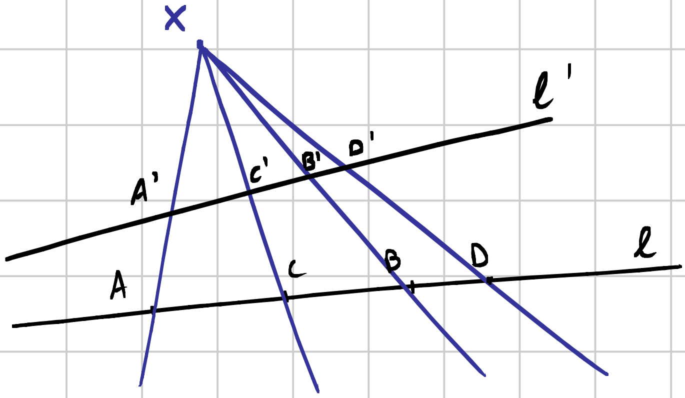
PREIMO 2013 - GP

Titolo nota 30/05/2013

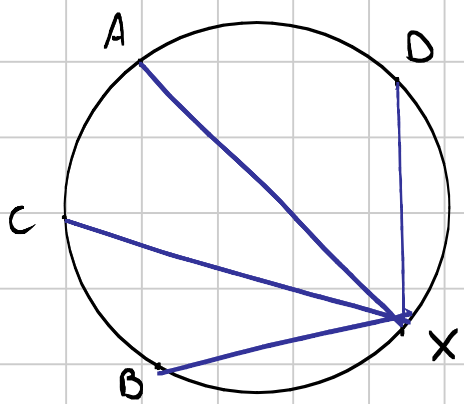
5.



$$(A,B,C,D) = \frac{AC}{BC} \Big/ \frac{AD}{BD} = \frac{AC}{BC} \cdot \frac{BD}{AD}$$

$$\frac{AC}{BC} \cdot \frac{BD}{AD} = \frac{\sin A\hat{X}C}{\sin B\hat{X}C} \cdot \frac{\sin B\hat{X}D}{\sin A\hat{X}D}$$

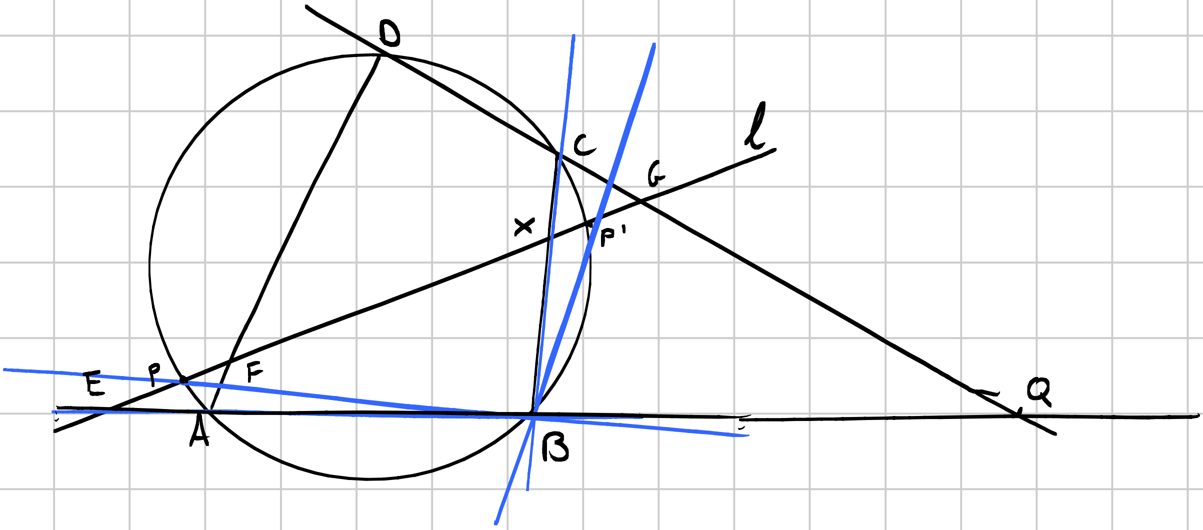
$$(A,B,C,D) = (AX, BX, CX, DX) = (A', B', C', D')$$



(AX, BX, CX, DX)

↓

non dipende da X !!

