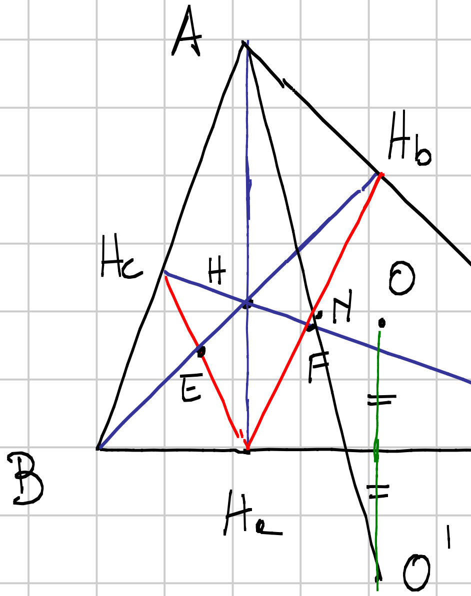
Pre IMO 2011 - Geometria Pomeriggio

Titolo nota 24/05/2011

5)



O' = circo di HBC

Th: a) N pt med di AO'

b) $EF \perp AN$

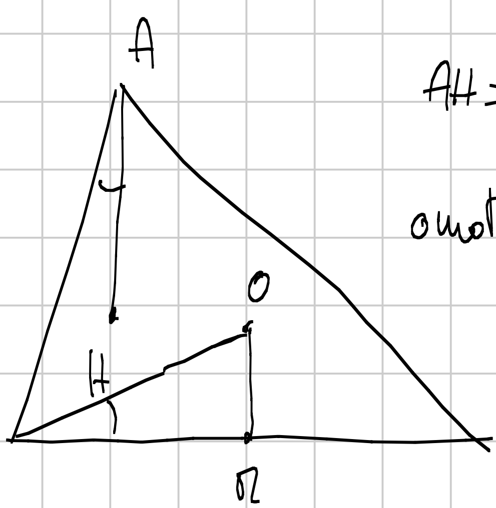
a) i. O' simm. di O rispetto a BC $\Leftrightarrow$ il simm di H risp a BC sta sulla circ. circo ad ABC

H' = simm di H risp. a BC $H'BC$ inscritto in Γ

$\Leftrightarrow$ HBC inscritto in Γ' = simm di Γ risp. a BC

ii. N è il pt. medio di OH (tendenzialmente inutile)

iii. $AH = OO'$



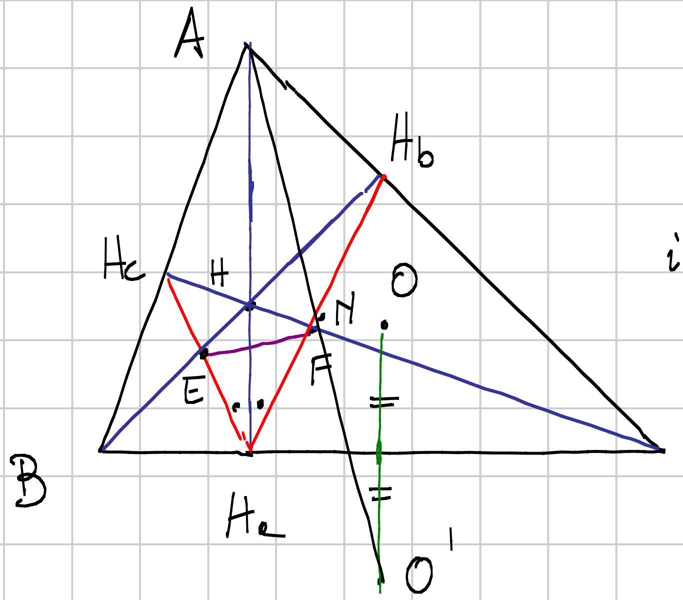
$AH = 2OM$

omot. centro G fatt. $-\frac{1}{2}$ $A \to M$, $H \to O$

$\Rightarrow AH = OO'$ e $AH \parallel OO'$ $\Rightarrow$ $AHO'O$ parall.

