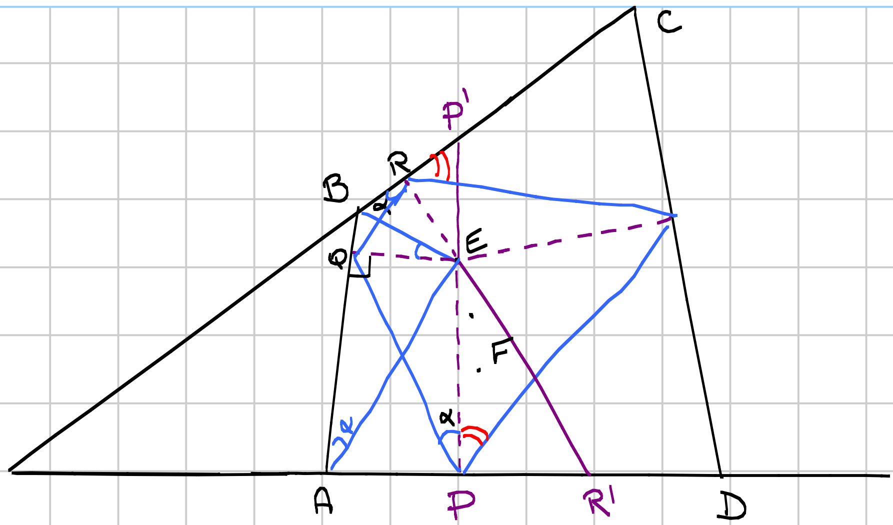
PRE IMO 2012 — GEOMETRIA (P)

Titolo nota 24/05/2012

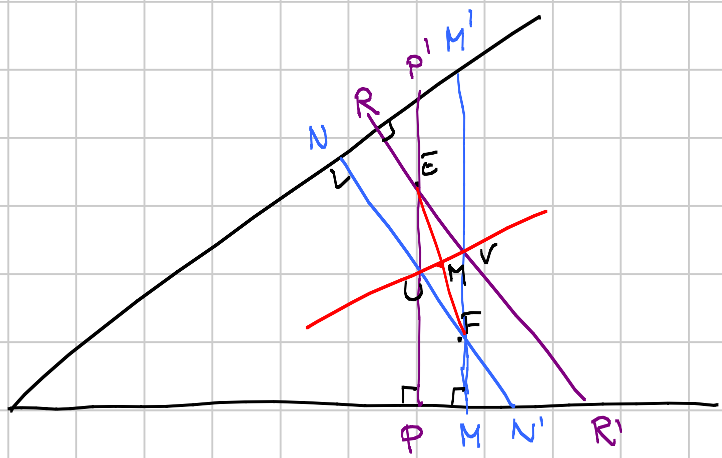
Es 5



Step 1: le proiez di E sui lati di ABCD sono concicliche. (ω_E è la circo)

Step 2: $P' \in \omega_E$

$P'PQR$ è ciclico $(P'\hat{P}Q = \alpha = Q\hat{R}B)$



ω_F passa per $MNM'N'$

ω_E passa per $PRP'R'$.

Tesi: M pto medio EF sta sull'asse radicale di ω_E, ω_F.

$M \in UV$. Vediamo che U, V stanno sull'asse radicale:

$UP \cdot UP' \overset{?}{=} UN \cdot UN'$ ok, $PP'NN'$ ciclico.

