PreIMO-18 - Geometrie - Afternoon

Note Title 5/24/2018

X_{YZ} := piede della $\perp$ da X ad YZ.

$ABCDEF$ esagono ciclico
le tre intersezioni di
$A_{FD}A_{ED}$, $B_{DE}B_{EF}$, $C_{DF}C_{EF}$ e dei corrispondenti sono concicliche.

$\mathbb{C}$ con $\odot(ABCDEF)$ unitario $a\bar{a}=1$ e cyc.

A_{BC} $\qquad BC:\ z+bc\bar{z}=b+c$

$$\frac{z-a}{\bar{z}-\bar{a}}=-\frac{b-c}{\bar{b}-\bar{c}}=bc$$

$$z=a+bc\bar{z}-\frac{bc}{a}$$

$$a_{bc}=z=\frac{1}{2}\left[a+b+c-\frac{bc}{a}\right]$$

a_{fd}, a_{de}

$$A_{FD}A_{DE}:\quad \frac{z-a_{fd}}{\bar{z}-\overline{a_{fd}}}=\frac{a_{fd}-a_{de}}{\overline{a_{fd}}-\overline{a_{de}}}$$

$$a_{fd}-a_{de}=\frac{1}{2}\left[\cancel{a}+\cancel{d}+f-\frac{fd}{a}-\cancel{a}-\cancel{d}-e+\frac{de}{a}\right]=\frac{1}{2a}\underbrace{\left[af+de-fd-ae\right]}_{(a-d)(f-e)}$$

$$\frac{(a-d)(f-e)}{2a}$$

$$\overline{a_{fd}}-\overline{a_{de}}=\frac{1}{2}\cancel{a}\frac{(a-d)(f-e)}{\cancel{a}dfe}=$$

$$A_{FD}A_{DE}:\quad \frac{z-a_{fd}}{\bar{z}-\overline{a_{fd}}}=\frac{def}{a}\iff z-a_{fd}=\frac{def}{a}\bar{z}-\frac{def}{a}\overline{a_{fd}}$$

$$\overline{a_{fd}}=\frac{1}{2}\left[\frac{1}{a}+\frac{1}{d}+\frac{1}{f}-\frac{a}{df}\right]=\frac{1}{2}\frac{df+ad+af-a^2}{adf}$$

$$z = \frac{def}{a}\bar{z} + \frac{1}{2}\left(a + d + f + \frac{df}{a}\right) - \frac{1}{2a^2}\left(def + ede + aef - a^2 e\right)$$

$$\begin{cases} a z = \frac{def}{\cancel{a}}\bar{z} + \frac{1}{2a^{\cancel{2}}}\left(a^3 + a^2 d + a^2 f + a^2 e - a(df + de + ef) - def\right) \\ b z = \frac{def}{\cancel{b}}\bar{z} + \frac{1}{2b^{\cancel{2}}}\left(b^3 + b^2\left(\sum d\right) - b\sum de - def\right) \end{cases}$$

