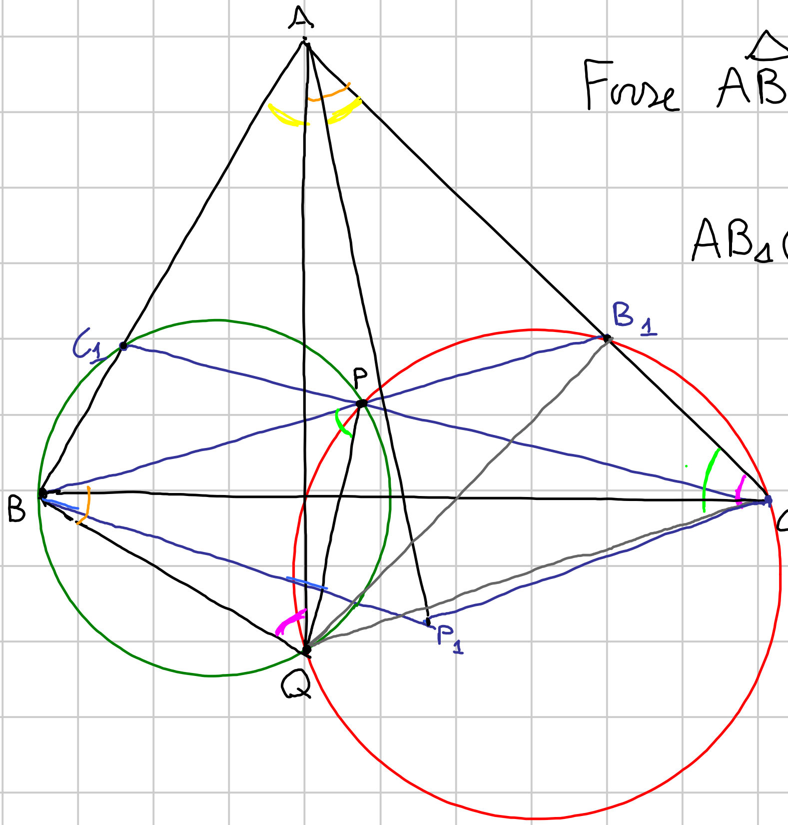
PreIMO '16 - GEOMETRIA (pomeriggio)

Es. 5



Forse $\overset{\triangle}{ABQ} \sim \overset{\triangle}{AP_1C}$

AB_1QB ciclico

$Q\hat{B_1}C = Q\hat{P}C$

$= 180° - Q\hat{P}C_1$

$= C_1\hat{B}Q = A\hat{B}Q$

$$B\hat{Q}A \underset{\substack{\uparrow \\ ABQB_1 \\ \text{ciclico}}}{=} B\hat{B_1}A \underset{\substack{\uparrow \\ CP_1 // BB_1}}{=} A\hat{C}P_1$$

Vogliamo provare a dimostrare $\dfrac{AQ}{BQ} = \dfrac{AC}{CP_1}$

Questa è vera $\Leftrightarrow \dfrac{AQ}{BQ} = \dfrac{AC}{PB} \Leftrightarrow \dfrac{AQ}{AC} = \dfrac{BQ}{BP}$

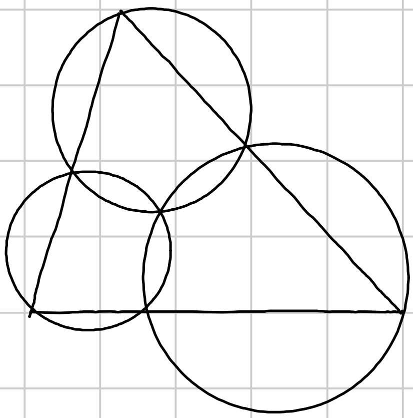
Vediamo $\overset{\triangle}{AQC} \sim \overset{\triangle}{BQP}$

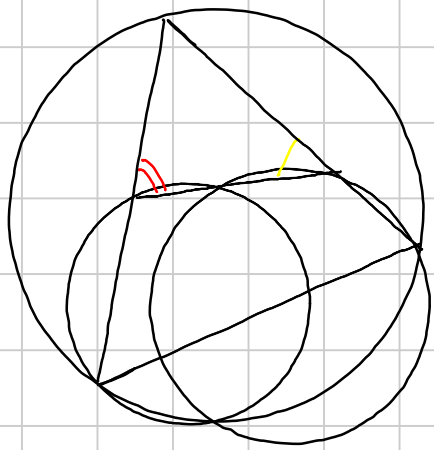
$Q\hat{B}P = Q\hat{A}C$ (per ciclicità $ABQB_1$)