Es 6

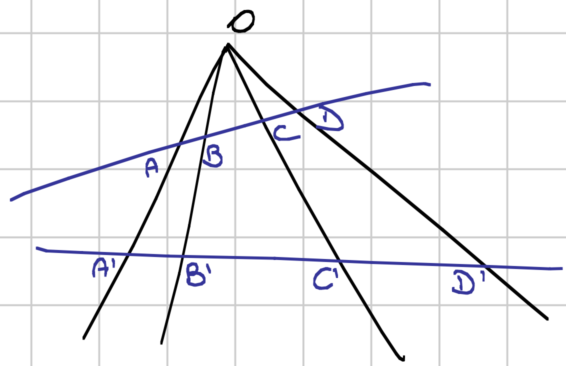
Birapporti

$$(A,B;C,D) = \frac{AC \cdot BD}{AD \cdot BC}$$

- segmenti con segno
- ordinati in qualunque modo
- pti ∞

Fatti

$$(A,B;C,D) = (A',B';C',D')$$



Dim

$$\frac{AC}{\sin A\hat{O}C} = \frac{OC}{\sin O\hat{A}C}$$

$$\frac{BC}{\sin B\hat{O}C} = \frac{OC}{\sin O\hat{B}C}$$

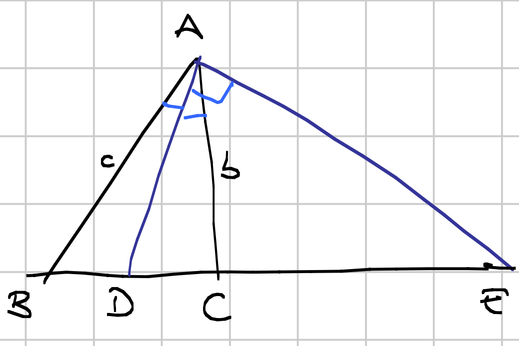
$$\frac{BD}{\sin B\hat{O}D} = \frac{OD}{\sin O\hat{B}C}$$

$$\frac{AD}{\sin A\hat{O}D} = \frac{OD}{\sin O\hat{A}D}$$

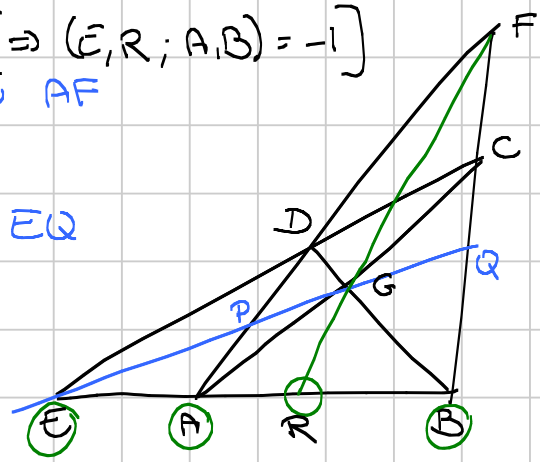
$$\frac{AC \cdot BD}{BC \cdot AD} = \dots = \text{espressione negli angoli in } O.$$

Oss: $(B,C;D,E) = -1$.