$$(b-a)z = \frac{(b-a)}{2ab}\left[ab(a+b) + ab\sum d + def\right]$$

BOH FARE ATTENZIONE AI SEGNI

$$c_1 = \frac{1}{2}\left(a + b + d + e + f + \frac{def}{ab}\right)$$

$$c_1 = \frac{1}{2}\left(\sum_{i=1}^{6} a + \frac{def - abc}{ab}\right)$$

ci metto anche la c

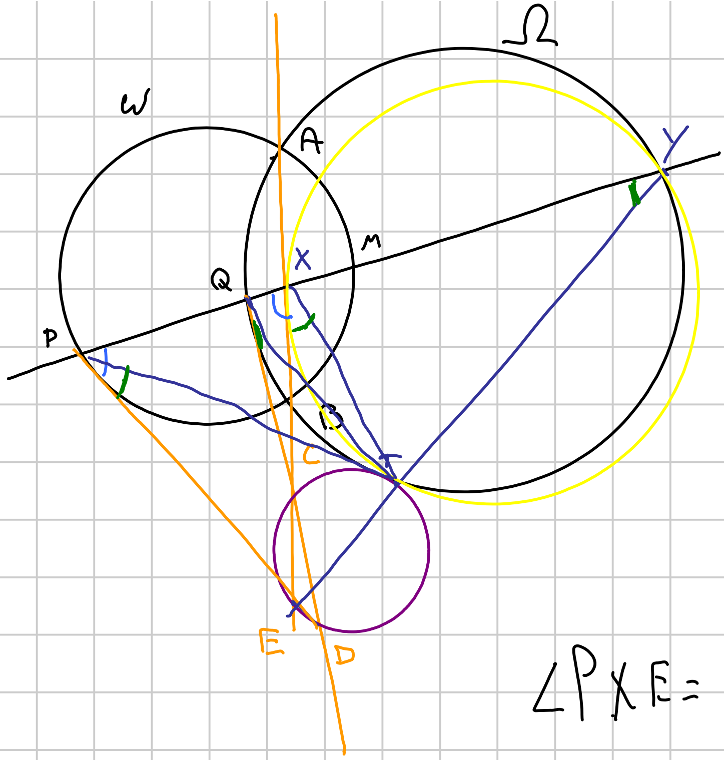
$$c_1 - \frac{1}{2}\sum_{i=1}^{6} a = \frac{1}{2}\,\frac{def - abc}{ab}$$

Basta anche solo dire $|ab| = 1$ e $|def - abc| = |abc - def|$ e uso la simmetria

$$\overline{c_1} - \frac{1}{2}\overline{\sum_{i=1}^{6} a} = \frac{1}{2}\,\frac{(abc - def)\cancel{ab}}{\cancel{ab}cdef} = \frac{1}{2}\,\frac{abc - def}{cdef}$$

$$\left(c_1 - \frac{1}{2}\sum \quad\right)\left(\overline{\quad\quad}\right) = -\frac{1}{4}\,\frac{(abc - def)^2}{abcdef}$$

e si verifica essere simmetrica



PROBLEMA 2

$\odot_{CDE}$ tange Ω

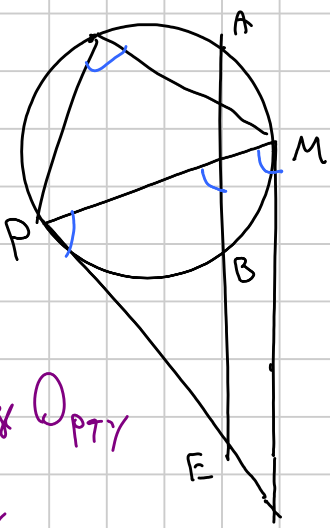
$C = AB \cap r_Q \qquad D = r_Q \cap r_P$

$E = AB \cap r_P$

$Y = PQ \cap \Omega \quad (Y \neq Q)$

$X = PQ \cap AB$

$\angle PXE = \angle XPE = \underset{Z}{\angle PZM} \quad \forall Z$ in ω



Dunque PXE è isoscele $\quad PE = XE$.

$T = EY \cap \Omega$

$Pow_\omega(E): PE^2 = EB \cdot EA = ET \cdot EY$

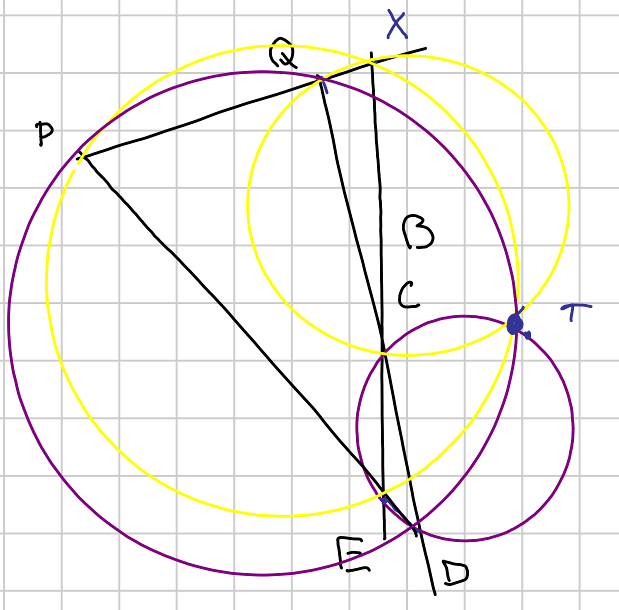
$PE = EX \Rightarrow EX^2 = ET \cdot EY$