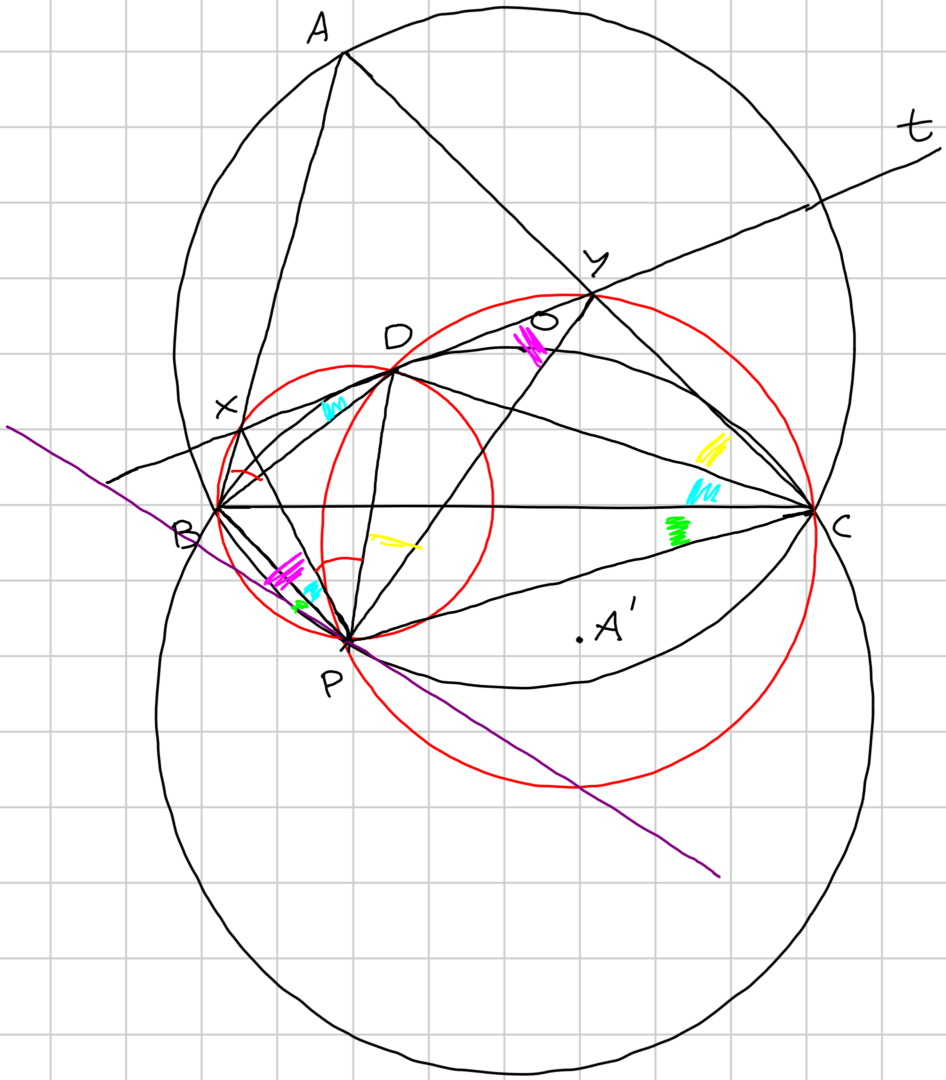
$$B\hat{P}Q = 180° - Q\hat{P}B_1 \underset{\substack{\uparrow \\ \text{ciclicità} \\ \text{rossa.}}}{=} Q\hat{C}B_1$$

$\Rightarrow \overset{\triangle}{BQP} \sim \overset{\triangle}{AQC} \Rightarrow \overset{\triangle}{ABQ} \sim \overset{\triangle}{AP_1C} \Rightarrow B\hat{A}Q = P_1\hat{A}C$

