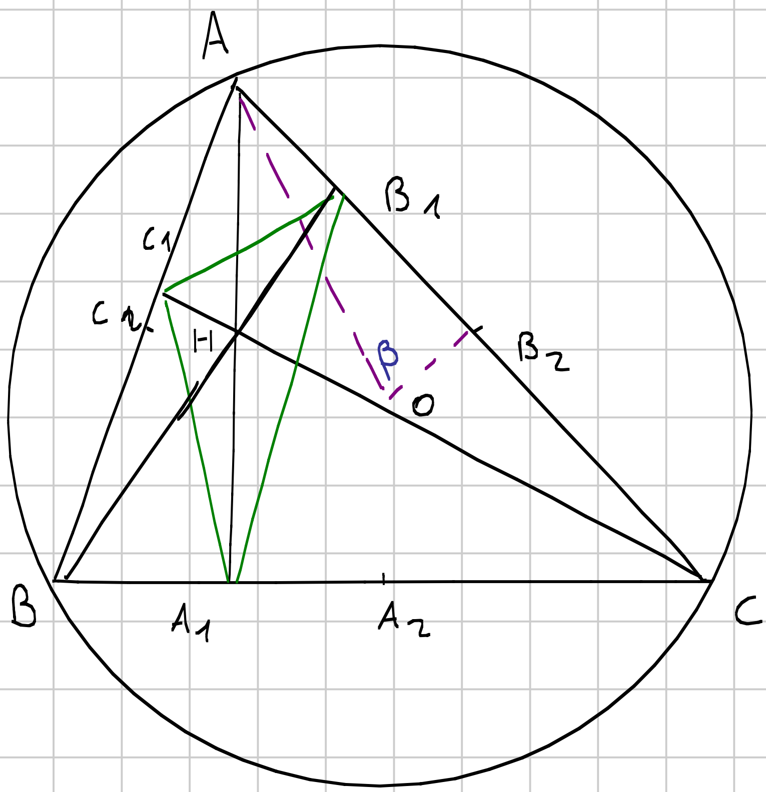
GEOMETRIA MATTINA -PREIMO 12

Titolo nota 24/05/2012



O e H coniugati isogonali

$B\hat{A}H = C\hat{A}O$

$\quad\| \qquad\quad \|$

$90-\beta \qquad 90-\beta$

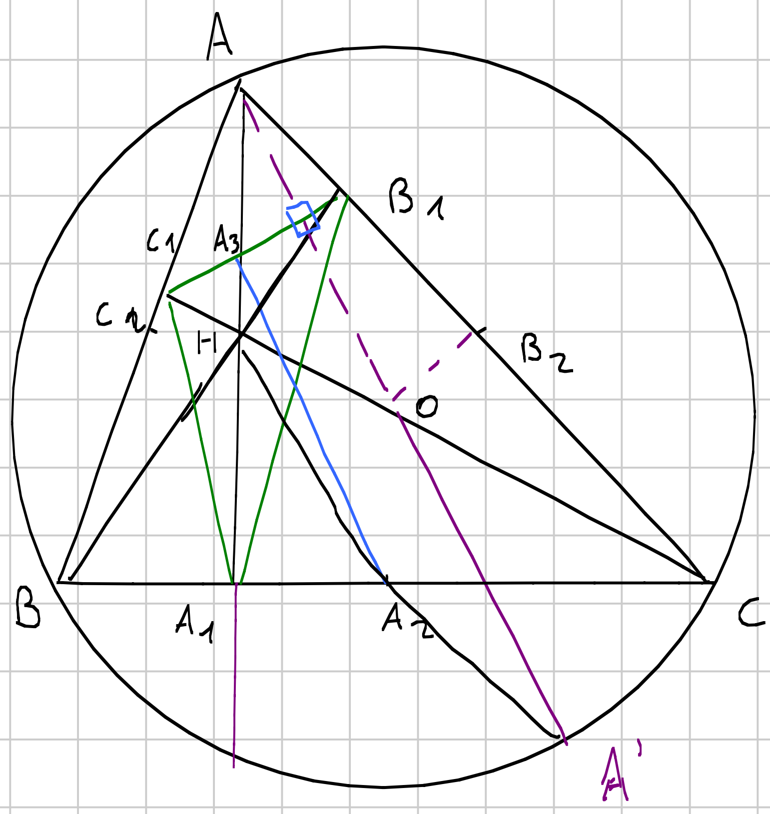
$AO \perp B_1C_1$

$ABC \to AB_1C_1$

$\downarrow$

$AH \to AO$

$AO \perp B_1C_1$!!

