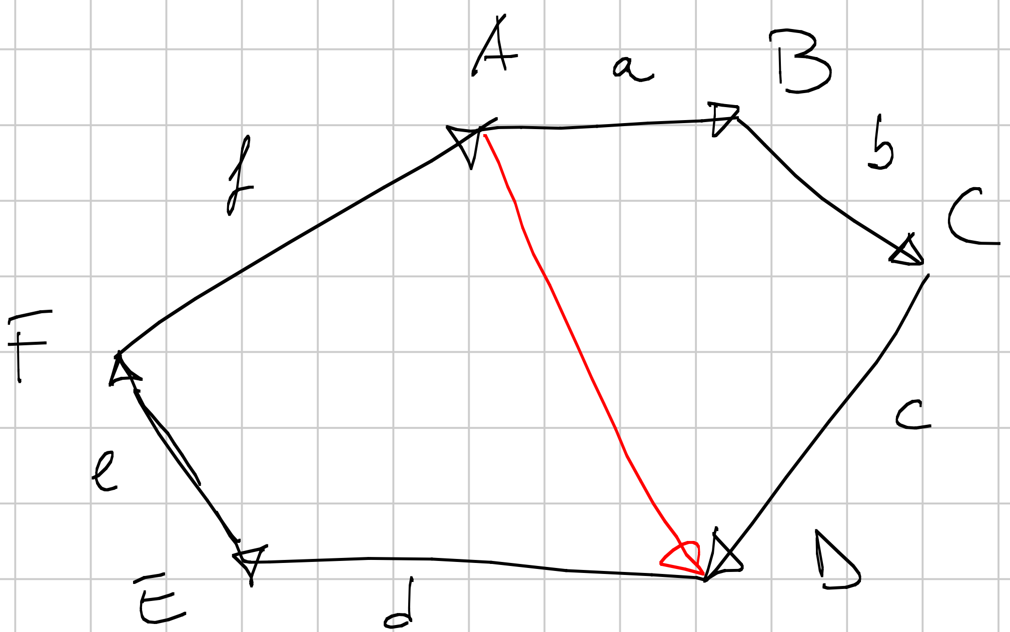
ABCDEF es. convesso

con $AD = BC + EF$

$BE = CD + FA$

$CF = AB + DE$

Dim che $\dfrac{AB}{DE} = \dfrac{EF}{BC} = \dfrac{CD}{FA}$



$a + b + c + d + e + f = 0$

$$\vec{AD} = \vec{a} + \vec{b} + \vec{c} = -\vec{d} - \vec{e} - \vec{f}$$

$$\vec{a} - \vec{d} = \vec{x}$$

$$\vec{e} - \vec{b} = \vec{y}$$

$$\vec{c} - \vec{f} = \vec{z}$$

$$\vec{AD} = \frac{1}{2}\left(\vec{a} + \vec{b} + \vec{c} - \vec{d} - \vec{e} - \vec{f}\right)$$

$$\left|\frac{\vec{x} - \vec{y} + \vec{z}}{2}\right| = |\vec{b}| + |\vec{e}| \geq |\vec{b} - \vec{e}| = |\vec{y}|$$

$$\vec{z} - \vec{y} + \vec{x} = \vec{m}$$

$$\left|\frac{\vec{y} - \vec{x} + \vec{z}}{2}\right| = |\vec{a}| + |\vec{d}| \geq |\vec{a} - \vec{d}| = |\vec{x}|$$

$$\vec{y} - \vec{x} + \vec{z} = \vec{e}$$

$$\left|\frac{\vec{x} + \vec{y} - \vec{z}}{2}\right| = |\vec{c}| + |\vec{f}| \geq |\vec{c} - \vec{f}| = |\vec{z}|$$