EP tang $\odot_{PTY}$

EX tang $\odot_{XTY}$

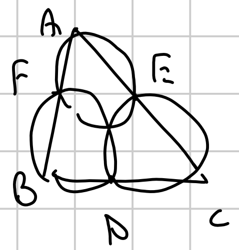
Siccome $\angle TXE = \angle TPE \to TPXE$ è ciclico

QC tange $\Omega \Rightarrow \angle CQT = \angle QYT = \angle CXT \Rightarrow QXCT$ ciclico



DIM VELOCE MIQUEL

MIQUEL △



$\bigcirc_{AFE}$, $\bigcirc_{BFD}$, $\bigcirc_{CED}$

concorrono

(conto d'angoli)

Applicando a $\triangle PXE$ con punti Q, C, D

le $\bigcirc_{XQC}$, $\bigcirc_{XDE}$, $\bigcirc_{CDE}$ concorrono.

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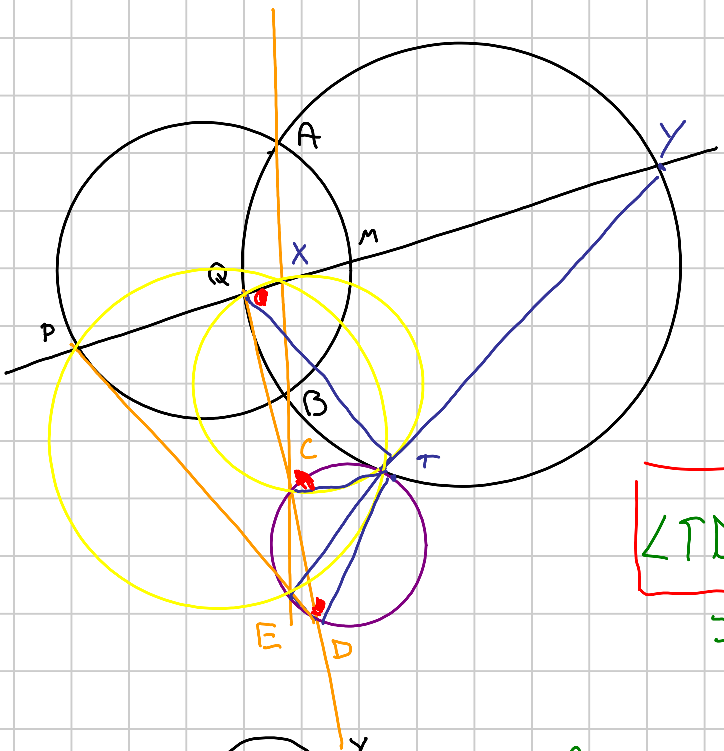