Problema 2







Miquel inverso $\to$ le circ. rosse si incontrano
su $\odot(ABC)$

STEP 1: $XYA'P$ ciclico

$\iff \widehat{XPY} = \alpha$

$\widehat{XPY} = \widehat{XPD} + \widehat{DPY} = \widehat{XBD} + \widehat{DCY} =$

$= \pi - \alpha - \widehat{DBC} - \widehat{BCD} =$

$\pi - \alpha - (\pi - 2\alpha) = \alpha$

STEP 2: r tangente $\odot(ABC)$ in P

Ts $\iff \widehat{rPX} = \widehat{PYX}$

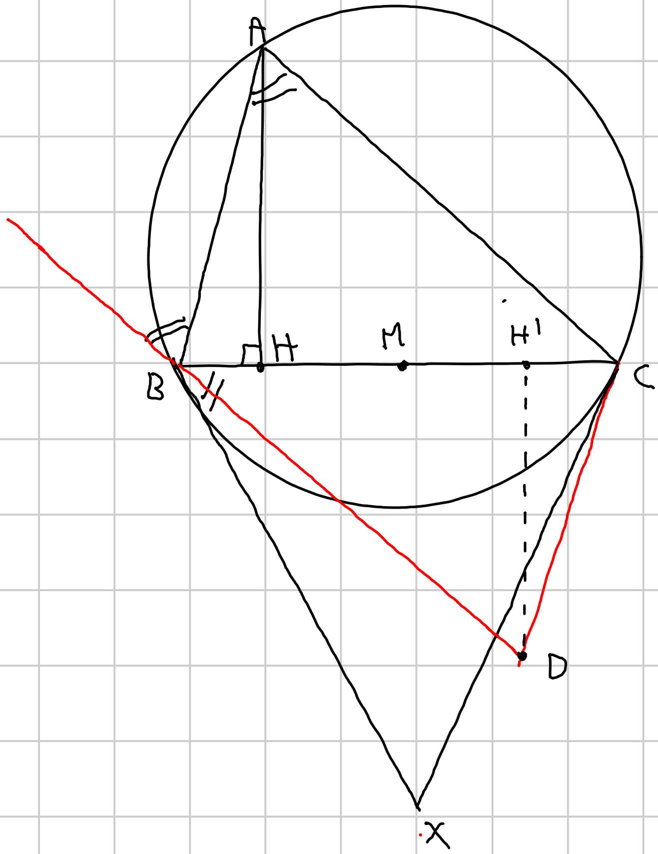
$\widehat{rPX} = \widehat{rPB} + \widehat{BPX}$