$$\vec{x} + \vec{y} - \vec{z} = \vec{k}$$

$|\vec{m}| \geq |\vec{e} + \vec{k}|$ e cicliche

$|\vec{m}|^2 \geq |\vec{e}|^2 + |\vec{k}|^2 + 2\,\vec{e}\cdot\vec{k}$ e cicliche

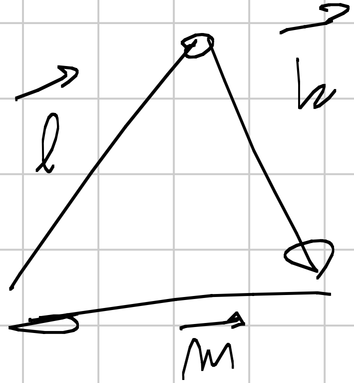
$$\cancel{|\vec{m}|^2 + |\vec{\ell}|^2 + |\vec{k}|^2} \geq \cancel{2}\left(|\vec{m}|^2 + |\vec{\ell}|^2 + |\vec{k}|^2\right) + 2\vec{\ell}\cdot\vec{k} + 2\vec{k}\cdot\vec{m} + 2\vec{\ell}\cdot\vec{m}$$

$$0 \geq |\vec{m} + \vec{\ell} + \vec{k}|^2 = |\vec{x} + \vec{y} + \vec{z}|^2$$

$\Rightarrow$ ① $\vec{x} + \vec{y} + \vec{z} = 0$

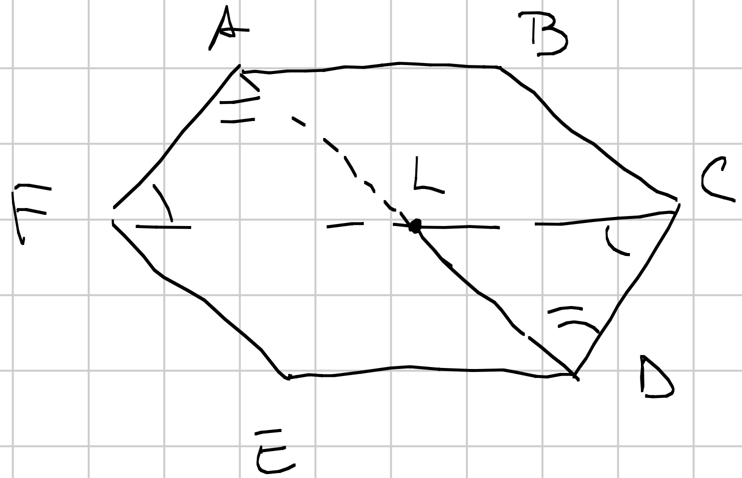
$$|\vec{b}| + |\vec{e}| = |\vec{b} - \vec{e}| \Rightarrow BC \parallel EF \text{ e così via.}$$

② $\vec{\ell} + \vec{k} + \vec{m} = 0$.



$$-\vec{\ell} = \vec{m} + \vec{k} = 2\vec{x} = 2(\vec{a} - \vec{d})$$

$CF \parallel AB \parallel DE$ e così via.