$\Rightarrow$ le diagonali si bisecano $\Rightarrow N$ pt. medio di AO'

b)



$E = H_cH_a \cap BH$

$F = H_bH_a \cap HC$

i. $\odot HH_cBH_a$ ciclico

$\Downarrow$

$HE \cdot EB = H_cE \cdot H_aE$ (I)

$\odot HH_aCH_b$ ciclico

$\Downarrow$

$HF \cdot FC = H_bF \cdot H_aF$ (II)

$\Gamma' =$ circ. circo a BHC

$\omega =$ circ di Feuerbach

H_cH_a è corda di ω

HB è corda di Γ'

$\Downarrow$

(I) $pow_{\Gamma'} E = pow_{\omega} E$

simil. (II) $pow_{\Gamma'} F = pow_{\omega} F$

$\Rightarrow EF$ asse rad di ω e Γ' $\Rightarrow EF \perp NO' = AO' = AN$

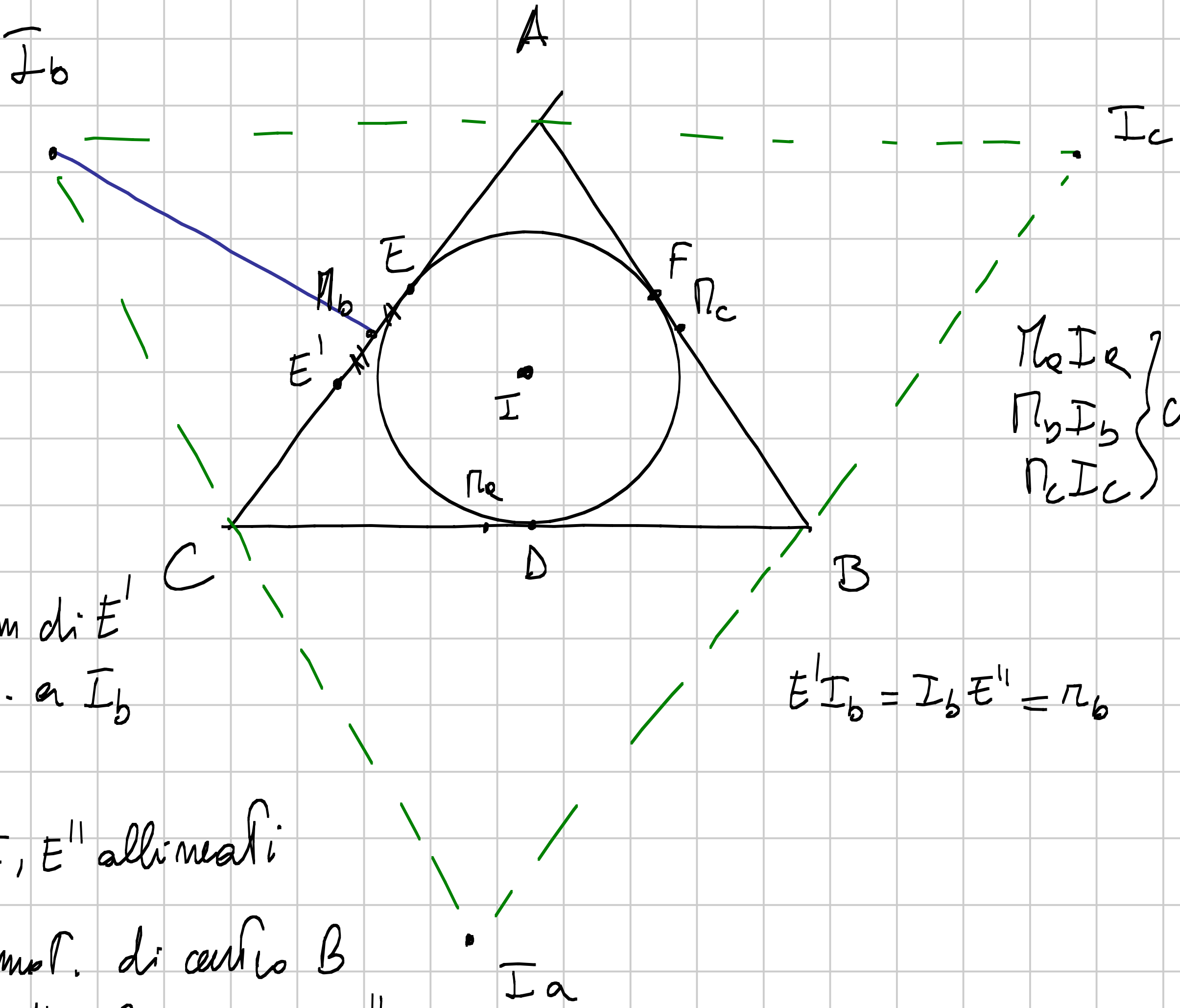
a) Reloaded $O' =$ simm. di O in BC

origine in O $\Rightarrow O' = 2\frac{B+C}{2} = B+C$

pt med di $AO' = \frac{A+O'}{2} = \frac{A+B+C}{2} = N$

6)

a)



r_aI_a, r_bI_b, r_cI_c } concor.

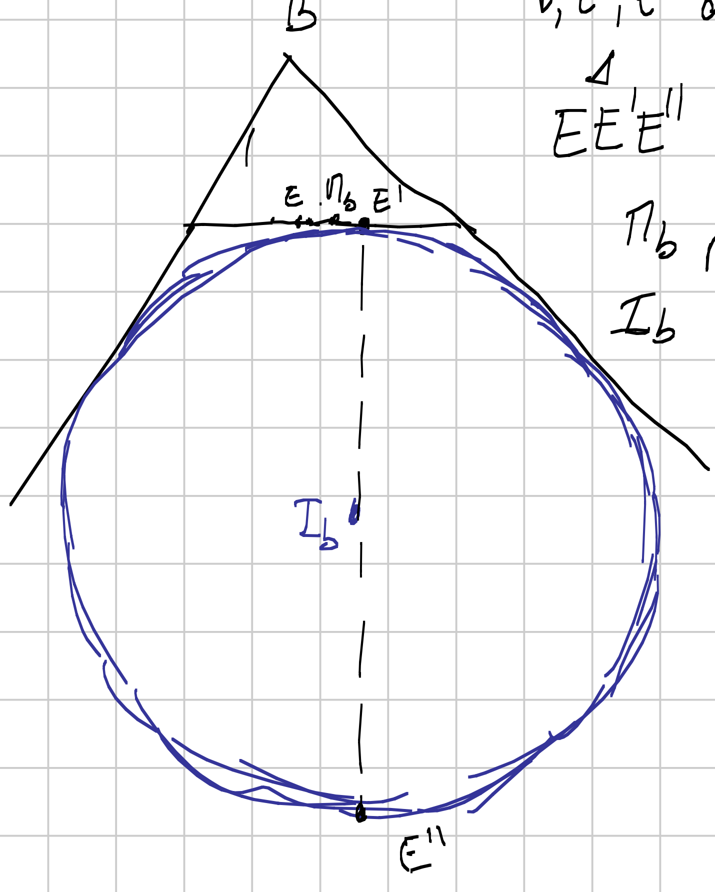
E'' = simm di E' risp. a I_b

⇓

B, E, E'' allineati

per omot. di centro B e fatt $\frac{r_b}{r}$ $E \to E''$

$E'I_b = I_bE'' = r_b$



B, E, E'' all.