Supponiamo TCED ciclico

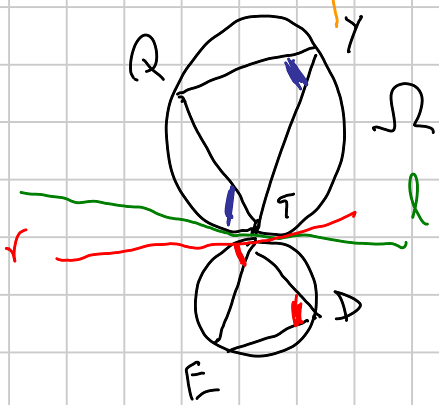
per ipotesi, serve la tangente in T

$\angle TDE = 180 - \angle TCE = \angle TCX =$

$= \angle TQX$

$\boxed{\angle TDE + \angle TYQ} = \angle TQY + \angle TYQ =$

$= 180 - \angle QTY = \boxed{\angle QTE}$



ℓ tang Ω $\quad \angle(TE, r) = \angle TDE$

r tang $\bigcirc_{TDE}$ $\quad \angle(TQ, \ell) = \angle TYQ$

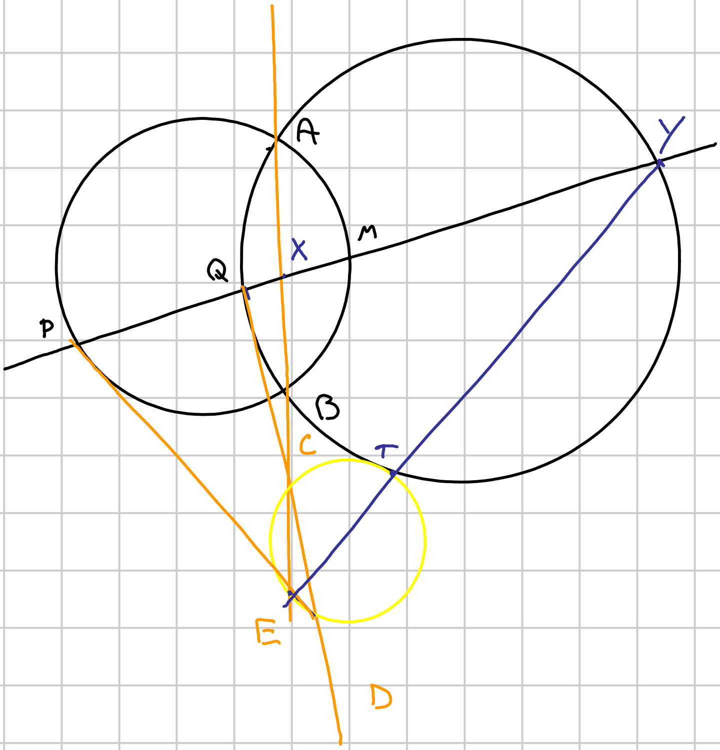
Facendo la somma degli angoli

$\angle QTE = \angle(QT, \ell) + \angle(r, \ell) + \angle(r, TE)$

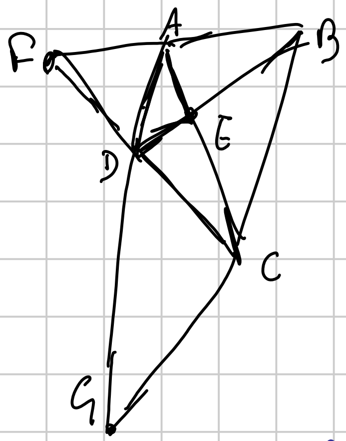
$= \angle QTE + \angle(r, \ell) \Rightarrow \angle(r, \ell) = 0$

$\Rightarrow$ r e ℓ coincidono =

$\ell = V$ Coze Sus a Ω de a Θ_{TCDE}


A
Y
M
X
Q
P
B
C
T
E
D

PROBLEMA [3]



COORDINATE BARICENTRICHE CON RIFERIMENTO USO $\boxed{EBC}$ (o EDC)

$BC=a,\ EC=b,\ EB=c$

$E=(1,0,0)\quad B=(0,1,0)\quad C=(0,0,1)$

$A=(\alpha,0,\gamma)\quad D=(\delta,\beta,0)$

$\sum_{cyc} a^2yz=\left(\sum_{cyc} x\right)\left(\sum_{cyc} ux\right)$ per opportuni $u,v,w\in\mathbb{R}$

circonferenza generica

Considero $\odot(ABCD)$, sicuramente $v=w=0$

$b^2\cancel{\alpha}\gamma=(\alpha+\gamma)u\cancel{\alpha}$

$c^2\cancel{\delta}\beta=(\delta+\beta)u\cancel{\delta}$

$$\boxed{\frac{b^2\gamma}{\alpha+\gamma}=\frac{c^2\beta}{\beta+\delta}}\quad (Eq\ 1)$$