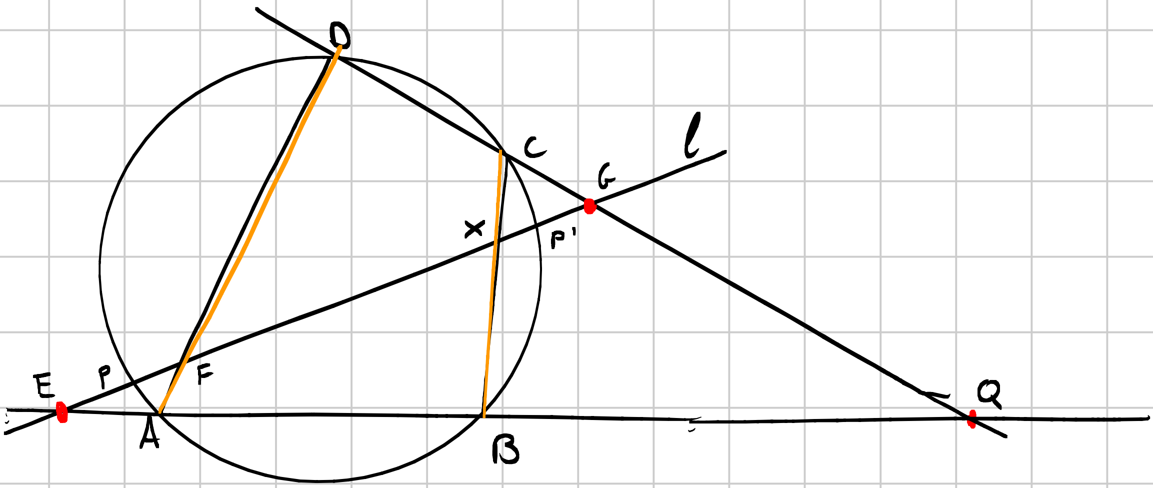


$$(P, P', A, C)_B = (P, P', A, C)_D$$

interseco con l

$$(P, P', E, X) = (P, P', F, G)$$

$$\frac{PE}{P'E} \cdot \frac{P'X}{PX} = \frac{PF}{P'F} \cdot \frac{P'G}{PG}$$



MENELAO ΔEQG rette AD e BC

$$\frac{EA}{AQ} \cdot \frac{QD}{DG} \cdot \frac{GF}{FE} = -1 \qquad \frac{EB}{BQ} \cdot \frac{QC}{CG} \cdot \frac{GX}{XE} = -1$$