$$\frac{BD}{DC} = \frac{c}{b} = \frac{BE}{CE}$$



Oss: $(E, G; P, Q) = -1$ $\quad [\Rightarrow (E, R; A, B) = -1]$



proietto da B su AF

$$(E, G; P, Q) = (A, D; P, F)$$

proietto da C su EQ

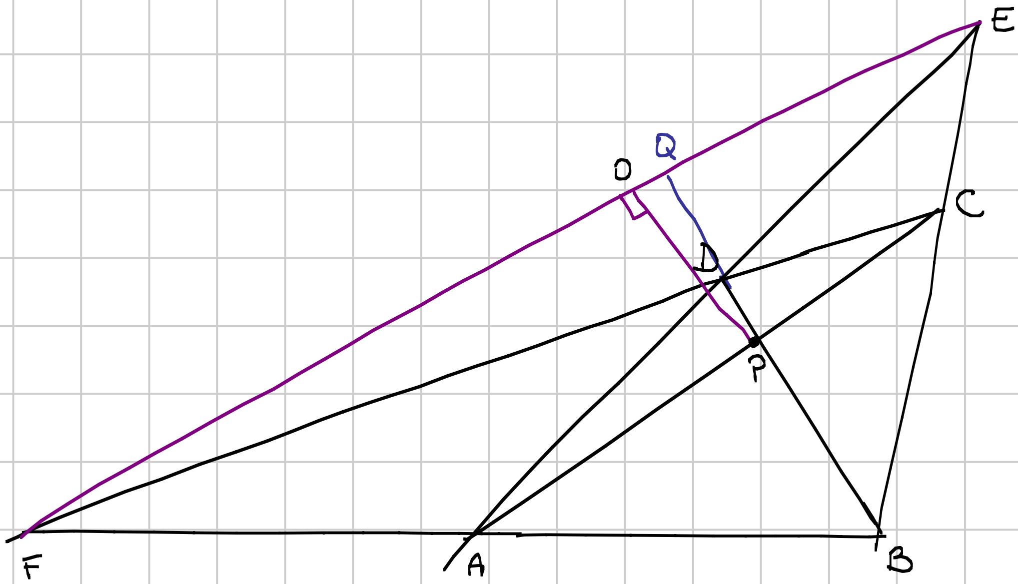
$$= (G, E; P, Q)$$

$$= \frac{GP \cdot EQ}{GQ \cdot EP}$$

$$= [(E, G; P, Q)]^{-1}$$

$$(E, G; P, Q) = \begin{cases} 1 \\ -1 \end{cases}$$

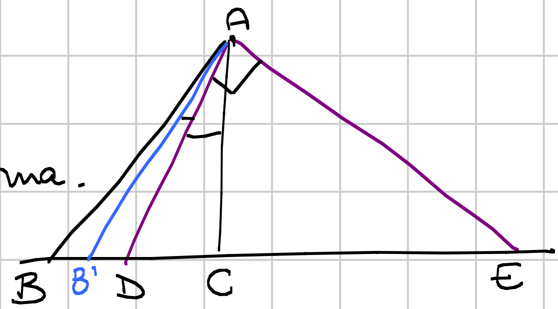
(ex: se il birapp è 1, due pti coincidono)



<u>Claim</u>: OP è bisettrice di $B\hat{O}D$ e di $A\hat{O}C$.

$Q = EF \cap BD$.

$(B, D; P, Q) = -1$. (quadrilatero $ACEF$, B)



<u>Oss</u>: $(B, C; D, E) = -1$ +

$D\hat{A}E = 90°$ $\Rightarrow$

AD è bis interna e AE è bis esterna.

<u>Dim</u>:

Sia B' t.c.

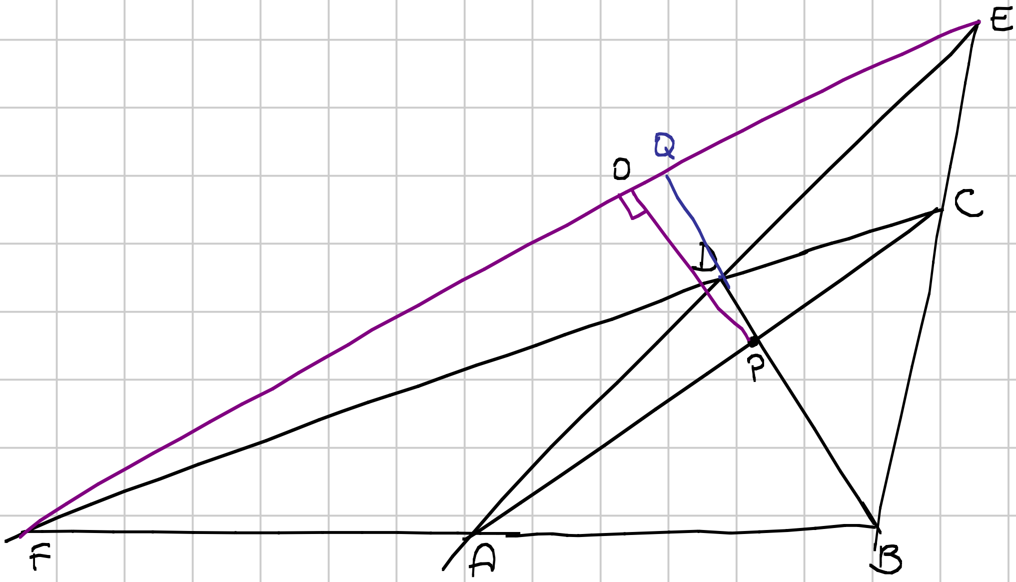
$B'\hat{A}D = D\hat{A}C$

AE è bis esterna di $AB'C$.

