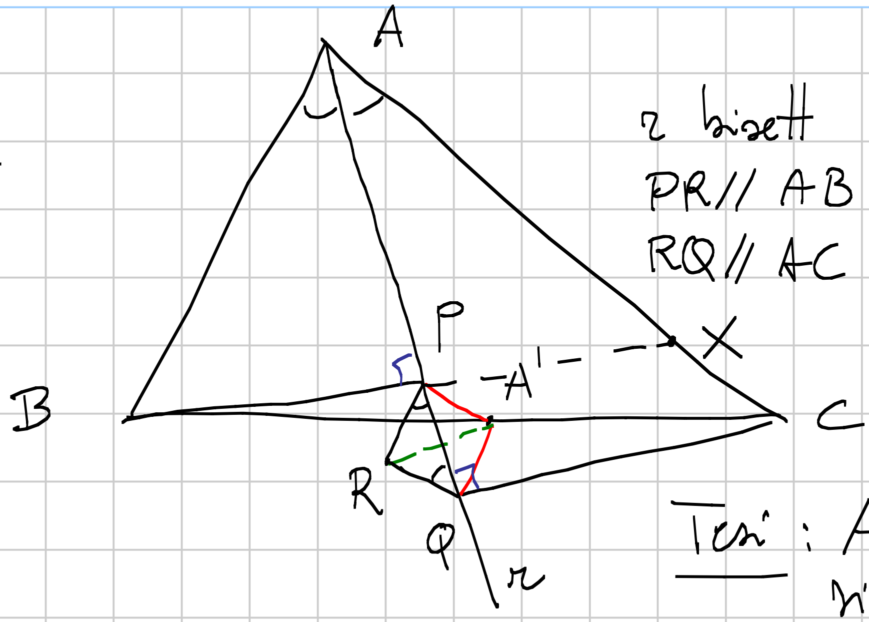
PRE IMO 2009 - GEOMETRIA - Mattino

Titolo nota 26/05/2009

Problema 1



r bisett

$PR \parallel AB$

$RQ \parallel AC$

Tesi: AR simmediana

A' punto medio di BC

AR e AA' sono simm. risp. a r

Se BP incontra AC in X, $BP = PX$

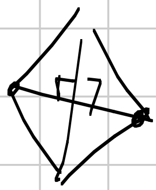
e $A'B = A'C \Rightarrow PA' \parallel CX = CA$

Allo stesso modo $A'Q \parallel BA$

$\Rightarrow PRQA'$ è un parallelogramma

Inoltre $B\hat{A}P = R\hat{P}Q$ e $P\hat{A}C = P\hat{Q}R$

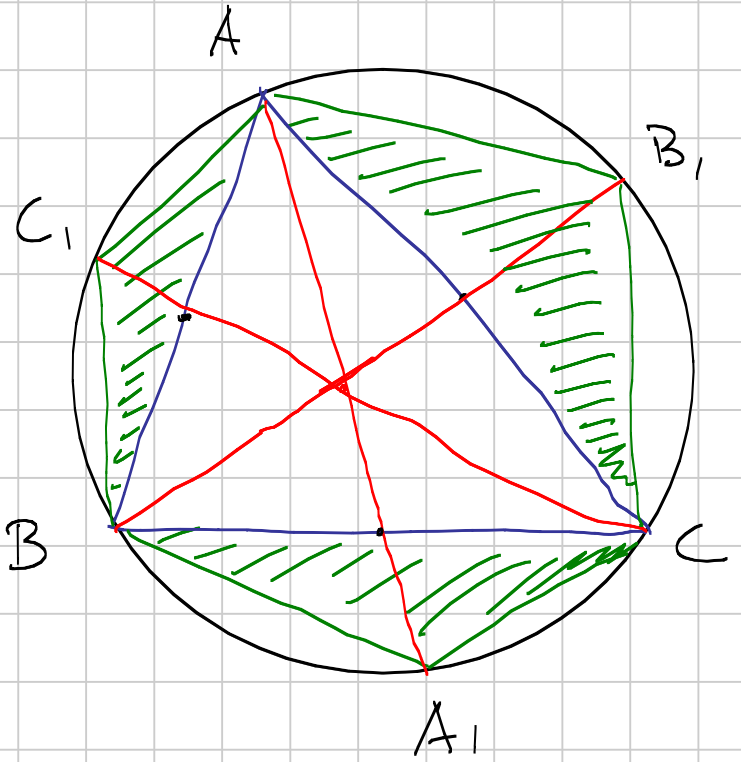
$\Rightarrow R\hat{P}Q = P\hat{Q}R \Rightarrow RPQ$ isoscele



$\Rightarrow PRQA'$ è un rombo.