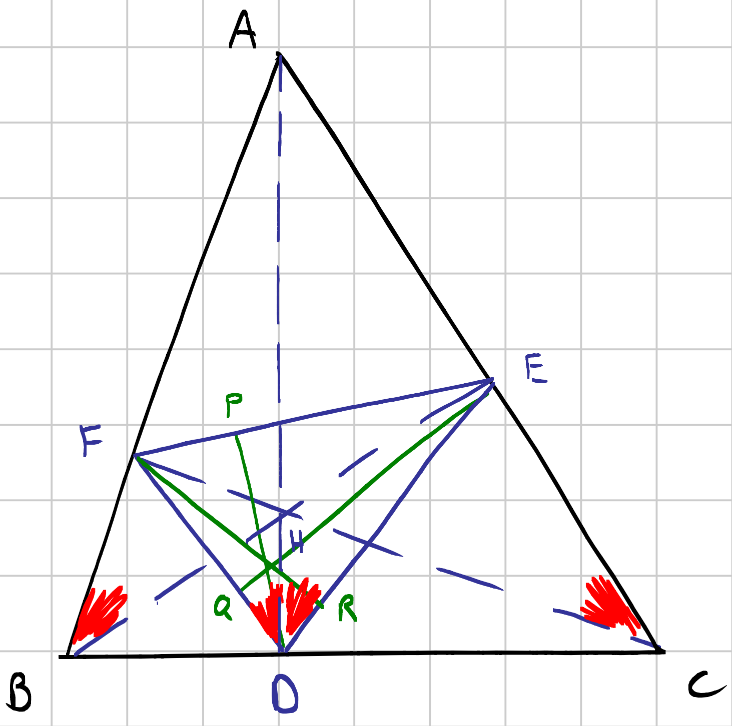
li moltiplico

$$\frac{EA \cdot EB}{CG \cdot DG} \cdot \frac{QD \cdot QC}{AQ \cdot BQ} \cdot \frac{GF}{FE} \cdot \frac{GX}{XE} = 1$$

$EA \cdot EB = EP \cdot EP'$

$CG \cdot DG = GP' \cdot GP$

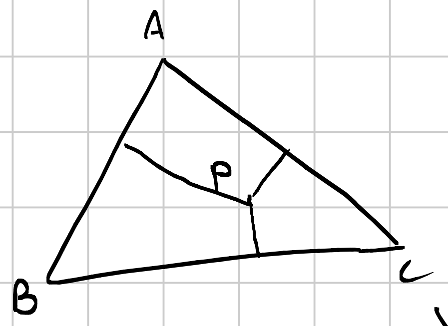
6.



Oss 1
H incentro di DEF

Oss 2
A, B, C excentri di DEF

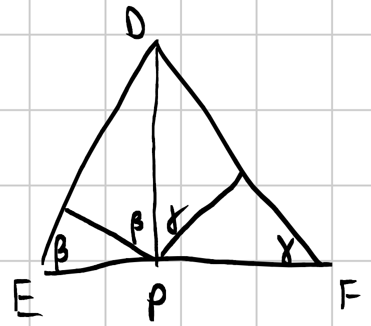
trilineari rispetto a DEF



$P = (d(P,BC), d(P,AC), d(P,AB))$

$A(-1, 1, 1)$ e cicliche

α, β, γ angoli di DEF $a = \cos\alpha$ e cicliche



$P = (0, \cos\gamma, \cos\beta) = (0, c, b)$

$r_1 : a_1 x + b_1 y + c_1 z = 0$

$r_2 : a_2 x + b_2 y + c_2 z = 0$

$X = (b_1 c_2 - b_2 c_1, c_1 a_2 - c_2 a_1, a_1 b_2 - a_2 b_1)$

$P = (a_1, b_1, c_1)$ $Q(a_2, b_2, c_2)$

$r : (b_1 c_2 - b_2 c_1) x + (c_1 a_2 - c_2 a_1) y + (a_1 b_2 - a_2 b_1) z = 0$

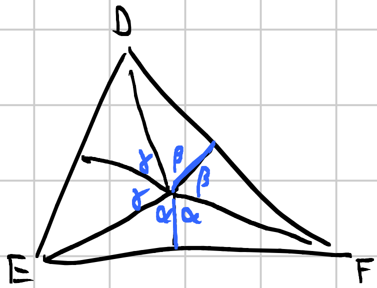
$r_{AP}: \ (b-c)x + by - cz = 0$

$r_{BQ}: \ -ax + (c-a)y + cz = 0$

$X = \big(bc + c(c-a), \ ac - c(b-c), \ (b-c)(c-a) + ab \big)$

$x = \big(\cancel{c}(\underline{b+c-a}), \ \cancel{c}(\underline{a+c-b}), \ \cancel{c}(\underline{a+b-c}) \big)$

OK → AP, BQ, CR concorrono!
(trilineari di X cicliche)