Δ $EE'E''$, r_bI_b

r_b pt. med di EE'

I_b " " di $E'E''$

⇓ $r_bI_b \parallel EE'' = BE$

$M_aI_a \parallel AD$
$M_bI_b \parallel BE$
$M_cI_c \parallel CF$

D, E, F Tg. delle cf. inscr. sui lati

⇓

omot. di centro G e fatt. $-\frac{1}{2}$

manda A, B, C in M_a, M_b, M_c

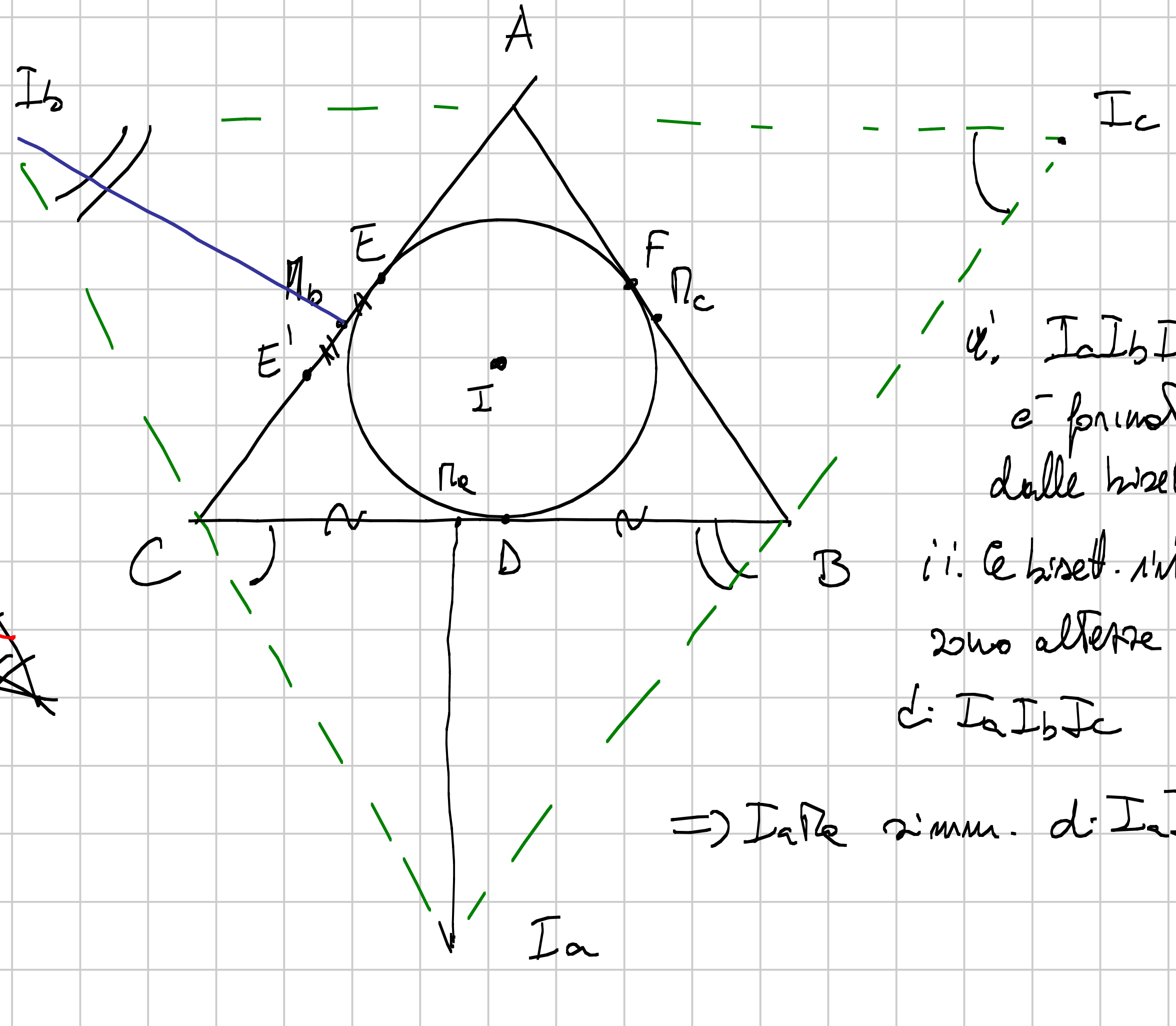
e conserva i parall. ⇒ manda AD, BE, CF (*)