$(B', C; D, E) = -1$

$\Rightarrow$ $B' = B$.

Sol 2



Menelao su $B\hat{D}E$ con P, A, C

$$\frac{BC}{CE} \cdot \frac{EA}{AD} \cdot \frac{DP}{PB} = -1$$

Ceva su $B\hat{D}E$ con F

$$\frac{BC}{CE} \cdot \frac{EA}{AD} \cdot \frac{DQ}{QB} = 1$$

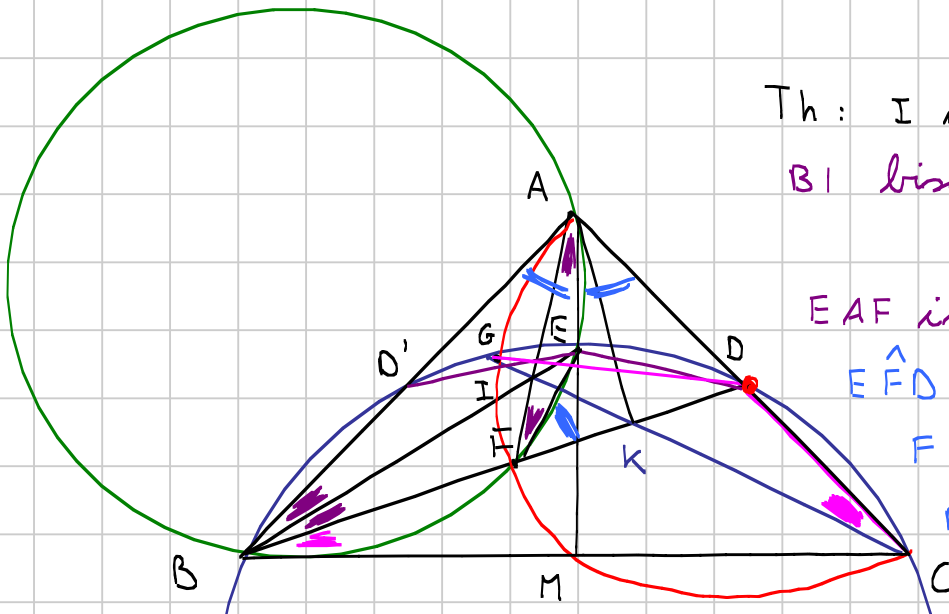
$$\Rightarrow \quad \frac{DP}{PB} = \frac{DQ}{QB} = K$$

P, Q stanno sulla circo di Apollonio con rapp K.
Siccome $P\hat{O}Q = 90°$ anche O ci sta.

$$\frac{DP}{PB} = \frac{DO}{OB}$$

$\Leftrightarrow$ OP bisettrice.

$\angle \vartheta$



Th: I incentro di ABK

BI bisettrice $A\hat{B}K$

EAF isoscele

$E\hat{F}D = B\hat{A}E = D\hat{A}E$

FDA isoscele

$DA = DF = DM = DC = DG$

$D\hat{A}K = 2\vartheta$

$B\hat{A}I = \frac{\alpha}{2} - \vartheta$

$K\hat{A}I = \frac{\alpha}{2} - D\hat{A}K + E\hat{A}I =$

$= \frac{\alpha}{2} - 2\vartheta + \vartheta = \frac{\alpha}{2} - \vartheta$

AF asse radicale

BE " "