$H(1, 1, 1)$

O circocentro di DEF

$R \cos\alpha$
$R \cos\beta$
$R \cos\gamma$

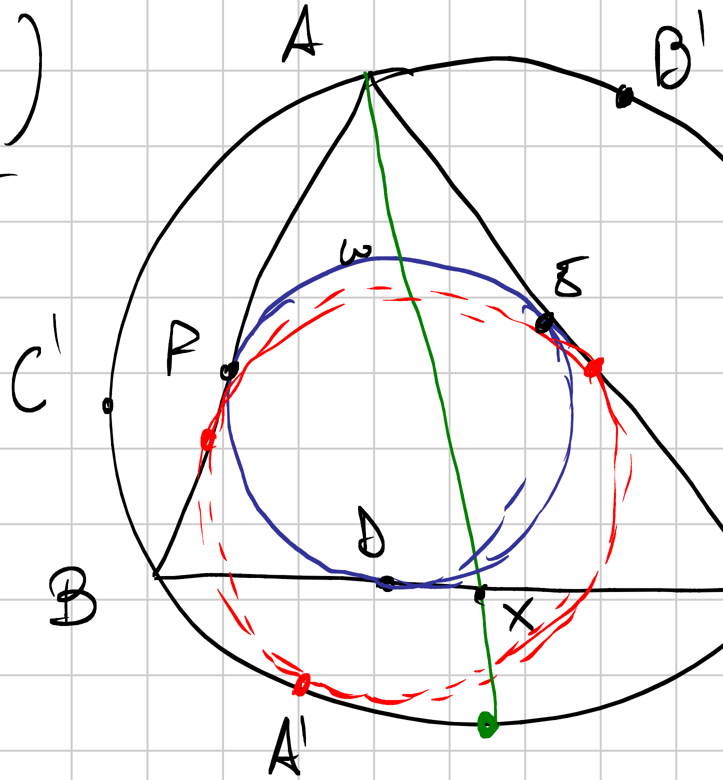
$O(a, b, c)$

$r_{HO}: \ (c-b)x + (a-c)y + (b-a)z = 0$

$$\begin{pmatrix} 1 & 1 & 1 \\ a & b & c \\ b+c-a & a+c-b & a+b-c \end{pmatrix}$$

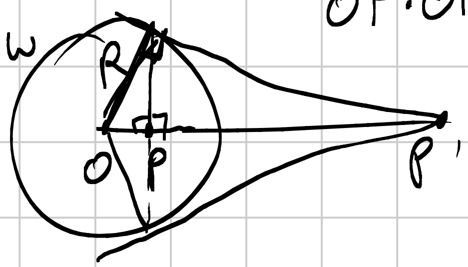
$(3^a \text{ riga}) + 2 \cdot (2^a \text{ riga}) - (a+b+c)(1^a \text{ riga}) = 0$

G7)



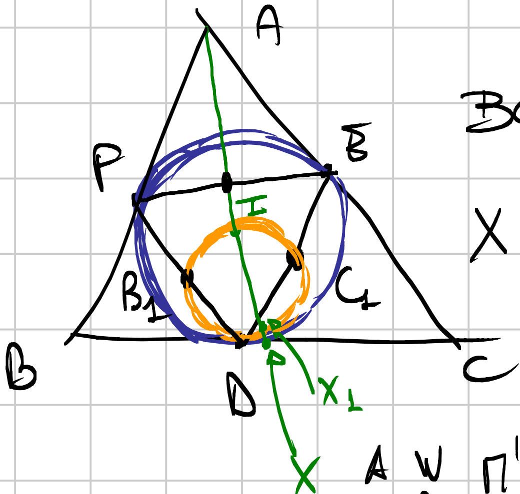
Th: $B'C'DX$ ciclico

Inversione in ω
- D, E, F fissi
- $A, B, C \to$ pt. medi di EF, FD, DE.



$OP \cdot OP' = R^2$

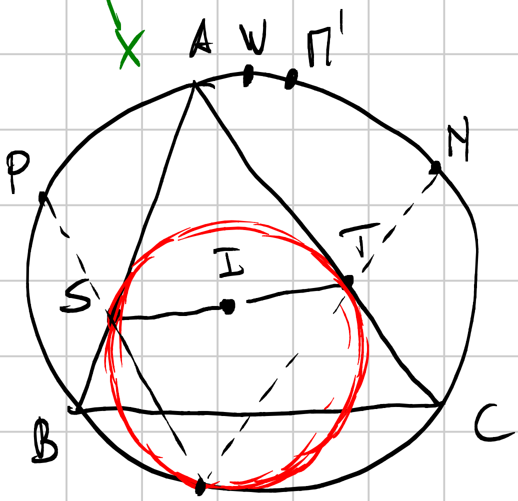
l'inv. di P' in ω è il pt. medio della corda tra i punti di tangenza delle tangenti a ω da P'.