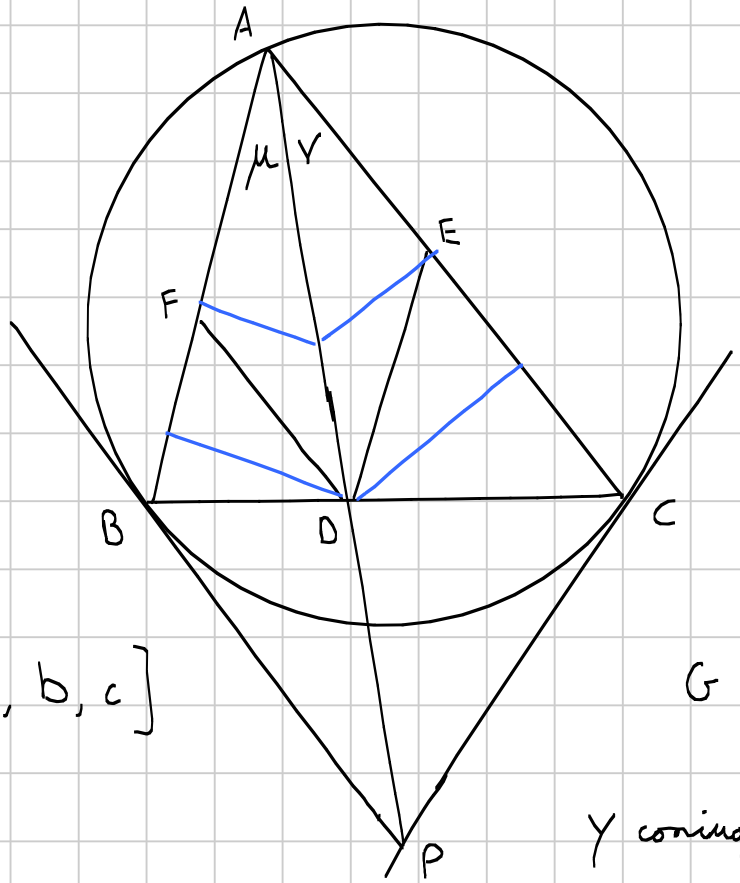


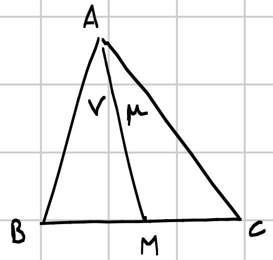
A_2 pt medio $A'H$

omotetia di centro H e fattore $\frac{1}{2}$

$A' \to A_2$

$\ell(AO) \to \ell(A_2A_3)$





AP simmediana

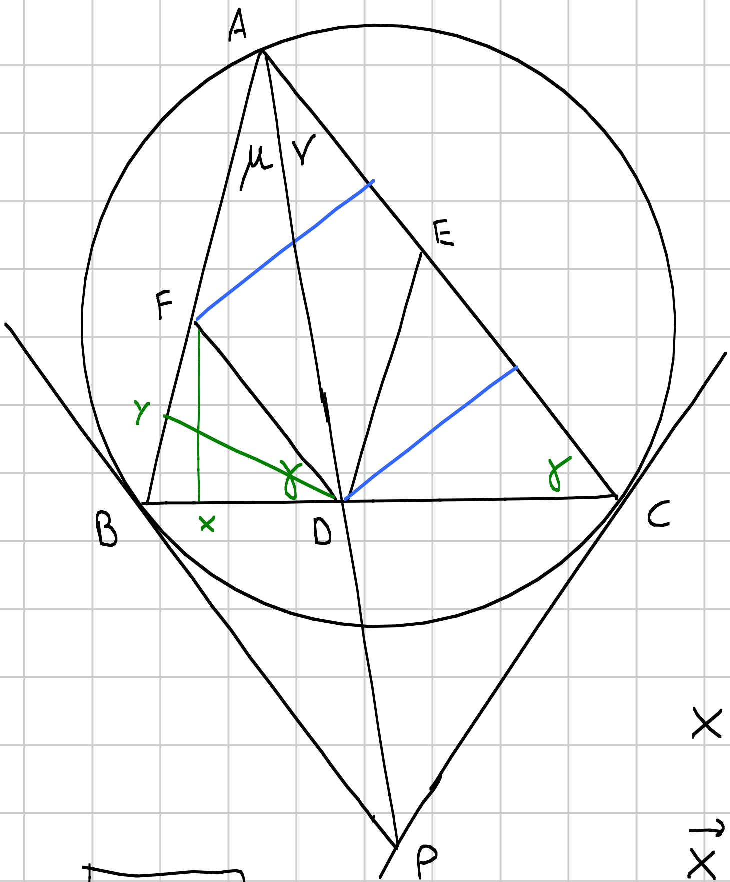
$$X = [\alpha, \beta, \gamma]$$

$$G = \left[\frac{1}{a}, \frac{1}{b}, \frac{1}{c}\right]$$

$$D[0, b, c]$$

Y coniug. isog. di X $\rightarrow Y = \left[\frac{1}{\alpha}, \frac{1}{\beta}, \frac{1}{\gamma}\right]$

L pt. di Lemoine $\rightarrow L = [a, b, c]$



$$D = [0, kb, kc]$$

$$F = \left[kc \cdot \frac{c}{a}, kb, 0\right]$$

$$\frac{FX}{\sin\gamma} = \frac{DY}{\sin\alpha} \qquad FX = DY \cdot \frac{c}{a}$$