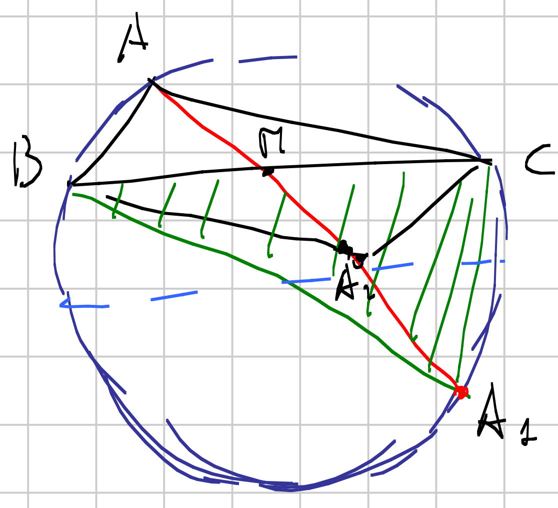
$\Rightarrow A'$ è simm di R risp. a PQ. $\square$

Problema 2



$$S_{ABC} \le S_{ABC_1} + S_{AB_1C} + S_{A_1BC}$$

I) ABC ottusangolo



A_2 simm di A risp. a M

ABA_2C è un parallelogramma

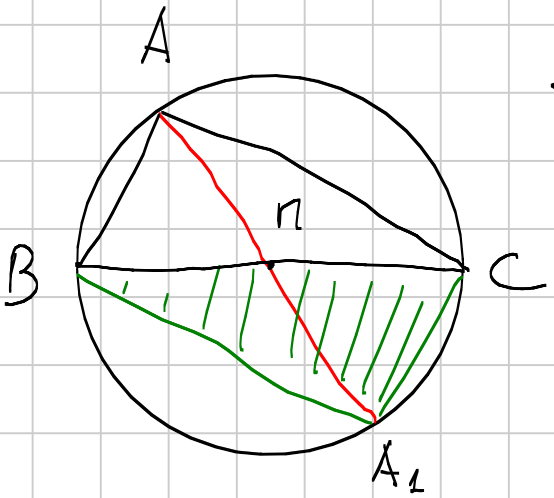
$\Rightarrow S_{ABC} = S_{A_2BC}$

Se A_2 sta tra M e A_1 abbiamo finito.