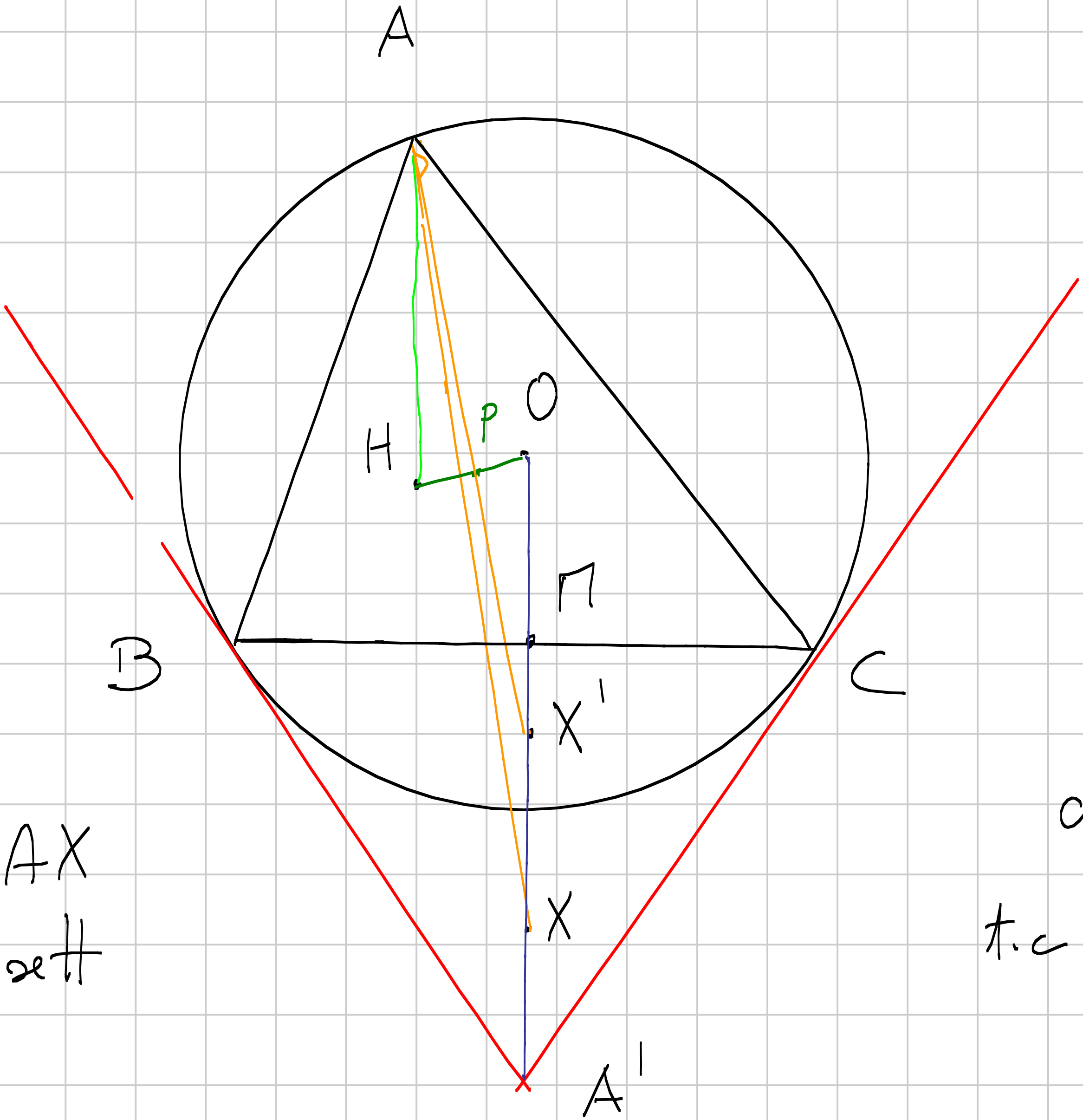


$$\begin{cases} FL = ED \\ LC = AB \end{cases}$$

$$\begin{cases} AL = BC \\ LD = FE \end{cases}$$

$$\frac{AB}{DE} = \frac{CL}{LF} = \frac{DC}{AF} = \frac{LD}{LA} = \frac{EF}{BC}$$

ES. 7



$$\frac{OX}{OA'} = t$$

AX, BY, CZ concorrono in P'

coniug. isog. di P

t.c. $\dfrac{OP}{PH} = \dfrac{1}{2t}$

AX' simm di AX risp. alla bisett

Oss: AHP e OPX' sono simili.