$$F = [c^2, ab, 0]$$

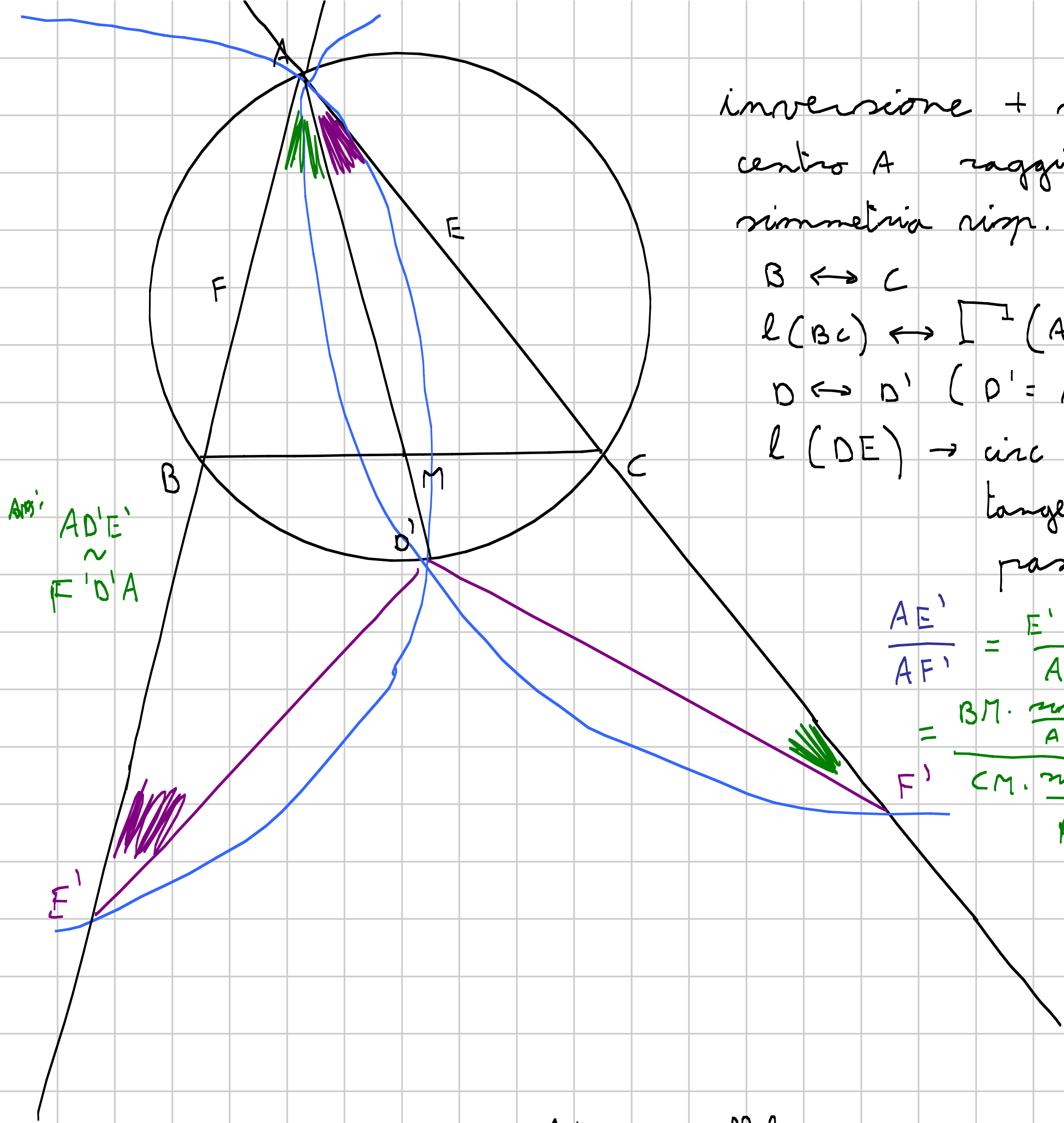
$$X = [\alpha, \beta, \gamma]$$

$$\vec{X} = \frac{a\alpha \vec{A} + b\beta \cdot \vec{B} + c\gamma \vec{C}}{a\alpha + b\beta + c\gamma}$$

$$\boxed{\vec{A} = 0}$$

$$\vec{F} = \frac{ac^2 \vec{A} + ab^2 \vec{B}}{ac^2 + ab^2} = \frac{b^2 \vec{B}}{b^2 + c^2}$$