$BC \to \Gamma_a$ cfr per I, B_1, C_1
(inv di B, inv di C)

$X \to X_1$ = seconda intersezione di $AI = A_1 I$ e Γ_a

Oss:



$A'I$ biseca $\widehat{BA'C}$

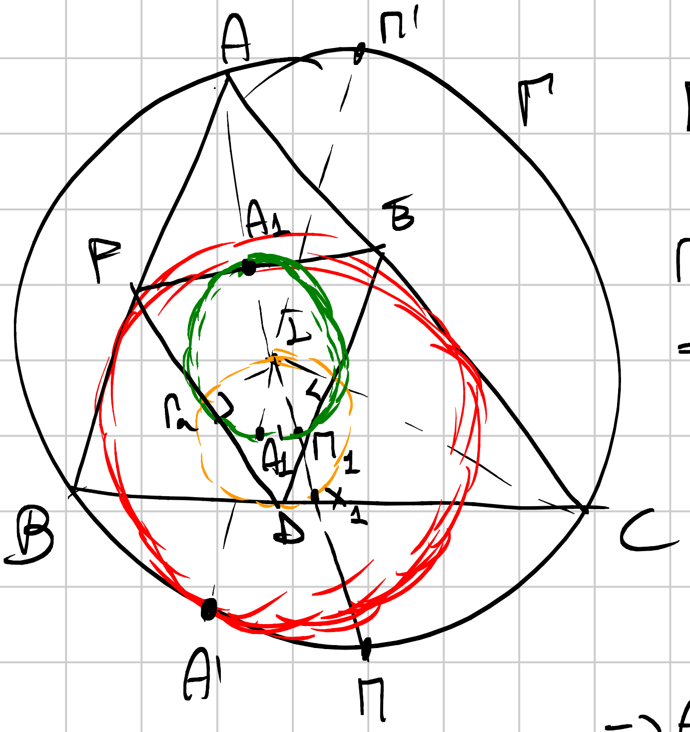
M' = pt. medio dell'arco BC che contiene A.

$\Rightarrow A', I, M'$ sono allineati.

SottoOss: S, I, T allineati e I pt medio ST.

P, S, A', A', T, M allineati; bisett. di $\widehat{PA'M}$ è $A'W$

W = pt medio di AM' ⇒ AA', $A'I$ simm. risp ad AW per il lemma delle simm. ⇒ A', I, M' allineati.