$\widehat{rPB} = \widehat{PCB}$

$\widehat{BPX} = \widehat{BDX} = \widehat{BCD}$

$\widehat{rPX} = \widehat{PCB} + \widehat{BCD} = \widehat{PCD}$

ESERCIZIO 7



$\angle(BD, AB) = \angle(AC, AB)$

$\Rightarrow BD \parallel AC$

$\Rightarrow ABDC$ é un parallelogrammo

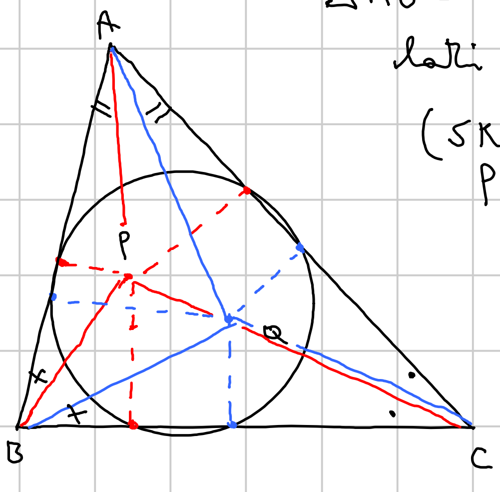
D simm. di A risp. a M

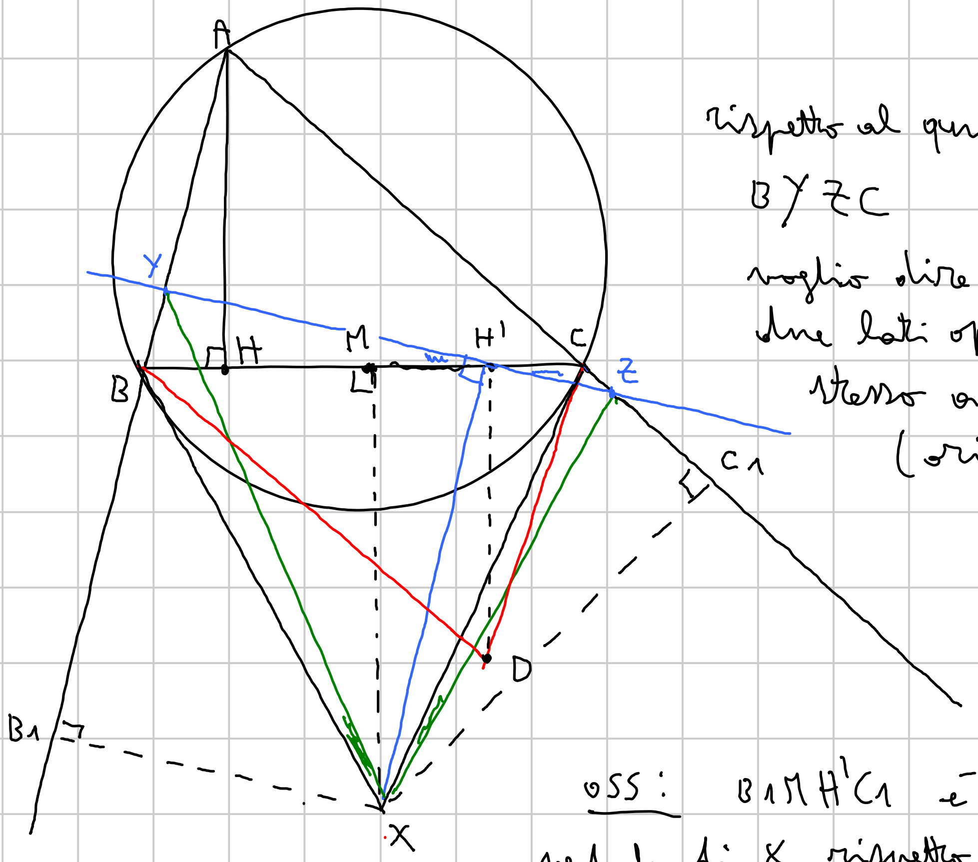
H' simm. di H risp. a M

allora $AH \perp BC \Rightarrow DH' \perp BC$

LEMMA: se P e Q sono coniugati isogonali in $\triangle ABC$ allora le loro proiezioni sui lati stanno su una circonferenza

(SKETCH: se faccio i simmetrici di P risp. a BC, CA, AB ho un triangolo di centro Q)





rispetto al quadrilatero

$BYZC$

voglio dire che X vede due lati opposti sotto lo stesso angolo (orientato)

OSS: $B_1MH'C_1$ è il quadrilatero pedale di X rispetto a $BYZC$

ma D è il coniugato isogonale di X in $\triangle ABC$

e $DH' \perp BC$

LEMMA $\Rightarrow$ $B_1MH'C_1$ è ciclico

Ora $$B\widehat{X}Y = B_1\widehat{X}Y - B_1\widehat{X}B$$

$$B_1\widehat{X}Y = B_1\widehat{H'}Y, \qquad B_1\widehat{X}B = B_1\widehat{M}B$$

$$B_1\widehat{H'}Y - B_1\widehat{M}B$$

$$B_1\widehat{H'}Y = B_1\widehat{H'}B + B\widehat{H'}Y$$

$$B_1\widehat{H'}B + B\widehat{H'}Y - B_1\widehat{M}B = B\widehat{H'}Y - H'\widehat{B_1}M$$

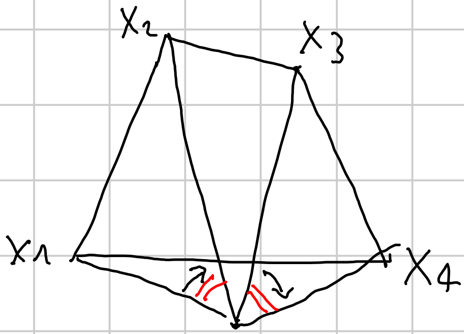
analogamente $C\widehat{X}Z = C\widehat{H'}Z - H'\widehat{C_1}M$