$$AF \cdot AB = \vec{F} \cdot \vec{B} = \frac{b^2}{b^2+c^2} \vec{B} \cdot \vec{B} = \frac{b^2 c^2}{b^2+c^2}$$



inversione + simmetria
centro A raggio $\sqrt{bc}$
simmetria risp. bisettrice

$B \leftrightarrow C$

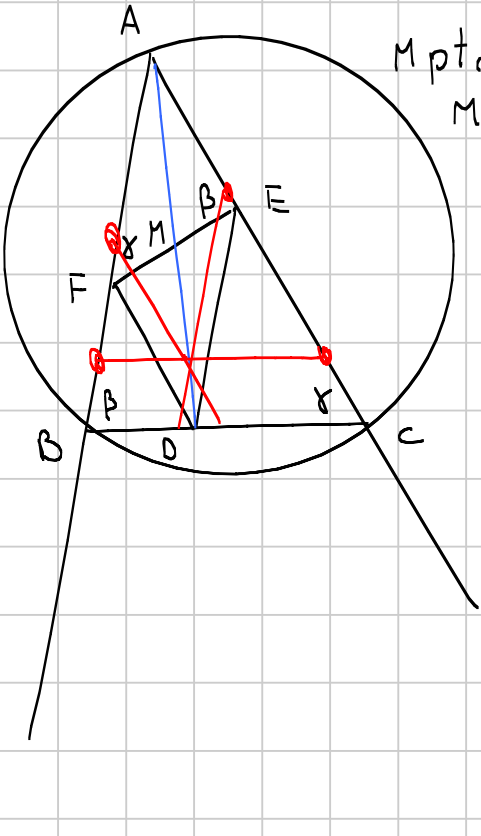
$\ell(BC) \leftrightarrow \Gamma(ABC)$

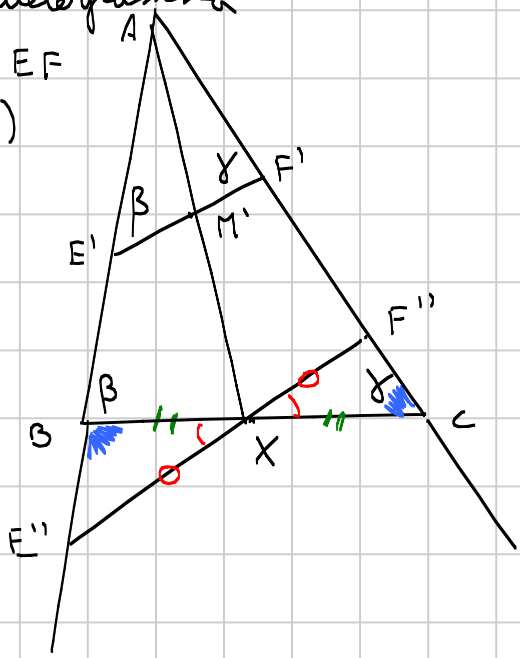
$D \leftrightarrow D'$ $\quad (D' = AM \cap \Gamma(ABC))$