$$\frac{OP}{PH} = \frac{OX'}{HA} = \frac{PX'}{AP}$$

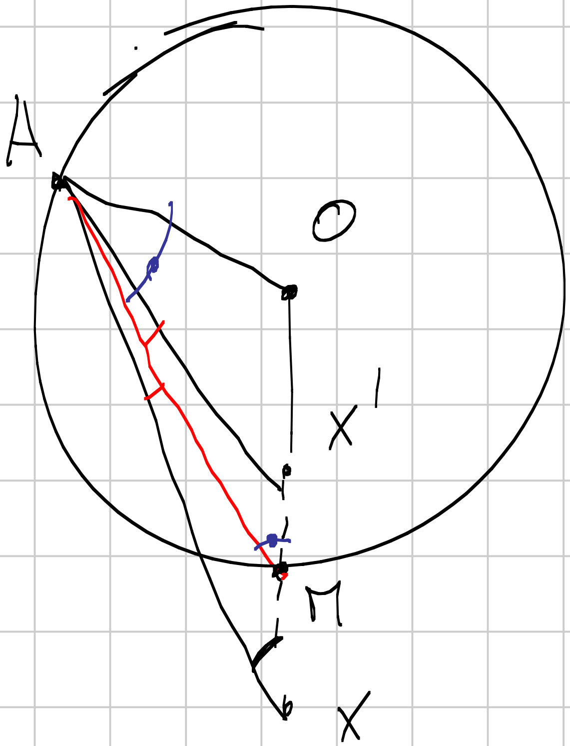
$$\frac{X'O}{OA} = \frac{OA}{OX} \Rightarrow OX \cdot OX' = OA^2$$

$$X'OA \simeq AOX$$

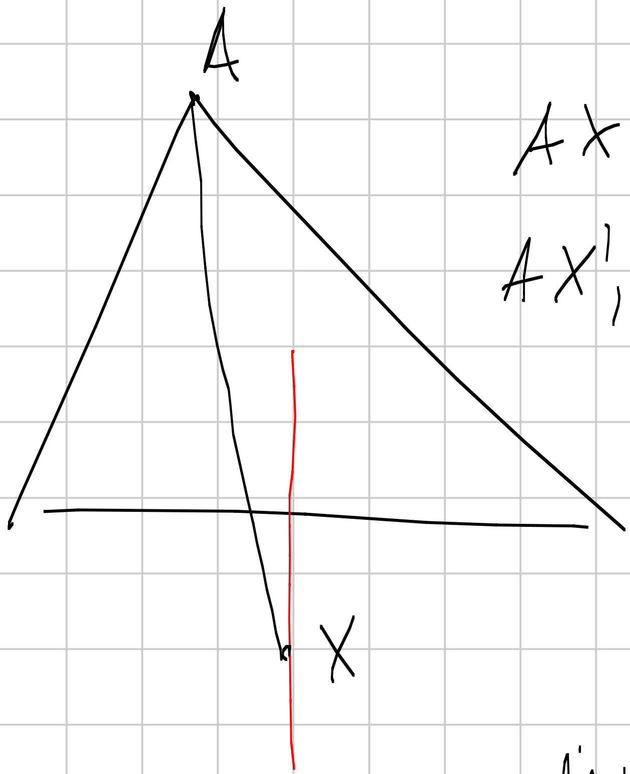
$$\widehat{AXO} = \widehat{A\Pi O} - \widehat{XA\Pi} =$$

$$= \widehat{A\Pi O} - \widehat{\Pi AX'} =$$

$$= \widehat{\Pi AO} - \widehat{\Pi AX'} = \widehat{OAX'}$$



Fatto + generale



$AX, BY, CZ \longrightarrow T$

$AX', BY', CZ' \longrightarrow T'$

con. isog. di T

X', Y', Z'

inversi di X, Y, Z nella circ. circ. ad ABC

Vogliamo calcolare $\frac{OX'}{AH}$

sappiamo che $\underbrace{OX' \cdot OX}_{} = R^2$ $\qquad \frac{OX}{OA'} = t$

$\underbrace{OX = t \cdot OA'}_{}$

$OM \cdot OA' = R^2$

$\underbrace{OA' = \frac{R^2}{OM}}_{}$ $\qquad \underbrace{t \cdot OX' \cdot OA' = R^2}_{}$