$I = AF \cap BE$

$\Downarrow$ centro radicale

CI asse rad

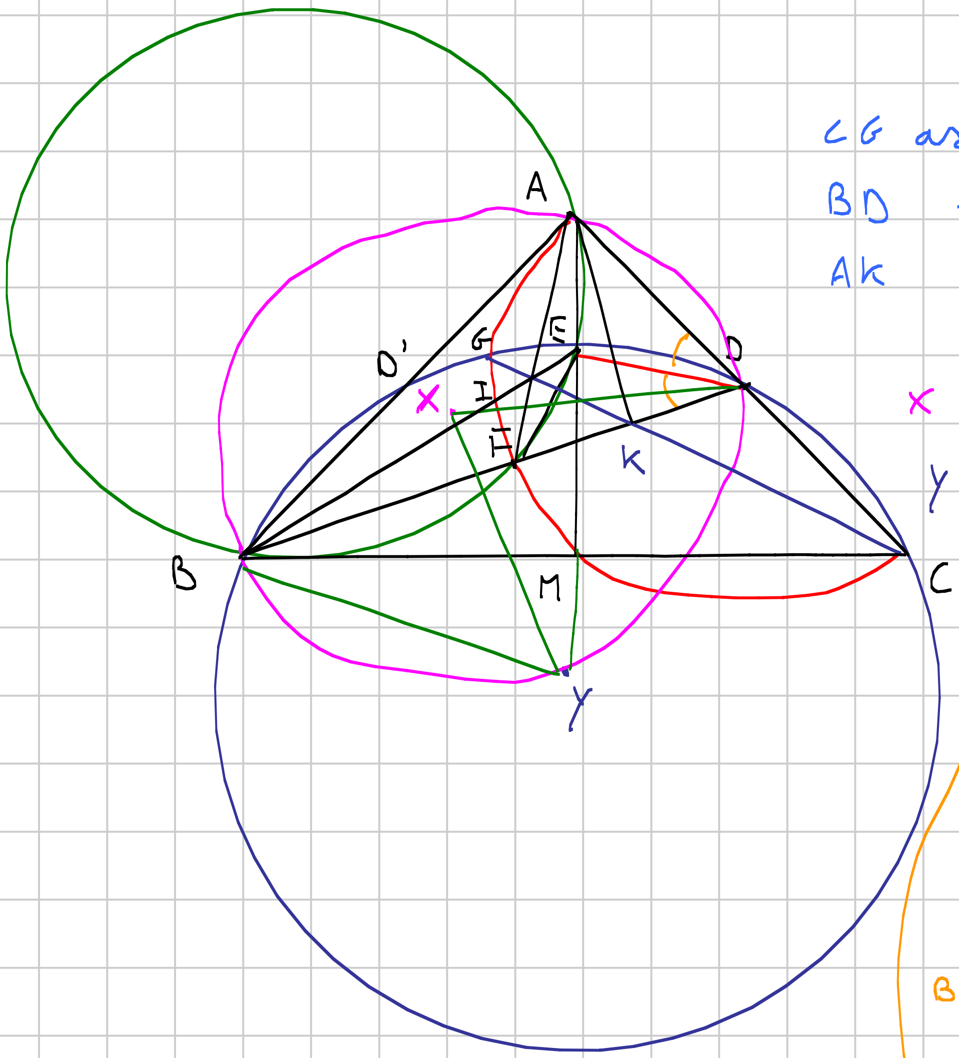
$G = \cap$

$D\hat{C}K = D\hat{B}C$

$D\overset{\triangle}{C}K \sim D\overset{\triangle}{B}C$

$D\overset{\triangle}{A}K \sim D\overset{\triangle}{B}A$

$DK \cdot DB = DC^2 = DA^2$



CG asse radicale

BD " "

AK " "

X centro cfr

Y " "

B Y D A ciclico

$B\hat{Y}E = 2 \cdot B\hat{D}E$

AEF isoscele

ADF isoscele

DE bisettrice di $F\hat{D}A$

$B\hat{D}E = \frac{1}{2} B\hat{D}A$

$B\hat{Y}E = B\hat{Y}A = B\hat{D}A$