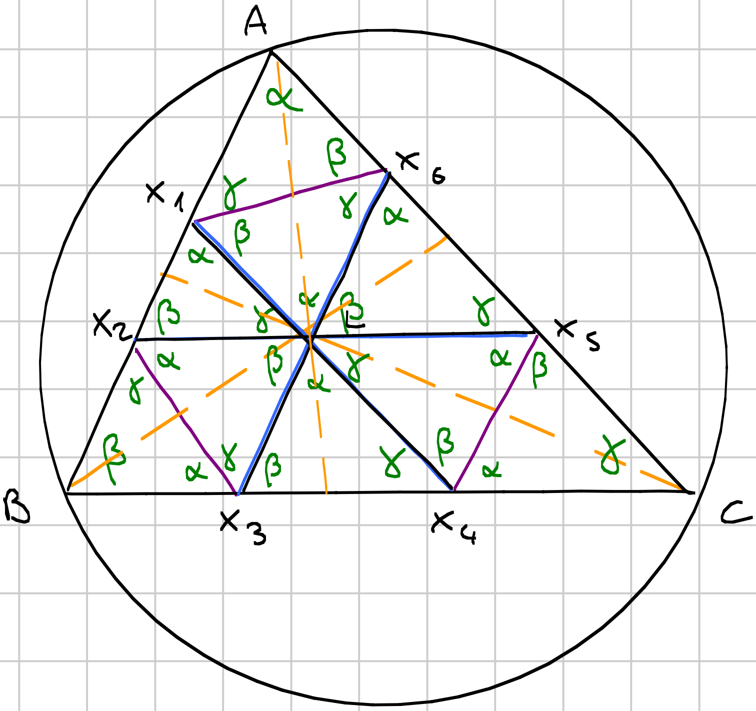
$\ell(DE) \to$ circ passante per A
tangente a AC
passante per D'

$$\frac{AE'}{AF'} = \frac{E'D'}{AD'} = \frac{\sin B\hat{A}M}{\sin C\hat{A}M} = \frac{BM \cdot \frac{\sin\beta}{AM}}{CM \cdot \frac{\sin\gamma}{AM}} = \frac{\sin\beta}{\sin\gamma} = \frac{b}{c}$$

AEDF parallelogramma
M pt medio di EF
$M \in \ell(AD)$





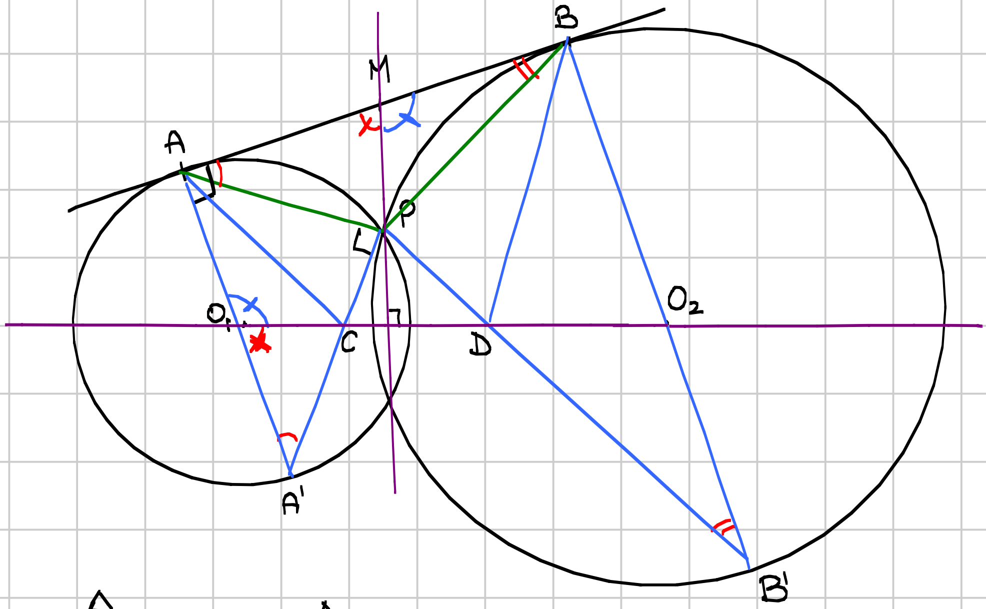


$X_1 X_2 X_5 X_6$ ciclici

$X_2 \widehat{X_3} X_6 = \gamma = X_2 \widehat{X_5} X_6$

X centro della circ. di Lemoine

X pt medio di OL



Claim: $\overset{\Delta}{AA'C} \sim \overset{\Delta}{APB}$.