in M_aI_a, M_bI_b, M_cI_c (+)

le prime (*) concorrono in Gergonne (K)

⇒ le seconde (+) concorrono in M t.c. $MG = \frac{1}{2} GK$

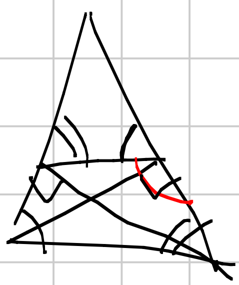
a-bis)



i. $I_aI_bI_c$ è formato dalle bisett. est.

ii. le bisett. int. sono altezze di $I_aI_bI_c$

⇒ I_aM_a simm. di $I_aI_bI_c$

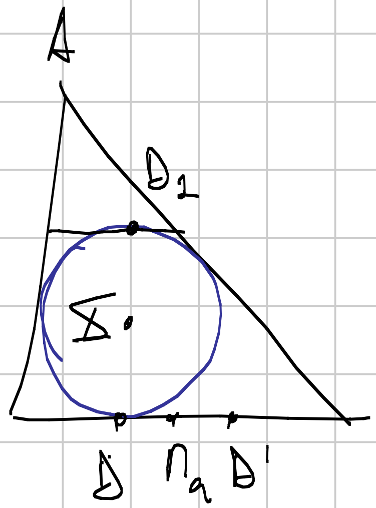


b) $HIK \sim ONM$?

$MG = \frac{1}{2} GK$ x omot.

$OG = \frac{1}{2} GH$ x omot.

<u>Hope</u> : $NG = \frac{1}{2} GI$ x omot



N_a = concorr. di AD', BE', CF'

D_1 = simm di D risp a I

A, D_1, D' sono all.

I pt. med di DD_1, N_a pt. med di DD'

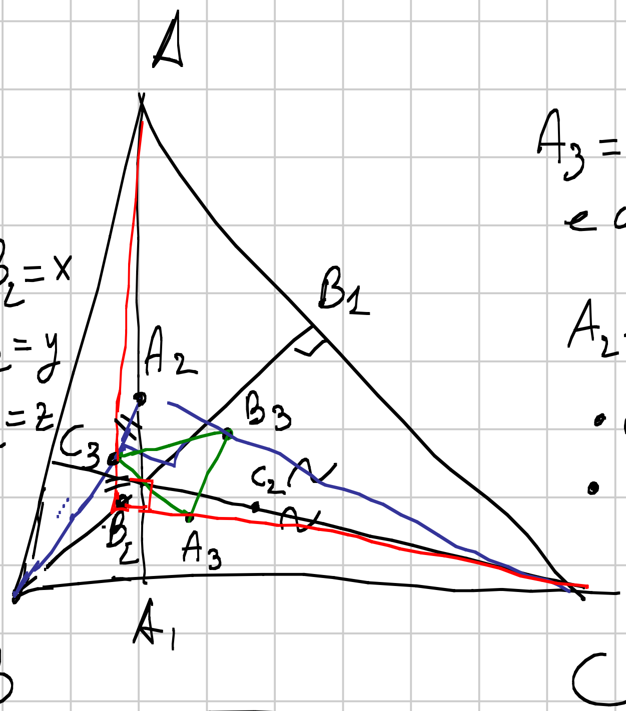
$\Rightarrow IN_a \,//\, D'D_1 = AD'$

7)