Γ_1 = cfr per i punti medi di $\triangle DEF$

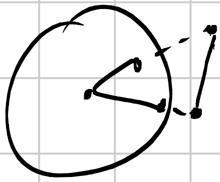
M pt medio di $\widehat{BC}$

⇒ M_1 = seconda int. di AI con Γ_a

$A_1\hat{M}_1A_1' = I\hat{M}_1A_1' = I\hat{A'}M =$

$= M'\hat{A'}M = \frac{\pi}{2}$

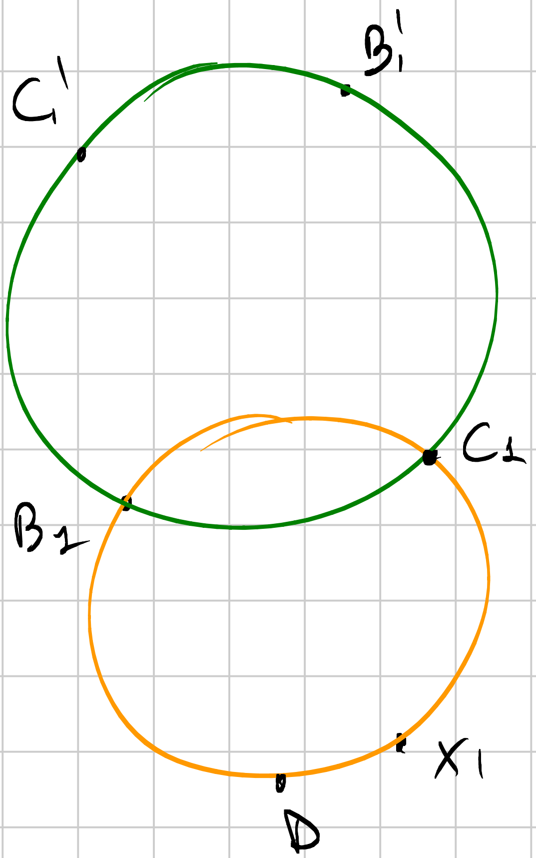
⇒ A_1' è diam. opp. a A_1 in Γ_1



⇒ $B_1'C_1' \parallel B_1C_1$

$I\hat{X}_1D = \frac{\pi}{2}$ ⇒ $X_1D \perp IX_1 = IA = IA_1 = IM_1$

⇒ $X_1D \parallel B_1C_1$



B_1C_1 corda comune

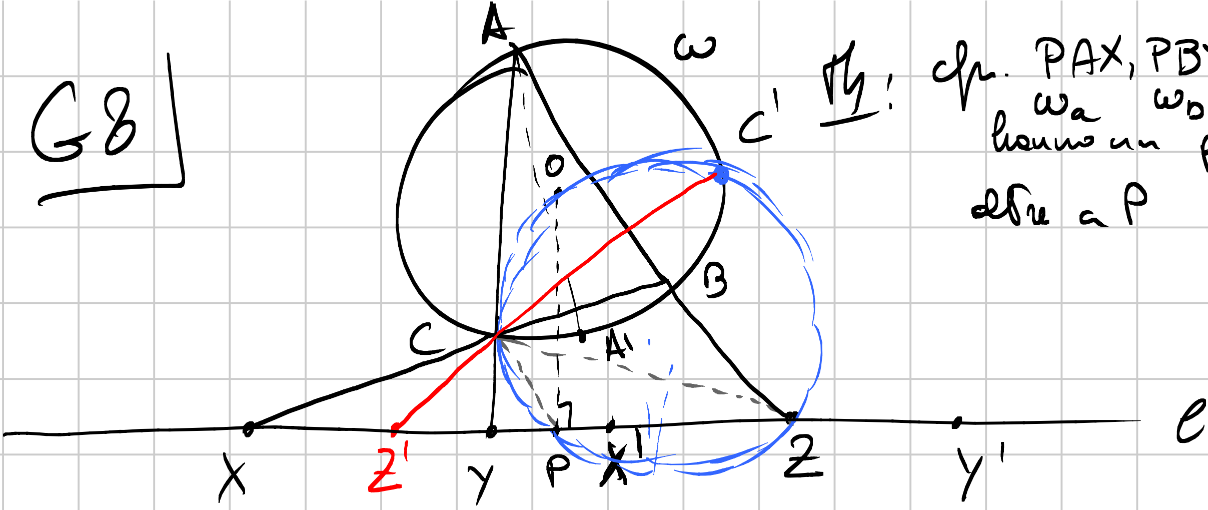
⇒ Asse di B_1C_1 contiene i centri

⇒ è asse anche di DX_1 e $B_1'C_1'$

$DX_1B_1'C_1'$ è un trapezio isoscele

⇒ ciclico.

G8



<u>Th</u>: cfr. PAX, PBY, PCZ (ω_a, ω_b, ω_c) hanno un punto in comune oltre a P

<u>Th</u> $\Rightarrow$ cfr sono coassiali $\Leftrightarrow$ gli assi radicali di $\{\omega_a, \omega\}$ $\{\omega_b, \omega\}$ e $\{\omega_c, \omega\}$ concorrono.