M pto medio di AB $\rightsquigarrow$ M sta sull'asse radicale

$\times +)$ $\Rightarrow$ $\overset{\Delta}{O_1A'C} \sim \overset{\Delta}{MAP}$

$$\frac{AA'}{A'C} = 2\,\frac{A'O_1}{A'C} = 2\,\frac{AM}{AP} = \frac{AB}{AP}$$

$\Rightarrow$ tesi

Sol 2: Rotazione + omotetia.

Es 3

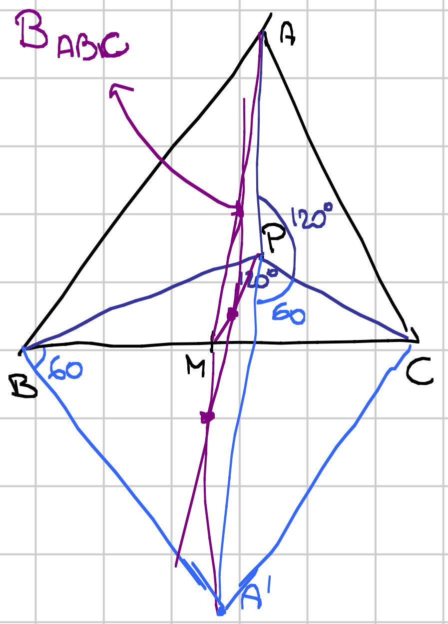
Sol 1:

Idea: A, P, A' sono allineati.

Dim:

$A'BPC$ è ciclico $\Rightarrow 60° = A'\hat{B}P = A'\hat{P}C$

$\Rightarrow A'\hat{P}C + C\hat{P}A = 180°$.



$\frac{B+C}{2} = M$ è l'origine

$$O_{BPC} = \text{Baric}_{A'BC} = \frac{A'+B+C}{3} = \frac{A'}{3}$$

$$B_{BPC} = \frac{P}{3}$$

$$B_{ABC} = \frac{A}{3}$$

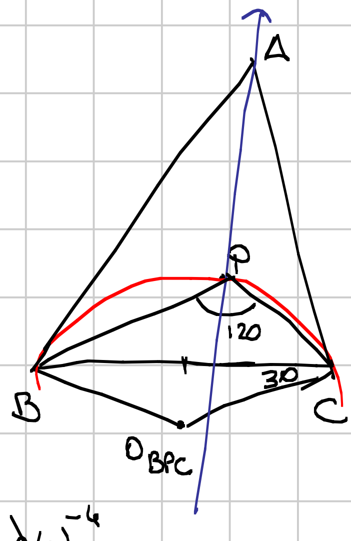
allineati perchè lo erano A, P, A'

Sol 2

Origine in P.

$A = 1 \qquad B = \mu\omega^4 \qquad$ dove $\omega^{12} = 1$

$\qquad\qquad\qquad\qquad C = \lambda\omega^{-4}$



$$B_{BPC} = \frac{\mu\omega^4 + \lambda\omega^{-4}}{3}$$

$$B_{ABC} = \frac{1 + \mu\omega^4 + \lambda\omega^{-4}}{3}$$

$$O_{BPC} = c + \frac{(b-c)}{\sqrt{3}}\omega = \lambda\omega^{-4} + \frac{\mu\omega^4 - \lambda\omega^{-4}}{\sqrt{3}}\omega$$

Sono allineati?

In generale a, b, c sono all $\Leftrightarrow a-b = \alpha(c-b)$, $\alpha\in\mathbb{R}$

$$\frac{1}{3} \overset{?}{=} \alpha\left(\underbrace{\lambda\omega^{-4} + \frac{\mu\omega^4 - \lambda\omega^{-4}}{\sqrt{3}}\omega - \frac{\mu\omega^4 + \lambda\omega^{-4}}{3}}_{z}\right)$$

$$\operatorname{Im} z = -\lambda \frac{\sqrt{3}}{2} + \cancel{\frac{1}{2\sqrt{3}}\mu} + \frac{\lambda}{\sqrt{3}} \cancel{- \frac{\sqrt{3}}{2\cdot 3}\mu} + \frac{\sqrt{3}}{2\cdot 3}\lambda$$

$$= 0$$

$\square$