$t \cdot \frac{OX'}{OM} = 1 \Rightarrow \frac{OX'}{OM} = \frac{1}{t}$

$\frac{OX'}{AH} = \frac{1}{2t}$

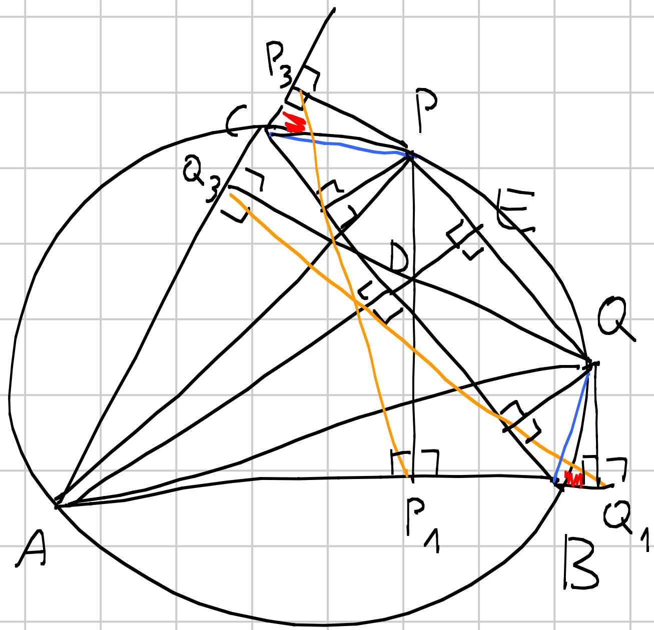
Fatto generale: $AH = 2 \cdot OM_A$

(5)

PP_3AP_1

AQ_1QQ_3

$P_3P_1Q_1Q_3$



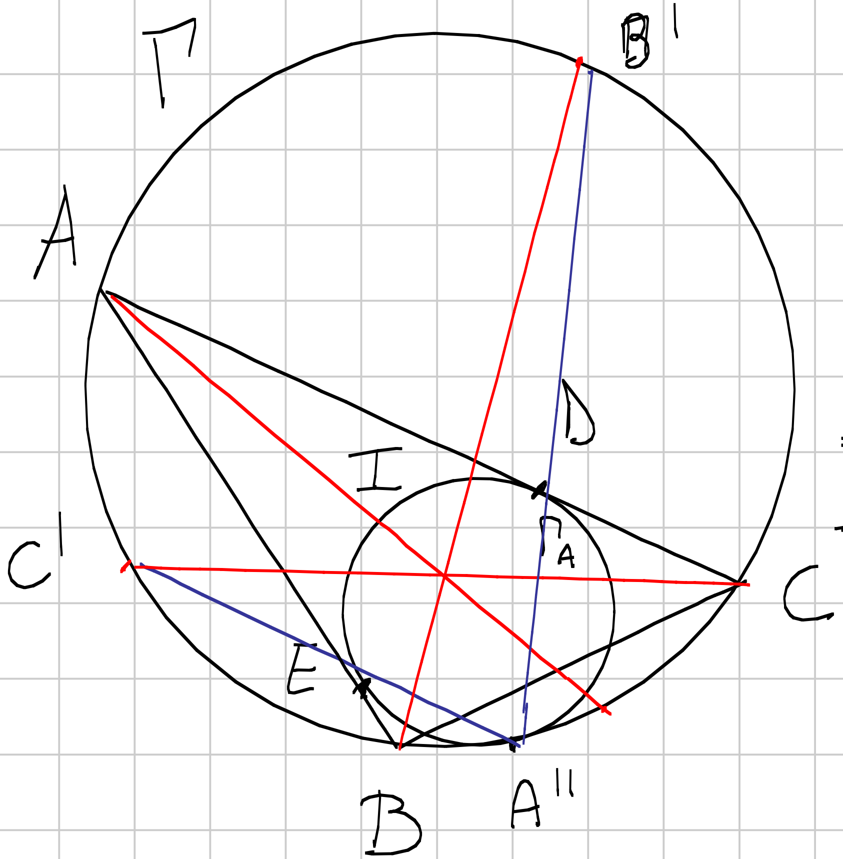
$$Q_3\hat{P}_3P_1 = P_1\hat{Q}_1Q_3$$

8 (a)