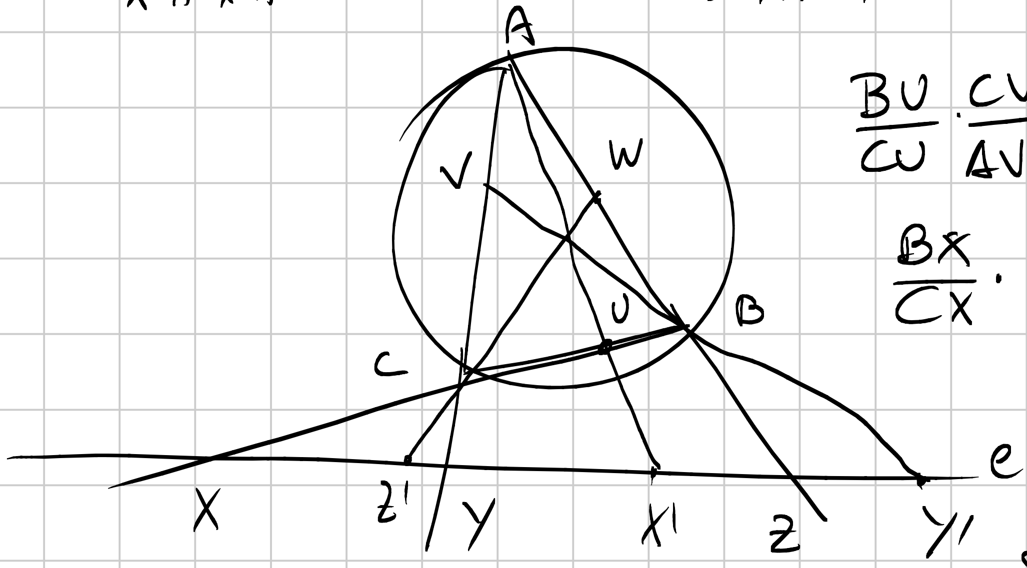
$\omega \cap \omega_a = \{A, A'\}$ $\quad \omega \cap \omega_b = \{B, B'\}$, $\quad \omega \cap \omega_c = \{C, C'\}$

$X' = AA' \cap \ell$ $\quad Y' = BB' \cap \ell$ $\quad Z' = CC' \cap \ell$

$$pow_{\omega_a}(X') = X'P \cdot X'X = X'P(X'P + PX) = (X'P)^2 + X'P \cdot PX$$

$$\| \qquad\qquad\qquad \|$$

$$X'A' \cdot X'A = X'O^2 - r^2 \quad \Rightarrow \quad X'P \cdot PX = OP^2 - r^2 = k^2$$



$$\frac{BU}{CU} \cdot \frac{CV}{AV} \cdot \frac{AW}{BW} \stackrel{?}{=} -1$$

$$\frac{BX}{CX} \cdot \frac{CY}{AY} \cdot \frac{AZ}{BZ} = 1$$

↘ vero x Menelao

$$\frac{BU}{CU} \Big/ \frac{BX}{CX} \cdot \frac{CV}{AV} \Big/ \frac{CY}{AY} \cdot \frac{AW}{BW} \Big/ \frac{AZ}{BZ} \stackrel{?}{=} -1$$

$\|$

(B, C, U, X) proietto da BC su ℓ da A

$\| \quad (Z, Y, X', X)$ $\qquad t \to -\frac{t}{k^2}$

$$\frac{zx'}{yx'} \cdot \frac{\cancel{yx}}{\cancel{zx}} \cdot \frac{xy'}{zy'} \cdot \frac{\cancel{zy}}{\cancel{xy}} \cdot \frac{yz'}{xz'} \cdot \frac{\cancel{xz}}{\cancel{yz}} =$$

$$= \frac{zx'}{yx'} \cdot \frac{yz'}{zz'} \cdot \frac{xy'}{zy'} \cdot \frac{zz'}{xz'} = -1$$

$$\frac{zx'}{yx'} \cdot \frac{yz'}{zz'} = (z, y, x', z') \qquad \frac{xy'}{zy'} \cdot \frac{zz'}{xz'} = (x, z, y', z')$$

$$t \to -\frac{t}{k^2}$$

$$(z', y', x, z)$$