$\odot ABE:\ \sum_{cyc} a^2yz=\left(\sum_{cyc} x\right)wz$ $\Leftarrow B,E$

$b^2\alpha\cancel{\gamma}=(\alpha+\gamma)w\cancel{\gamma}$ $\Leftarrow A$ $\qquad w=\frac{b^2\alpha}{\alpha+\gamma}$

$BC:\ x=0$

$a^2y\cancel{z}=\frac{b^2\alpha}{\alpha+\gamma}\cancel{z}(y+z)$

$y(a^2(\alpha+\gamma)-b^2\alpha)=zb^2\alpha\qquad P=(0,\ b^2\alpha,\ a^2(\alpha+\gamma)-b^2\alpha)$

$\odot ADE:\ \sum_{cyc} a^2yz=\left(\sum_{cyc} x\right)(vy+wz)$ $\Leftarrow E$

$b^2\alpha\cancel{\gamma}=(\alpha+\gamma)w\cancel{\gamma}$ $\Leftarrow A$ $\qquad w=\frac{b^2\alpha}{\alpha+\gamma}$

$$c^2 \cancel{\beta} \delta = (\beta+\delta) v \cancel{\beta} \quad A - D \qquad v = \frac{c^2\delta}{\beta+\delta}$$

CD: $x\beta = y\delta$

Strada 1: sostituisco $x=\delta, y=\beta$ e trovo z

Strada 2:

$$a^2\beta y z + b^2\delta y z + c^2\delta y^2 = (y\delta + y\beta + z\beta)\left(\frac{b^2\alpha}{\alpha+\gamma} z + \frac{c^2\delta}{\beta+\delta} y\right)$$

$$y[a^2\beta + b^2\delta] = y\left[\frac{\beta+\delta}{\alpha+\gamma} b^2\alpha\right] + y \underbrace{\frac{c^2\beta\delta}{\beta+\delta}}_{\frac{b^2\gamma\delta}{\alpha+\gamma}} + z\frac{b^2\alpha\beta}{\alpha+\gamma}$$

$$y\left[a^2\beta(\alpha+\gamma) + b^2\delta(\cancel{\alpha+\gamma}) - b^2\alpha(\beta+\cancel{\delta}) - \cancel{b^2\gamma\delta}\right] = z b^2\alpha\beta$$

$$y[a^2\beta(\alpha+\gamma) - b^2\alpha\beta] = z b^2\alpha\beta$$

$$Q = (b^2\alpha\delta,\ b^2\alpha\beta,\ a^2\beta(\alpha+\gamma) - b^2\alpha\beta)$$

—o—

AB: $x\gamma = z\alpha$ CD: $x\beta = y\delta$

Allora $F = (\alpha\delta, \alpha\beta, \gamma\delta)$

BC: $x = 0$ AD: $\det\begin{pmatrix} x & y & z \\ \alpha & 0 & \gamma \\ \delta & \beta & 0 \end{pmatrix} = 0$ AD: $-x\beta\gamma + y\delta\gamma + z\alpha\beta = 0$

$$G = (0, \alpha\beta, -\gamma\delta)$$

$$FP: \det\begin{pmatrix} x & y & z \\ \alpha\delta & \alpha\beta & \gamma\delta \\ 0 & b^2\alpha & a^2(\alpha+\gamma) - b^2\alpha \end{pmatrix} = 0$$

$$FP: x\left[a^2(\alpha+\gamma)\cancel{\alpha}\beta - b^2\cancel{\alpha}(\alpha\beta+\gamma\delta)\right] - y\cancel{\alpha}\delta\left(a^2(\alpha+\gamma) - b^2\alpha\right) + z b^2\alpha^{\cancel{2}}\delta = 0$$

$$GQ: \det\begin{pmatrix} x & y & z \\ 0 & \alpha\beta & -\gamma\delta \\ b^2\alpha\delta & b^2\alpha\beta & a^2\beta(\alpha+\gamma) - b^2\alpha\beta \end{pmatrix} = 0$$