$A_2C_3 = C_3B_2 = x$

$A_2B_3 = B_3C_2 = y$

$C_2A_3 = A_3B_2 = z$



$A_3 = BC_2 \cap B_2C$
e cicliche

A_2A_3, B_2B_3, C_2C_3 concorrono

• ceviane in $A_3B_3C_3$

• " in $A_2B_2C_2$

A_2A_3 risp. B_2C_2
$\quad$ // B_3C_3

$CA_2 \overset{?}{=} CB_2$

$$\begin{cases} A_2A_1^2 = A_1B \cdot A_1C \\ A_2C^2 = A_1C \cdot BC \end{cases}$$

$$\begin{cases} B_2B_1^2 = B_1C \cdot B_1A \\ B_2C^2 = B_2C \cdot AC \end{cases}$$

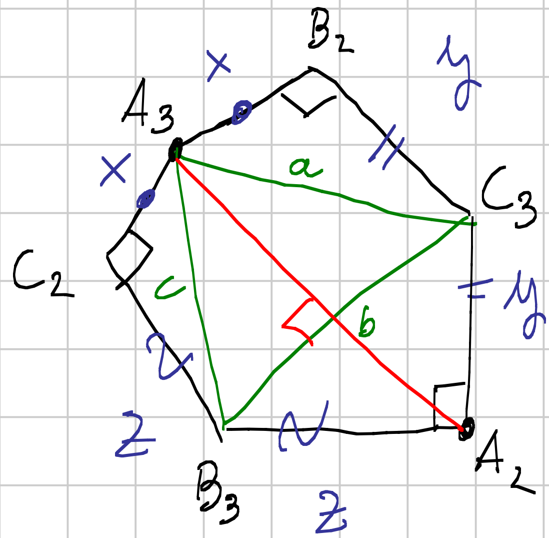
AB_1A_2B ciclico
||

$$\Updownarrow$$