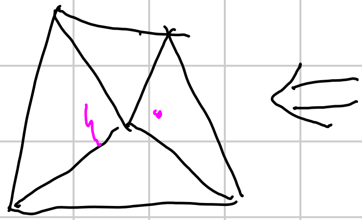
sapendo che $H'B_1C_1M$ è ciclico ottengo

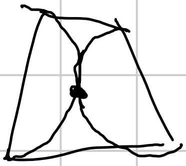
$$H'\widehat{B_1}M = H'\widehat{C_1}M$$

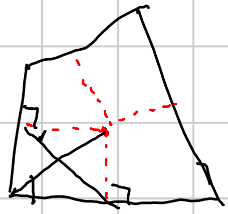
e quindi $B\widehat{X}Y = C\widehat{X}Z$

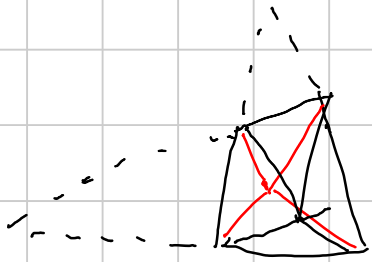
IN GENERALE:

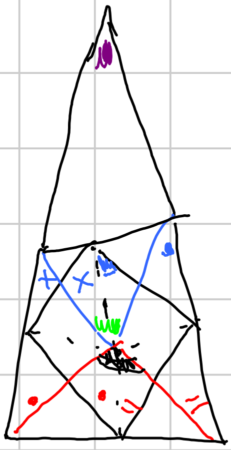




a)


b)




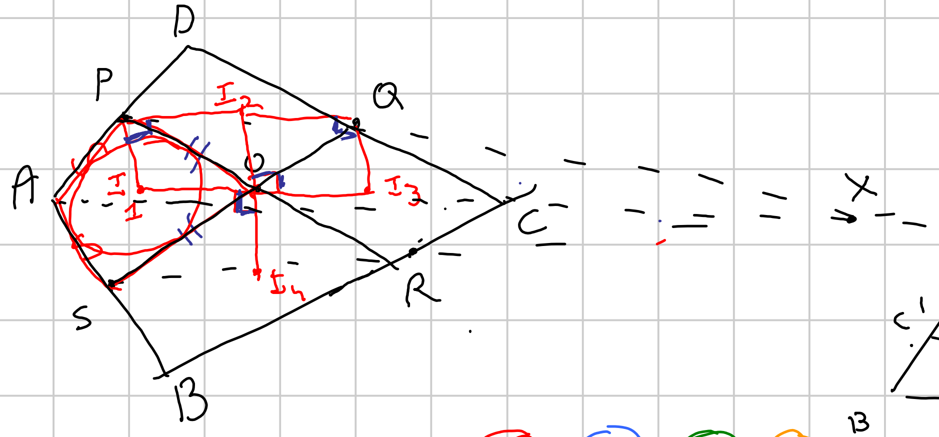


$\angle + \angle + \angle = \angle$

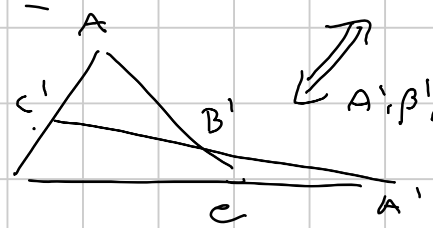
e ho $\angle + \angle = 180^\circ$

Esercizio 8

PQ, AC, RS concorrono (o sono parallele)



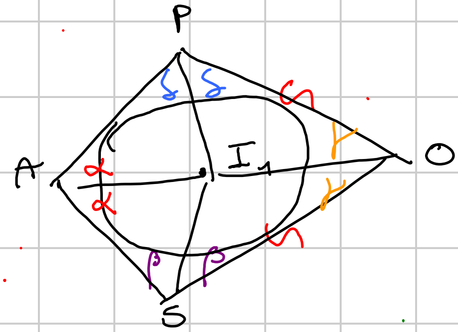
$$\frac{AC'}{C'B}\cdot\frac{BA'}{A'C}\cdot\frac{CB'}{B'A}=1 \iff A', B', C' \text{ allin}$$