$\mathcal{E}Q: \; x\cancel{\alpha}\beta\left[\alpha^2\beta(\alpha+\gamma)-b^2\alpha\beta+b^2\gamma\delta\right]-yb^2\cancel{\alpha}\gamma\delta^2-zb^2\alpha^{\cancel{2}}\beta\delta=0$

Voglio la retta per A che è $\perp$ $AC \equiv EC$

$\infty = (S_C, -b^2, S_A)$ e A

$$r: \det\begin{pmatrix} x & y & z \\ \alpha & 0 & \gamma \\ S_C & -b^2 & S_A \end{pmatrix} = 0$$

$$r: xb^2\gamma + y(\gamma S_C - \alpha S_A) - z\alpha b^2 = 0$$

Tesi equivale a: (le 3 rette concorrono)

$$\det\begin{pmatrix} b^2\gamma & \gamma S_C-\alpha S_A & -\cancel{b^2}\cancel{\alpha} \\ \beta[\alpha^2\beta(\alpha+\gamma)-b^2\alpha\beta+b^2\gamma\delta] & -b^2\gamma\delta^2 & -\cancel{b^2}\cancel{\alpha}\beta\delta \\ \beta[\alpha^2\beta(\alpha+\gamma)-b^2(\alpha\beta+\gamma\delta)] & -\delta(\alpha^2(\alpha+\gamma)-b^2\alpha)\beta & \cancel{b^2}\cancel{\alpha}\delta\cdot\beta \end{pmatrix} \overset{?}{=} 0$$

$III \to III - II$

$$\det\begin{pmatrix} 2\beta[b^2\gamma] & 2\beta[\gamma S_C-\alpha S_A] & -1\cdot 2\beta \\ \beta[\alpha^2\beta(\alpha+\gamma)-b^2\alpha\beta+b^2\gamma\delta] & -b^2\gamma\delta^2 & -\beta\delta \\ -2b^2\beta\gamma\cancel{\delta} & -\beta\cancel{\delta}\alpha^2(\alpha+\gamma)+b^2\alpha\beta\cancel{\delta}+b^2\gamma\delta^{\cancel{2}} & 2\beta\cancel{\delta} \end{pmatrix} \overset{?}{=} 0$$

$III \to III + I$

$$\det\begin{pmatrix} b^2\gamma & \gamma S_C-\alpha S_A & -1 \\ \beta[\quad] & -b^2\gamma\delta^2 & -\beta\delta \\ 0 & \boxed{} & 0 \end{pmatrix} \overset{?}{=} 0$$

Basta oppure $\boxed{} = 0$ oppure $b^2\cancel{\beta}\gamma\delta \overset{?}{=} \cancel{\beta}[\alpha^2\beta(\alpha+\gamma)-b^2\alpha\beta+b^2\gamma\delta]$

Vediamo m, è falso!

Voglio $\boxed{} \overset{?}{=} 0$

$$-\alpha^2\cancel{\beta}(\cancel{\alpha+}\gamma)+b^2\alpha\beta+b^2\gamma\delta+\beta\left[\gamma(\cancel{a^2}+b^2-c^2)-\alpha(b^2+c^2-\cancel{a^2})\right] \overset{?}{=} 0$$