$$CB_1 \cdot CA = CA_1 \cdot CB$$

$$B_2C^2 = A_2C^2$$

$$\Rightarrow C_3B_2 = C_3A_2$$



$$a = \sqrt{x^2+y^2}$$

$$b = \sqrt{y^2+z^2}$$

$$c = \sqrt{z^2+x^2}$$

$$c^2 + y^2 = a^2 + z^2$$

$$c^2 - a^2 = z^2 - y^2$$

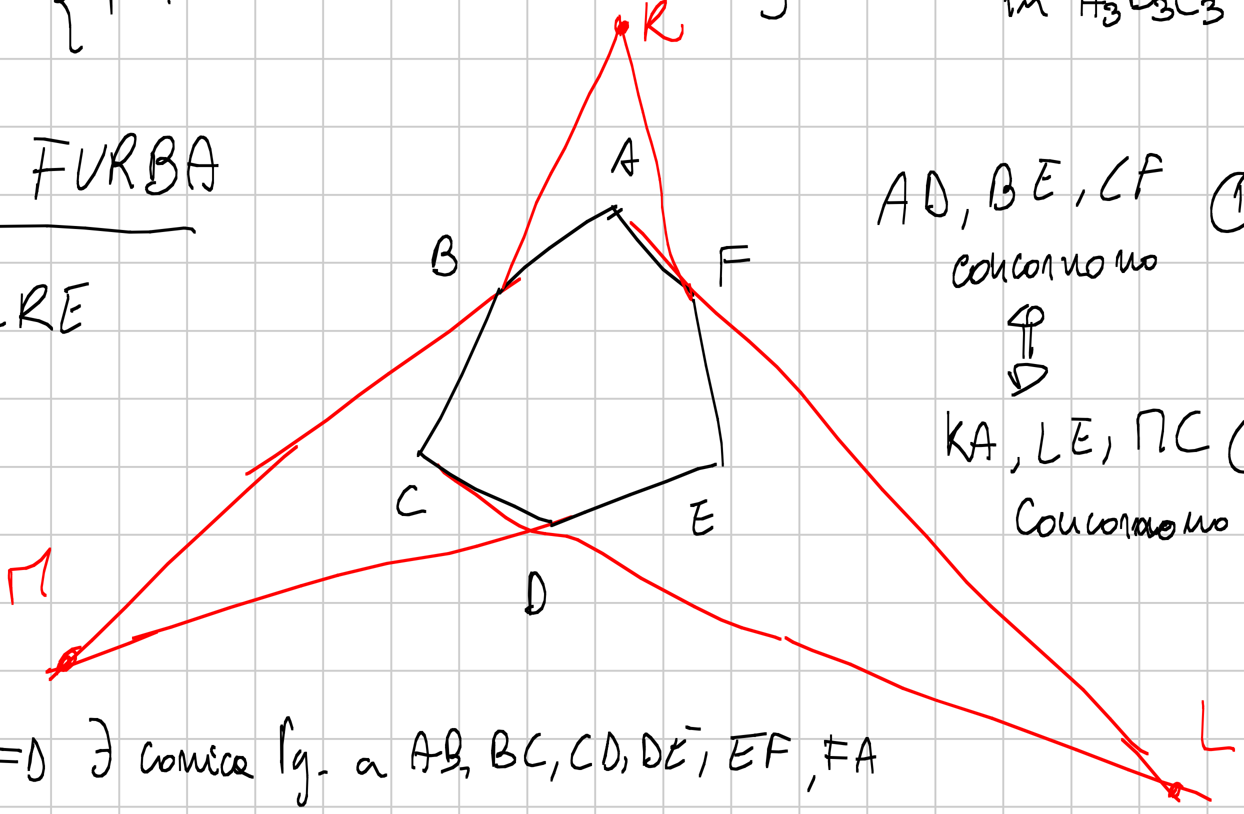
La perp. a XY per Z

$$= \{P : PX^2 - PY^2 = ZX^2 - ZY^2\}$$

$\Rightarrow A_3A_2$ è altezza in $A_3B_3C_3$

Sol FURBA

MALCRE



AD, BE, CF concorrono (I)

$\Updownarrow$

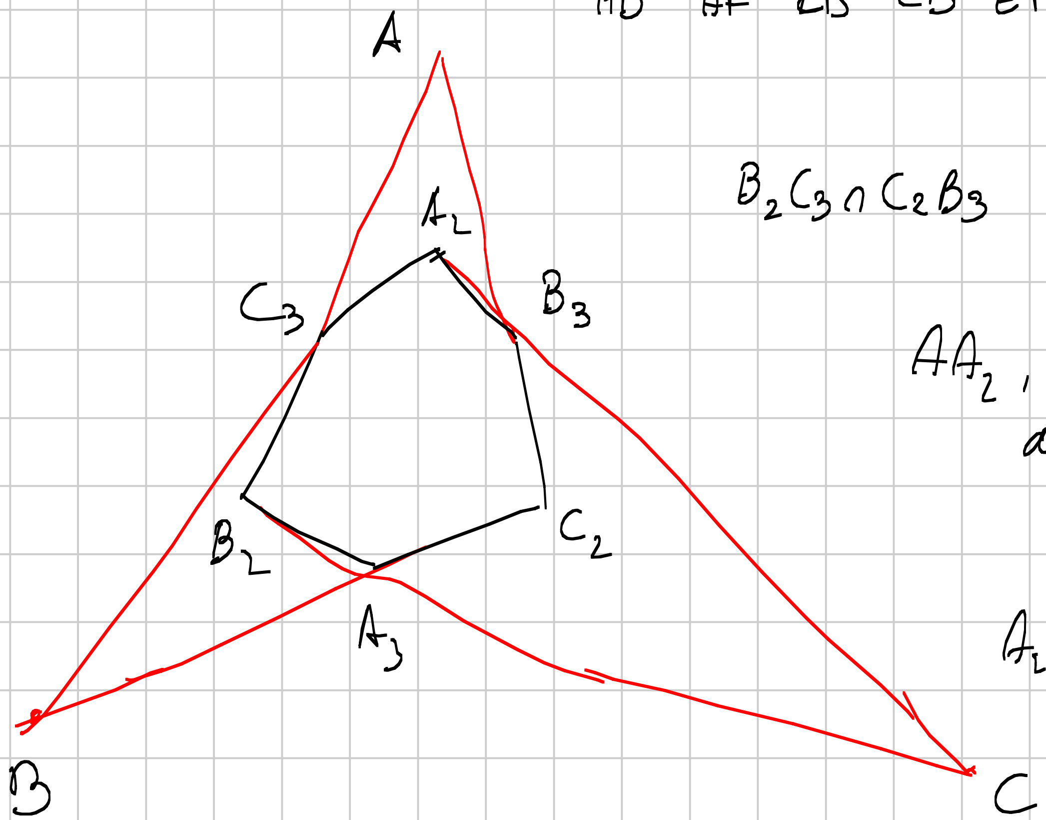
KA, LE, MC concorrono (II)