Concorrenza rette $\iff \underline{\frac{AP}{PD}\cdot\frac{DQ}{QC}\cdot\frac{CR}{RB}\cdot\frac{BS}{SA}=1}$

$$\frac{AP}{SA}=\frac{\sin(\alpha+\delta)\,PI_1}{\underbrace{\sin(\alpha+\beta)}\,SI_1}$$

$$\frac{OP}{SO}=\frac{\sin(\gamma+\delta)\,PI_1}{\underbrace{\sin(\beta+\gamma)}\,SI_1}$$



$$\sin x\sin y=\frac{\cos(x-y)-\cos(x+y)}{2}$$

$$\sin(\alpha+\delta)\sin(\gamma+\delta)=\frac{\cos(\alpha-\gamma)-\cos(\alpha+\gamma+2\delta)}{2}$$

$$\sin(\alpha+\beta)\sin(\beta+\gamma)=\frac{\cos(\alpha-\gamma)-\cos(\alpha+\gamma+2\beta)}{2}$$

$$\boxed{\frac{AP}{SA}\cdot\frac{OP}{SO}=\frac{PI_1^2}{SI_1^2}}$$

Identità

$$\frac{AP}{SA}=\frac{PI_1^2}{SI_1^2}\cdot\frac{SO}{OP}$$

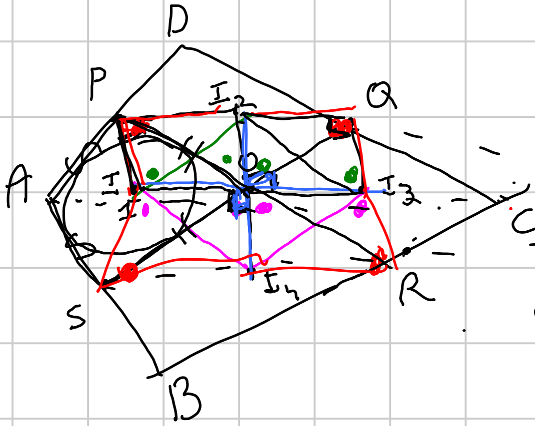
$$\frac{DQ}{PD}=\frac{QI_2^2}{PI_2^2}\cdot\frac{OP}{OQ}$$

$$\frac{CR}{QC}=\frac{RI_3^2}{QI_3^2}\cdot\frac{OQ}{OR}$$

$$\frac{BS}{RB}=\frac{SI_4^2}{RI_4^2}\cdot\frac{OR}{OS}$$

Th $\iff \cdot\cdot\cdot\cdot = 1$

$$\iff \frac{PI_1^2}{SI_1^2}\cdot\frac{QI_2^2}{PI_2^2}\cdot\frac{RI_3^2}{QI_3^2}\cdot\frac{SI_4^2}{RI_4^2}=1$$



Th $\iff \cdot\cdot\cdot\cdot = 1$

$$\iff \frac{PI_1^2}{SI_1^2}\cdot\frac{QI_2^2}{PI_2^2}\cdot\frac{RI_3^2}{QI_3^2}\cdot\frac{SI_4^2}{RI_4^2}=1$$

Oss. QI_3OI_2 ciclico $\Rightarrow$

$\Rightarrow Q\hat{I_3}I_2 = Q\hat{O}I_2 = I_2\hat{O}P = I_2\hat{I_1}P$

$\downarrow$

PI_2OI_1 ciclico

Quindi $PI_1I_2 \sim QI_3I_2$

Quindi $\frac{PI_1}{PI_2}=\frac{QI_3}{QI_2} \Rightarrow \boxed{\frac{PI_1}{PI_2}\cdot\frac{QI_2}{QI_3}=1}$

Analogamente $RI_3I_4 \sim SI_1I_4$

$\frac{RI_3}{RI_4}=\frac{SI_1}{SI_4} \Rightarrow \boxed{\frac{RI_3}{RI_4}\cdot\frac{SI_4}{SI_1}=1}$

È simple ■ + ■ $\longrightarrow$ th.

II "IDEA" di soluzione

Se vi interessa ... In realtà anche ABCD è circoscrivibile.

Il punto di concorrenza è il centro di omotetia fra "altre" due circonferenze...