I incentro di ABC

$DE \perp AI$

$I \in DE$



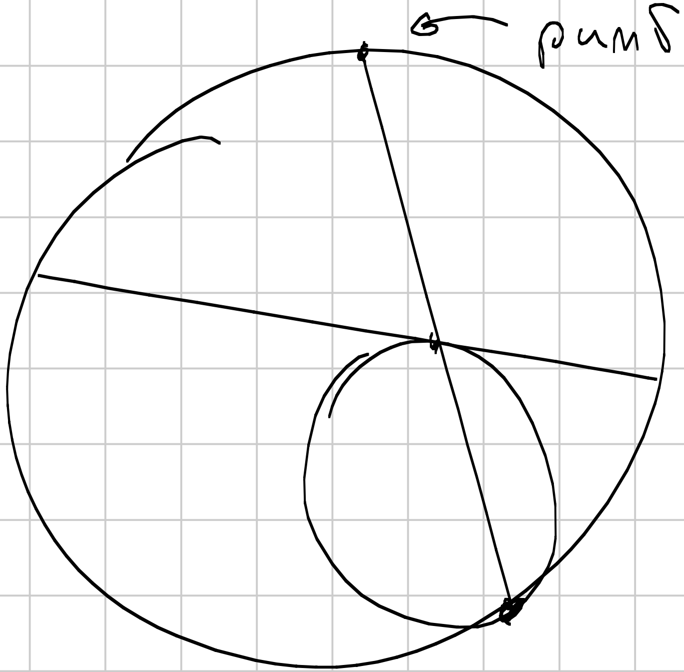
la cfr Γ_A tang.:

- ad AC in D
- ad AB in E

tange Γ.

$\Updownarrow$

Equiv.: se Γ_A tange Γ, AB, AC, passa per D, E

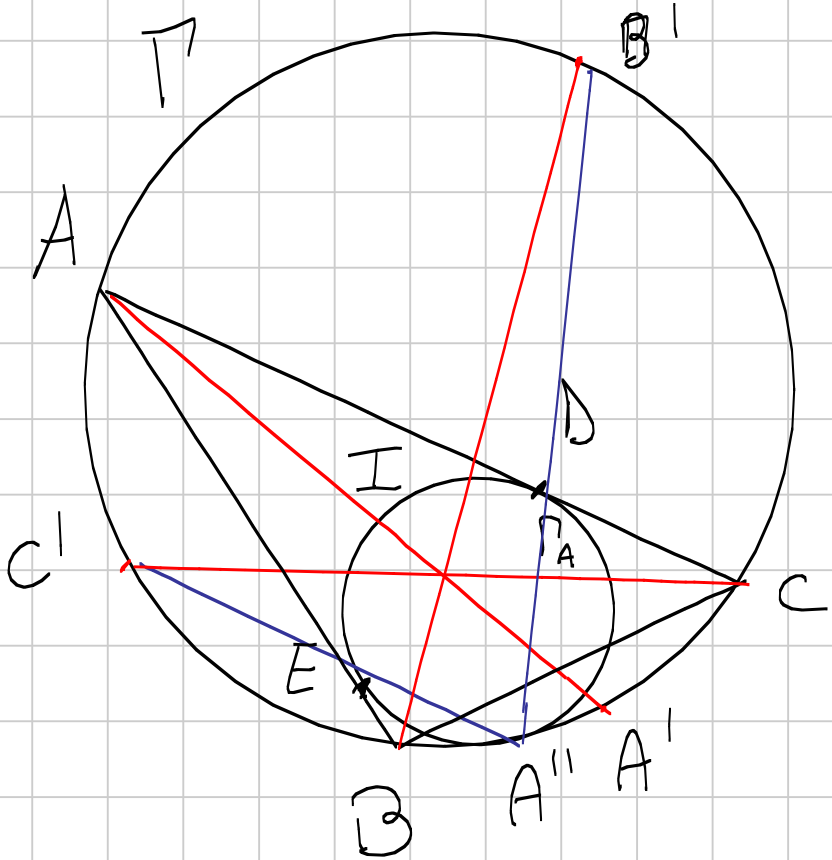


(homothety)