$B\hat{A}_2C = B\hat{A}C$ mentre $B\hat{A}_1C = \pi - B\hat{A}C$

$\Rightarrow B\hat{A}_1C < B\hat{A}_2C$

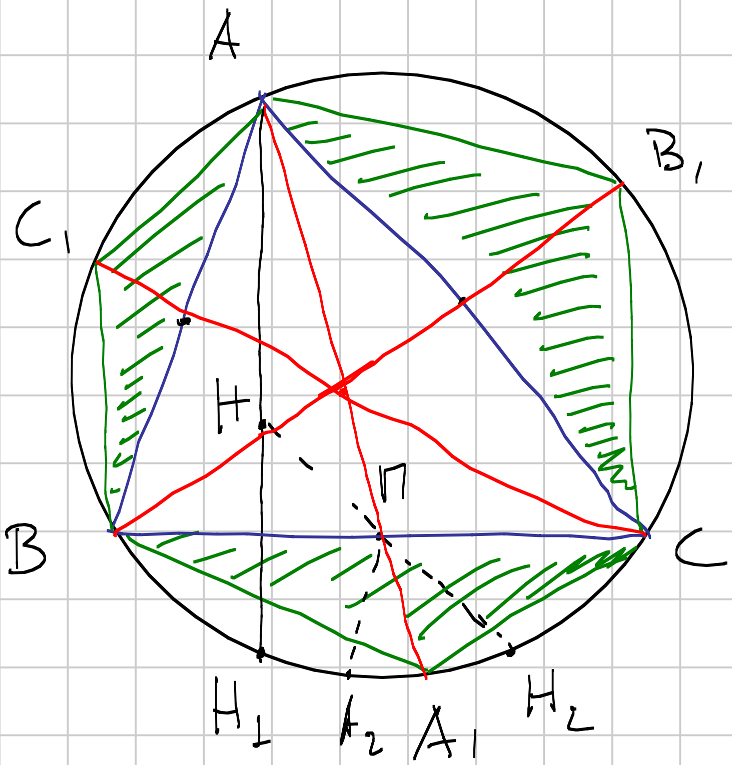
II) Rettangolo

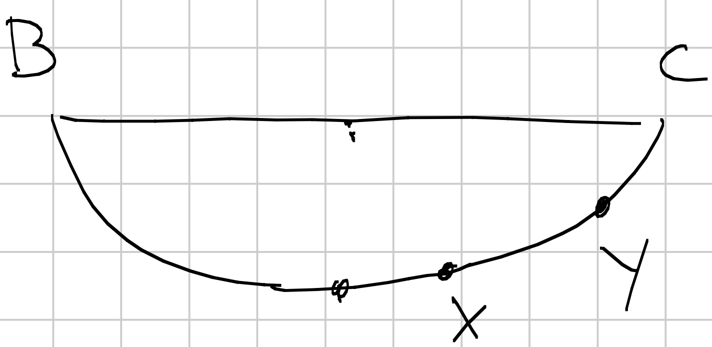


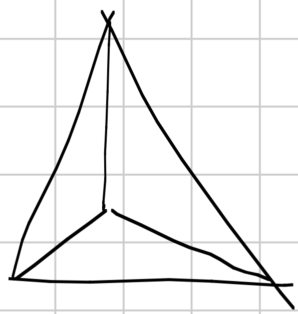
$$S_{ABC} = S_{A_1BC}$$

$\Rightarrow$ $S_{ABC} < S_{A_1BC}$ + roba positiva

III) Acutangolo

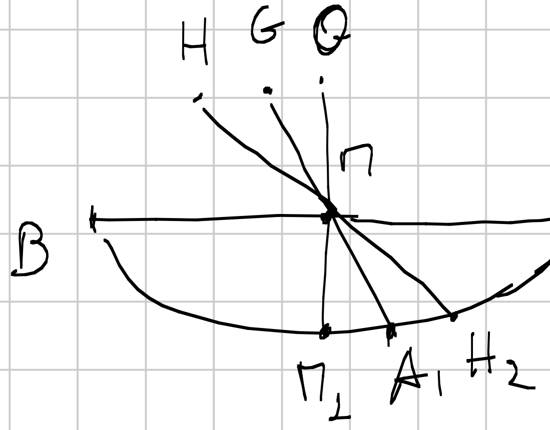






$$S_{BCY} < S_{BCX}$$

Prendo O, G, H (sono allineati)



$\Rightarrow A_1$ sta tra M_1 e H_2

$\Downarrow$

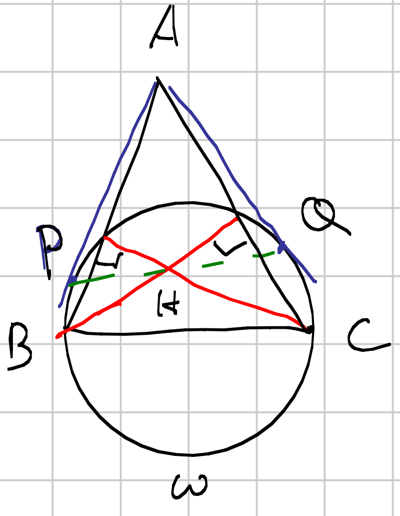
$$S_{A_1BC} > S_{H_2BC} = S_{HBC}$$

$$\Rightarrow S_{A_1BC} + S_{AB_1C} + S_{ABC_1} > S_{HBC} + S_{HAC} + S_{HAB} =$$