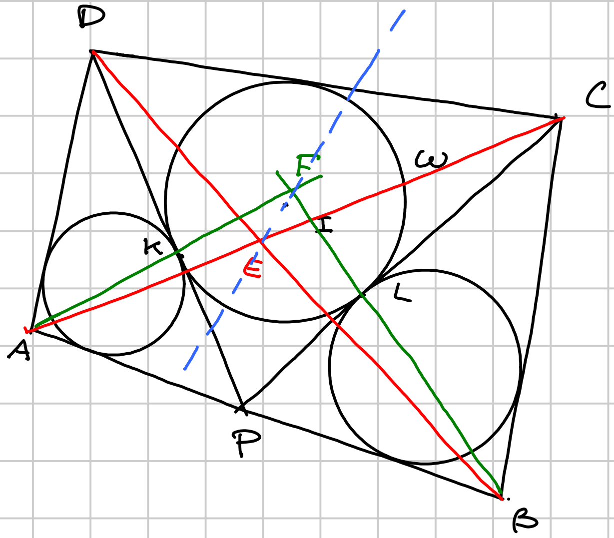
OSS (In un colore felice) NON c'è la α !!

Tesi iff $b^2\cancel{\alpha\beta} + b^2\gamma\delta + b^2\beta\gamma - c^2\beta\gamma - \cancel{\alpha\beta b^2} - \alpha\beta c^2 \overset{?}{=} 0$

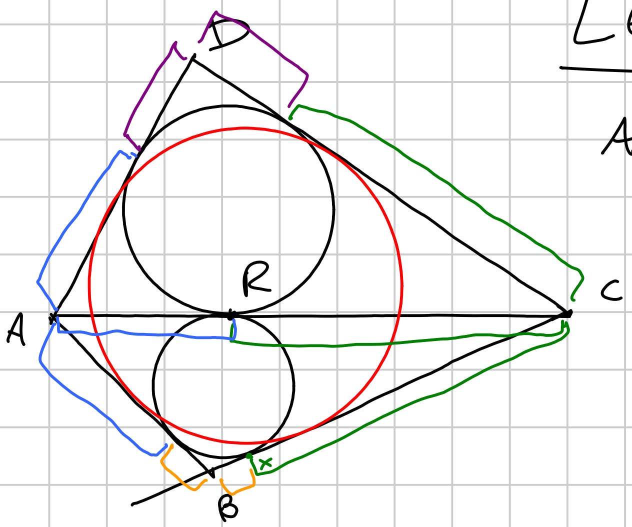
$\iff b^2\gamma(\beta+\delta) = c^2\beta(\alpha+\gamma)$ Vero!