(I) $\Leftrightarrow$ $\exists$ conica tg. a AB, BC, CD, DE, EF, FA

(11) $\Leftrightarrow$ $\exists$ conica tg a MA, AL, LC, CK, KE, EM

$\quad\quad\quad\quad\quad\quad\quad\quad\quad \| \quad \| \quad \| \quad \| \quad \| \quad \|$

$\quad\quad\quad\quad\quad\quad\quad\quad\quad AB \quad AF \quad CD \quad CB \quad EF \quad ED$

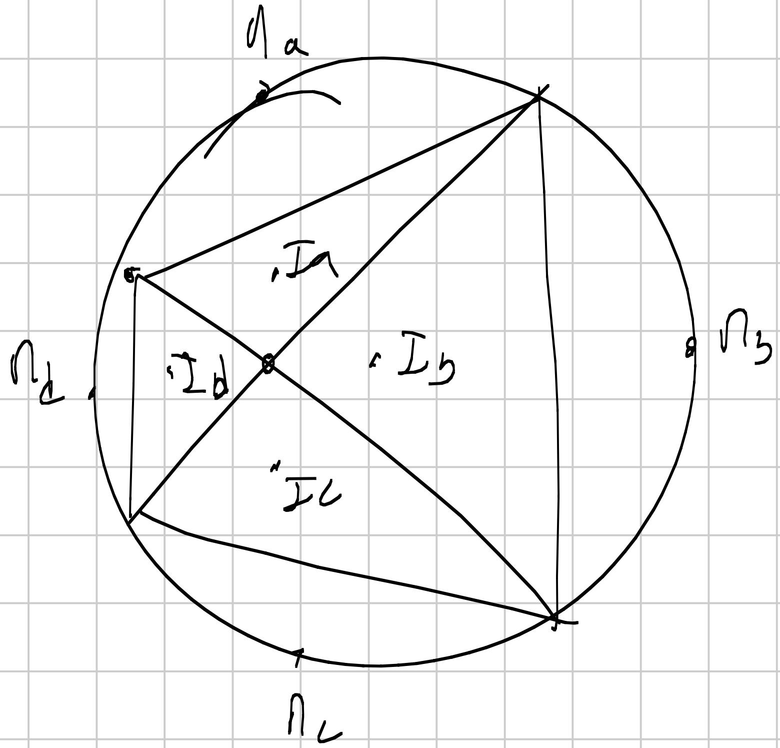


$B_2C_3 \cap C_2B_3$

AA_2, BB_2, CC_2 concorrenti alterne

$\Updownarrow$

A_2A_3, B_2B_3, C_2C_3 concorrenti.



$M_aI_a, M_bI_b, M_cI_c, M_dI_d$ concorrenti