$\Downarrow$

A'', D, B' allineati

A'', E, C' allineati



$A''C'CABB'$

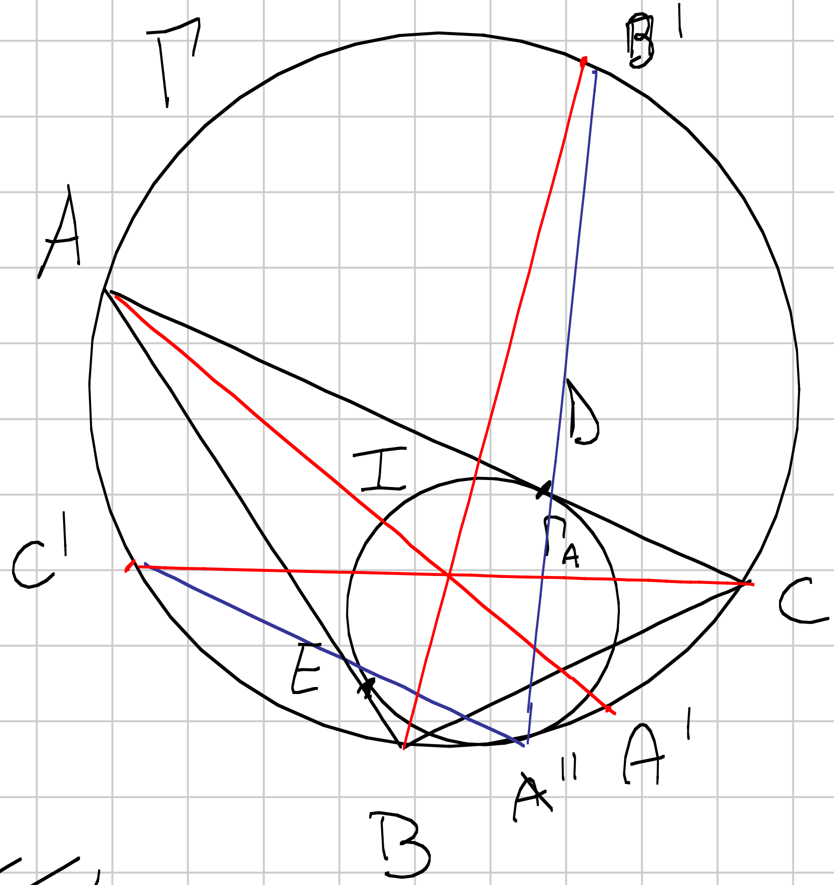
$\Rightarrow$ (Pascal)

$A''C' \cap AB = E$

$A''B' \cap AC = D$

$CC' \cap BB' = I$

sono allineati.



Invertendo in A

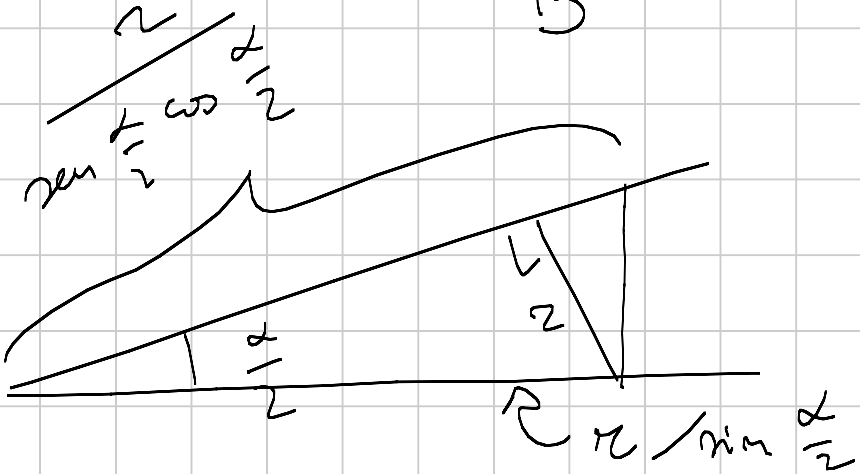
con raggio $\sqrt{b \cdot c}$

+ simm. risp. alla bisett. di A

1) $\{A, B, C\}$ fissi.

$\Gamma \longrightarrow BC$

$\Gamma_A \longrightarrow$ cfr. exinscritta opposta ad A



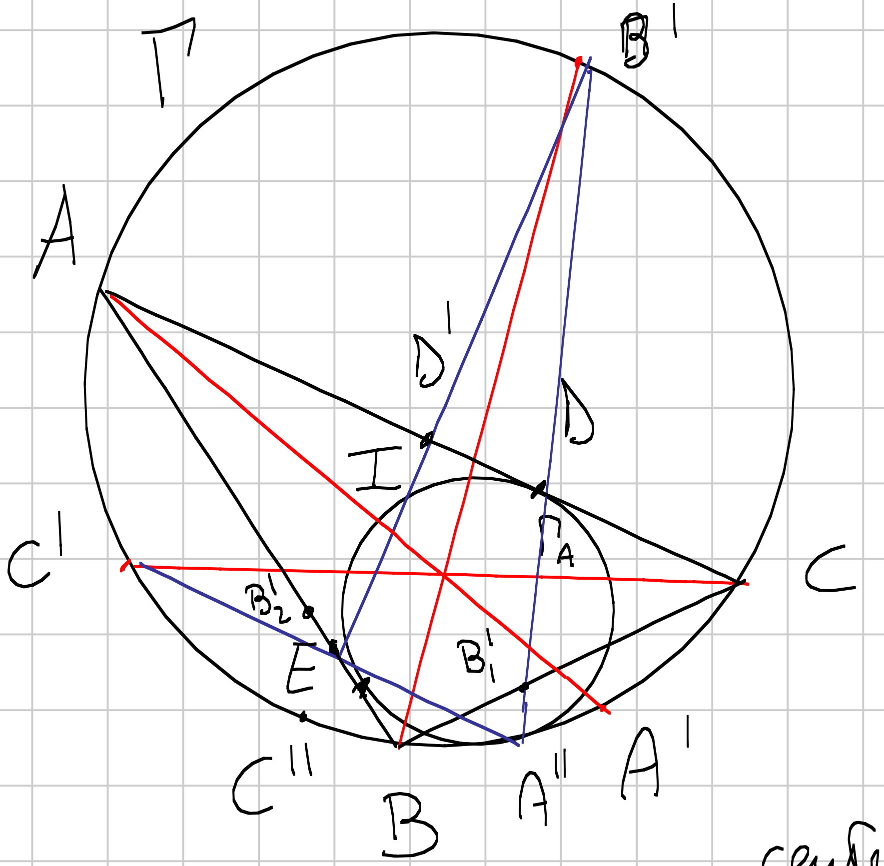
$$r_a = \frac{r}{\cos^2 \frac{\alpha}{2}}$$

$$\frac{r}{\operatorname{sen}\frac{\alpha}{2}\cos\frac{\alpha}{2}}\cdot\left(\frac{a+b+c}{2}\right) = \frac{\frac{\cancel{2}\Delta}{\cancel{a+b+c}}\cdot\frac{\cancel{a+b+c}}{\cancel{2}}}{\frac{\operatorname{sen}\alpha}{2}} =$$

$$= \frac{\frac{1}{2}bc\operatorname{sen}\alpha}{\frac{1}{2}\operatorname{sen}\alpha} = bc$$

(b) fatto dimostrando (a)

(c)



AA'' etc. concorrono

ω = cfr. inscritta

centro di similitudine tra ω e Γ_A = A

centro di sim. tra Γ_A e Γ = A''

$\Downarrow$

centro est. di sim tra ω e Γ $(S_e) \in AA''$

$\Rightarrow S_e = AA'' \cap BB'' \cap CC''$.

(d) $B_1' = CB \cap B'A''$

$B_2' = AB \cap B'C''$

$S_C = AA'' \cap CC''$

$AA''B'C''CB$

(Pascal)

sono allineati.