G-4

SL2007 - G8

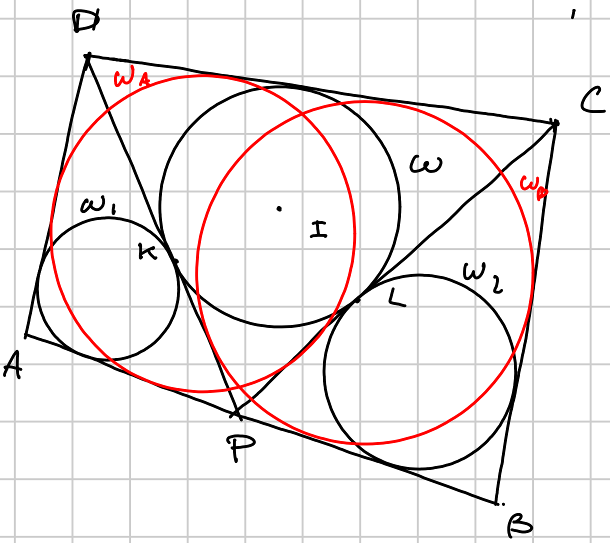


Ts E, F, I allineati



Lemma

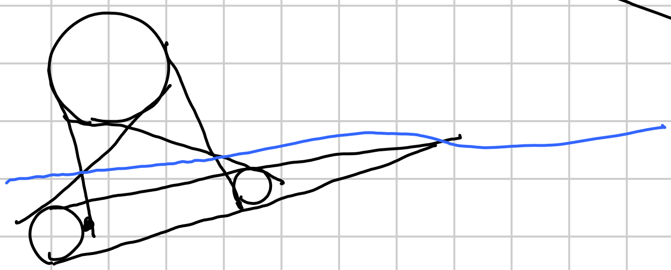
ABCD circos $\iff$ $AD + BC = AB + CD$

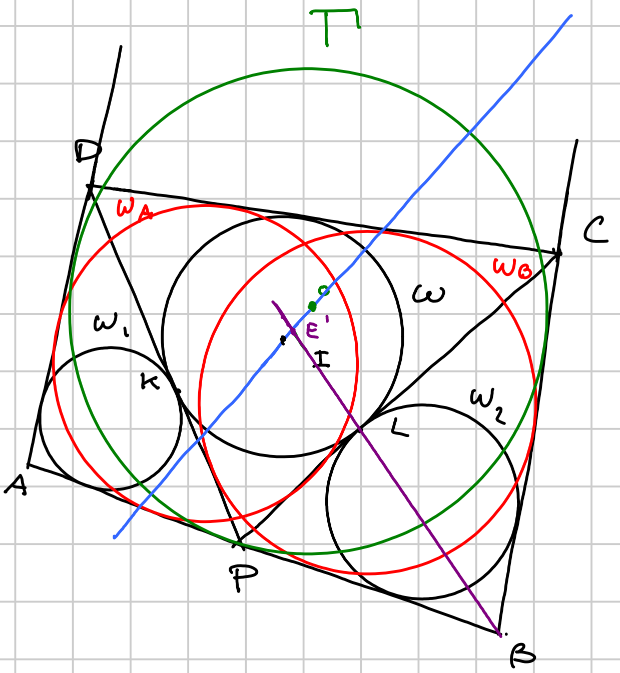


Monge

B è c.d.s ext di ω_2 e ω_B

D è c.d.s. ext di ω e ω_B





Monge

ω_1 $\quad$ ω

A e c.d.s ext

ω_1 $\quad$ ω_A

$\boxed{\omega_1 \quad \Gamma}$

Monge su ω_2, ω, Γ

c d s INT $\quad$ ω $\quad$ ω_2 $\quad$ L

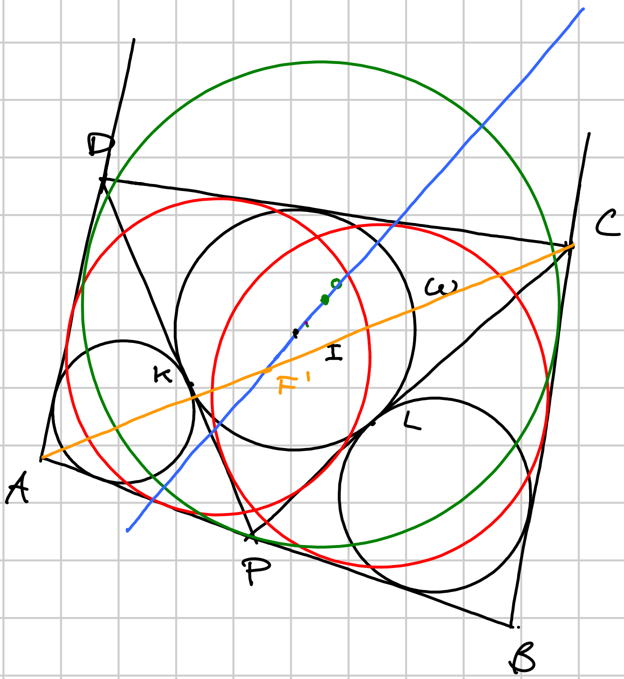
c d s EXT $\quad$ Γ $\quad$ ω_2 $\quad$ B

$BL \cap OI$ = c.d.s. INT di ω e Γ, E'

Monge ω_1, ω, Γ

$AK \cap OI$ = c.d.s. INT di ω e Γ, E'

$E' = E$ $\quad$ $E \in OI$



Monge su ω_A, Γ, ω

A c.d.s. Ext ω_A e Γ

C c.d.s Ext ω e ω_A

$\Rightarrow$ $AC \cap OI$ è c.d.s. EXT tra ω e Γ

$BD \cap OI$ è c.d.s Ext ω e Γ

$F' = AC \cap OI$ $\quad$ $F = F' \in OI$