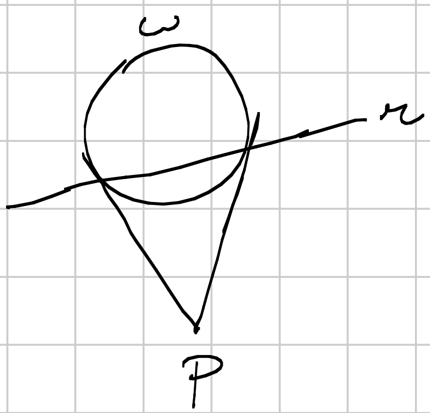
$$= S_{ABC}$$

Problema 3

Sol. di chi sa geom. proiettiva



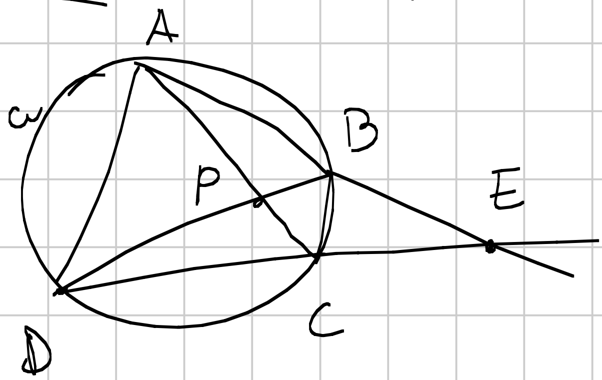
Tesi: P, H, Q allineati



$\text{pol}_\omega P = r =$ retta polare di P rispetto a ω

$H \in pol_\omega A$

Lemma : (? delle polari ?)

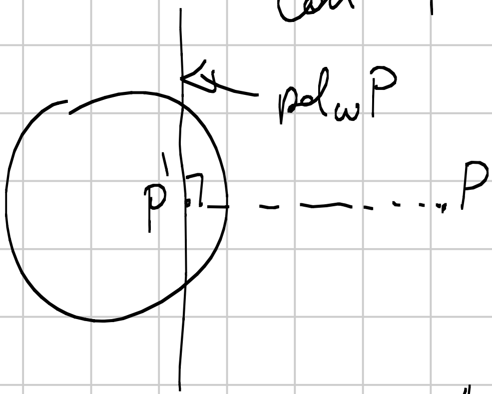


$pol_\omega(E) \ni P$

(Via alternativa :

$pol_\omega P$ = retta per $P' \perp PP'$

con P' = inv di P risp. a ω



inversione di centro A.)

Se uno non sa nulla : numeri complessi.

$b = -1 \qquad c = 1$

(i) Troviamo p $\qquad p \in$ cf. unit $\Rightarrow \bar{p} = \frac{1}{p}$

$pa \perp pO \Rightarrow \frac{p-a}{\bar{p}-\bar{a}} = -\frac{p-O}{\bar{p}-O} = -p^2$

$$\Rightarrow \quad p - q = -\bar{p}^2 \bar{p} + p^2 \bar{q}$$

$x^2 - sx + p$

$\underset{\text{SOMMA}}{\uparrow} \quad \underset{\text{PRODOTTO}}{\uparrow}$

$$\Rightarrow \quad \bar{a}p^2 - 2p + a = 0$$

$$p + q = \frac{2}{\bar{a}} \qquad pq = \frac{a}{\bar{a}}$$

h = intersezione tra pq e la perp. a bc da a

Tesi $\Leftrightarrow$ h ortocentro. $\Leftrightarrow$ $ch \perp ab$

$$\frac{h-c}{\bar{h}-\bar{c}} = -\frac{a-b}{\bar{a}-\bar{b}} = -\frac{a+1}{\bar{a}+1}$$

h sta su pq $\quad \bullet \; \bar{h} = \dfrac{p+q-h}{pq} = \dfrac{2-\bar{a}h}{a}$

$ah \perp bc$

$$\bullet \; \bar{h} = a + \bar{a} - h$$

$$h = \frac{a\bar{a} + a^2 - 2}{a - \bar{a}}$$

$h \in pq$

$h = p + t(q-p) \quad t \in \mathbb{R}$

$\dfrac{h-p}{q-p} \in \mathbb{R} \quad \dfrac{h-p}{q-p} = \dfrac{\bar{h}-\bar{p}}{\bar{q}-\bar{p}}$

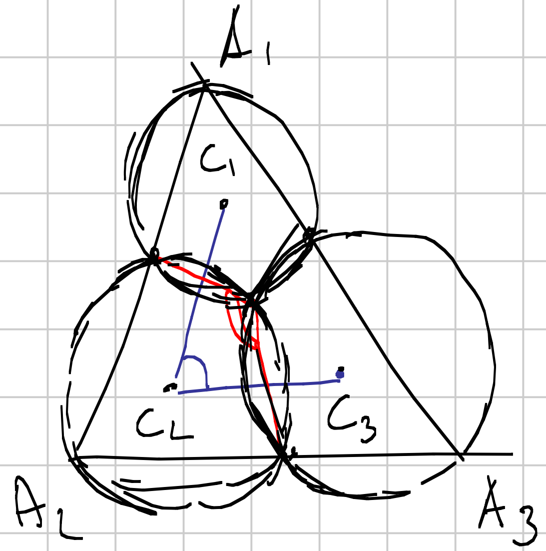
$$h - 1 = \frac{(a+1)(a+\bar{a}-2)}{a-\bar{a}}$$

$(a+\bar{a}-2) \rightarrow$ Reale

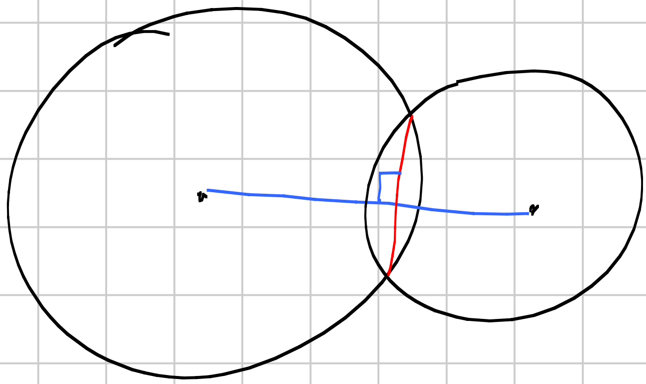
$a-\bar{a} \rightarrow$ immaginaria

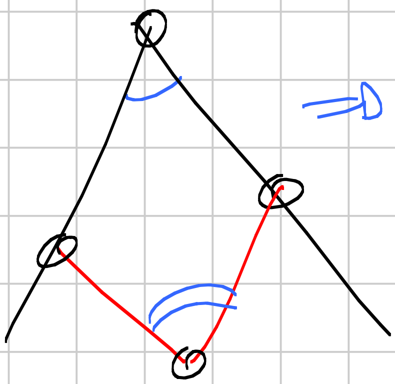
$$\frac{h-1}{\bar{h}-1} = -\frac{a+1}{\bar{a}+1} \Rightarrow \text{fine.} \;\square$$

Problema 4



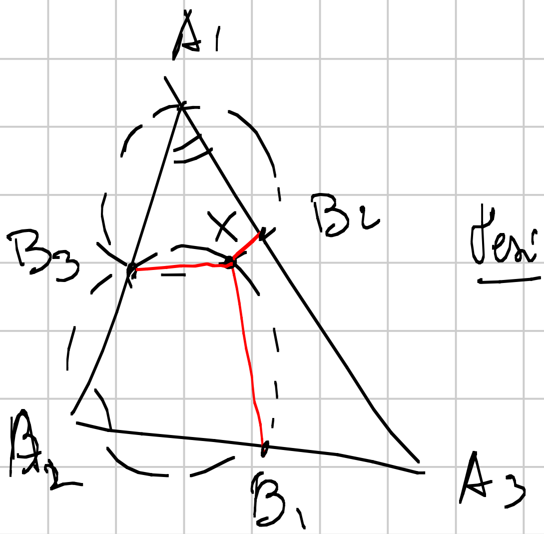
$\overset{\triangle}{C_1C_2C_3} \cong A_1A_2A_3$





$\Rightarrow$ ciclico

è vero se le 3 cfr passano per l'incontro dei tre assi radicali.



Tesi: $B_1 A_3 B_2 X$ ciclico

$B_1\hat{X}B_2 = \pi - B_2\hat{A_3}B_1$

$$B_1\hat{X}B_2 = 2\pi - B_1\hat{X}B_3 - B_3\hat{X}B_2 = 2\pi - (\pi - B_1\hat{A_2}B_3) - (\pi - B_3\hat{A_1}B_2)$$

$= B_1\widehat{A_2}B_3 + B_3\widehat{A_1}B_2 = \pi - B_2\widehat{A_3}B_1$ ok

$\Rightarrow$ i tre assi radicali si incontrano con angoli supp. a quelli del tri $A_1A_2A_3$

$\Rightarrow \overset{\triangle}{C_1C_2C_3} \sim \overset{\triangle}{A_1A_2A_3}$

Oss: