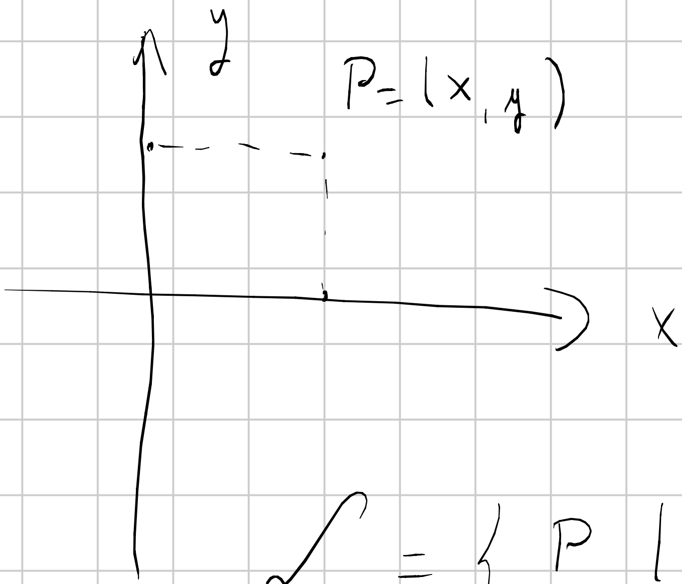
Sessione 2006 - Geometria 2 (S.M.)

Titolo nota 12/09/2006



$P = (x, y)$

$A = (a_1, a_2) \quad B = (b_1, b_2)$

$$M = \left(\frac{a_1 + b_1}{2}, \frac{a_2 + b_2}{2} \right)$$

$$d^2 = (b_1 - a_1)^2 + (b_2 - a_2)^2$$

$$\mathcal{L} = \{ P \mid \underline{PA = PB} \} \quad \text{asse di } AB$$

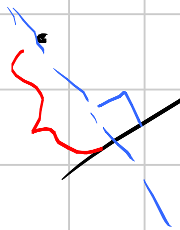
$$\sqrt{(x - a_1)^2 + (y - a_2)^2} = \sqrt{(x - b_1)^2 + (y - b_2)^2}$$

$$\cancel{x^2} - 2a_1 x + a_1^2 + \cancel{y^2} - 2a_2 y + a_2^2 = \cancel{x^2} - 2b_1 x + b_1^2 + \cancel{y^2} - 2b_2 y + b_2^2$$

$$2x(b_1-a_1)+2y(b_2-a_2)+a_1^2-b_1^2+a_2^2-b_2^2=0 \quad // \quad +(x^2+y^2)-(x^2+y^2)$$

Date r, s rette incidenti

$$\mathcal{L}=\{P \mid d(r,P)=d(s,P)\}$$



$r: a_1x+b_1y+c_1=0$

$s: a_2x+b_2y+c_2=0$

$$\ell=\{ax+by+c=0\}$$

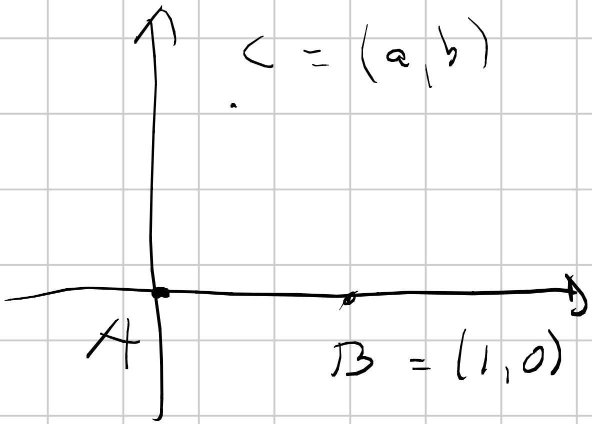
$$Q=(x_0,y_0)$$

$$d(\ell,Q)=\frac{|ax_0+by_0+c|}{\sqrt{a^2+b^2}}$$

$$\frac{|a_1x+b_1y+c_1|}{\sqrt{a_1^2+b_1^2}}=\frac{|a_2x+b_2y+c_2|}{\sqrt{a_2^2+b_2^2}}$$

| RHS = LHS

| RHS = − LHS



$L \in AB \mid LC$ bisett.

Teo bisett : $\dfrac{AL}{LB} = \dfrac{AC}{BC}$

1) Troviamo $L = (c,d)$

2) Troviamo la retta per C e L

$$\alpha(x-a) + \beta(y-b) \qquad \alpha = k(d-b) \qquad k \in \mathbb{R}^*$$

$$\alpha(c-a) = -\beta(d-b) \qquad \beta = k(a-c)$$

$$(d-b)(x-a) + (a-c)(y-b) = 0$$

$$\mathcal{L} = \{ P \mid d(A,P) = \lambda\, d(B,P) \}$$

$A = (a_1, a_2)$ $B = (b_1, b_2)$

$$P_\lambda = \lambda A + (1-\lambda) B =$$

$$= \left(\lambda a_1 + (1-\lambda) b_1 ,\ \lambda a_2 + (1-\lambda) b_2 \right)$$

$$\begin{cases} x = \\ y = \end{cases}$$

$\lambda \in \mathbb{R}$

A L B

x $AB - x$

$$\frac{x}{AB - x} = \lambda$$

$$x = (AB - x)\lambda$$

$$x(1+\lambda) = \lambda AB$$

$$x = \frac{\lambda}{1+\lambda} AB$$

$$d(P_\lambda, A)^2 =$$

$$= (\lambda a_1 + (1-\lambda) b_1 - a_1)^2 +$$

$$(\lambda a_2 + (1-\lambda b_2 - a_2)^2$$

$$= (1-\lambda)^2 a_1^2 + (1-\lambda)^2 b_1^2 - 2 a_1 b_1 (1-\lambda)^2 + (1-\lambda)^2 a_2^2 + (1-\lambda) b_2^2 - 2 a_2 b_2 (1-\lambda)^2$$

$$(1-\lambda)^2\left[(a_1-b_1)^2+(a_2-b_2)^2\right]=(1-\lambda)^2 AB^2$$

$$d(P_\lambda, B)^2 = \lambda^2 AB^2$$

$$\frac{P_\lambda A}{P_\lambda B} = \frac{1-\lambda}{\lambda}\,\frac{\cancel{AB}}{\cancel{AB}}$$

$$P_\lambda = (\lambda A + (1-\lambda)B)$$

$0 \le \lambda \le 1$ interno

A .... B

$\lambda < 0$

$\lambda > 1$ esterno

$AP = \lambda BP$ $\quad \cancel{\lambda} \neq 1$ $\quad \cancel{\lambda} \neq 0$ $\quad k > 0$

$$\sqrt{(x-a_1)^2+(y-a_2)^2} = K\sqrt{(x-b_1)^2+(y-b_2)^2}$$

$$x^2-2a_1x+a_1^2+y^2-2a_2y+a_2^2 = k^2x^2-2k^2b_1x+k^2b_1^2+k^2y^2-2k^2b_2y+k^2b_2^2$$

$$x^2(1-k^2)+y^2(1-k^2)+2x(k^2b_1-a_1)+2y(k^2b_2-a_2)+a_1^2+a_2^2-k^2b_1^2-k^2b_2^2=0$$

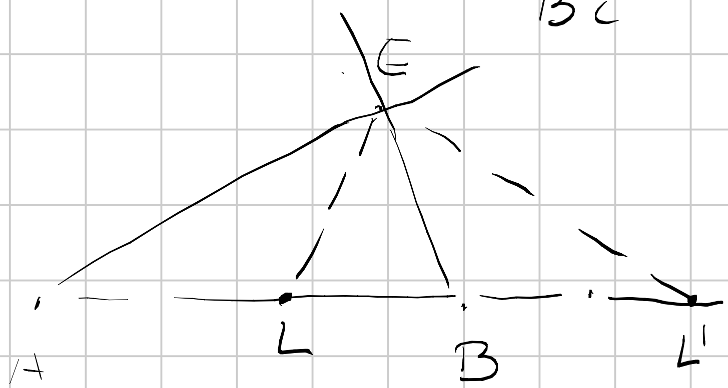
Circonferenza di Apollonio

$$x^2 + y^2 + 2x\frac{(k^2 b_1 - a_1)}{1-k^2} + 2y\frac{(k^2 b_2 - a_2)}{1-k^2} + \frac{a_1^2 + a_2^2 - kb_1^2 - kb_2^2}{1-k^2} = 0$$

$$\left(-\frac{k^2 b_1 - a_1}{1-k^2}, -\frac{k^2 b_2 - a_2}{1-k^2}\right) = \left(\frac{a_1}{1-k^2} - \frac{k^2}{1-k^2}b_1, \frac{a_2}{1-k^2} - \frac{k^2 b_2}{1-k^2}\right)$$

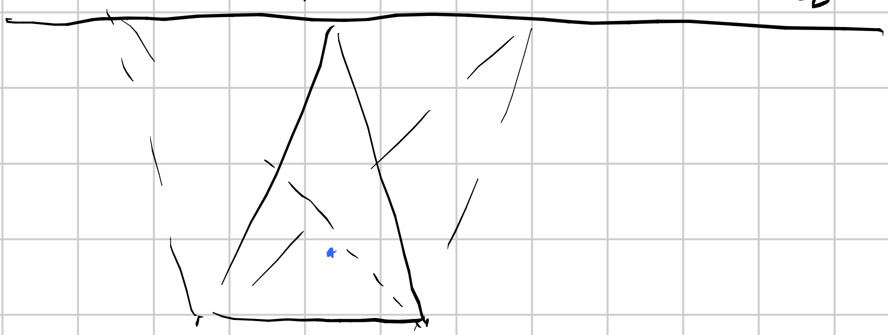
$$h = \frac{1}{1-k^2} \qquad = \left(h a_1 + (1-h) b_1, h a_2 + (1-h) b_2\right) = C$$

$$\frac{AC}{BC} = \frac{1-h}{h} = \frac{\frac{-k^2}{\cancel{1-k^2}}}{\frac{1}{\cancel{1-k^2}}} = -k^2$$



$$\frac{AE}{EB} = k$$

$$\frac{AL}{LB} = \frac{AE}{EB} = k$$



$AB \parallel r$

$A = (-1, 0) \qquad B = (1, 0)$

$r : \quad y = c$

$C = (k, c) \qquad k \in \mathbb{R}$

$$\begin{cases} \text{perp da } C \text{ a } AB : \quad x = k \\ \text{" da } B \text{ a } CA : \end{cases}$$

$\cdot)\ CA : \ y = (x+1)\dfrac{c}{k+1}$

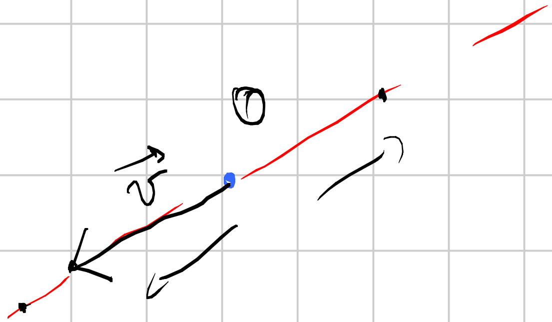
$m = -\dfrac{k+1}{c}$

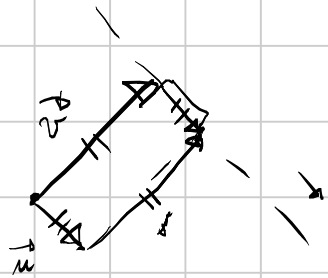
$y = (x-1)\left(-\dfrac{k+1}{c}\right)$

$$\begin{cases} x = k \\ y = (x-1)\left(-\dfrac{k+1}{c}\right) \end{cases}$$

$$\begin{cases} x = k \\ y = -\dfrac{k^2-1}{c} \end{cases}$$

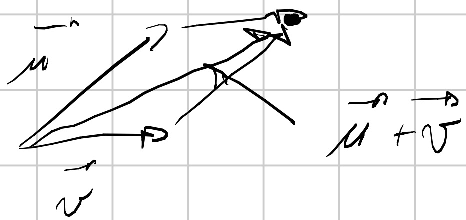
$$y = -\frac{x^2}{c} + \frac{1}{c}$$

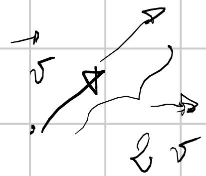




$\vec{u}+\vec{v} = \vec{v}+\vec{u}$

$\vec{v}+\vec{v}$





$\vec{v}$

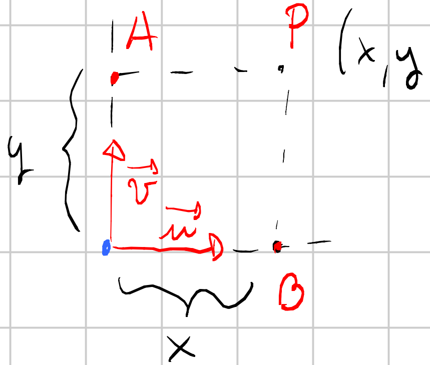
$\lambda \vec{v}$ è

- nella direzione di $\vec{v}$
- con lo stesso verso se $\lambda > 0$
  con verso opposto se $\lambda < 0$
- con lunghezza $= |\lambda| \cdot$ lunghezza di $\vec{v}$

$\lambda \in \mathbb{R}$

$|\vec{v}|$ lunghezza di $\vec{v}$ norma di $\vec{v}$

$\|\vec{v}\|$



$\vec{A}$ = vettore da O a A

$\vec{A} = y\,\vec{v}$

$\vec{B} = x\,\vec{u}$

$\vec{P} = x\,\vec{u} + y\,\vec{v} = (x, y)$

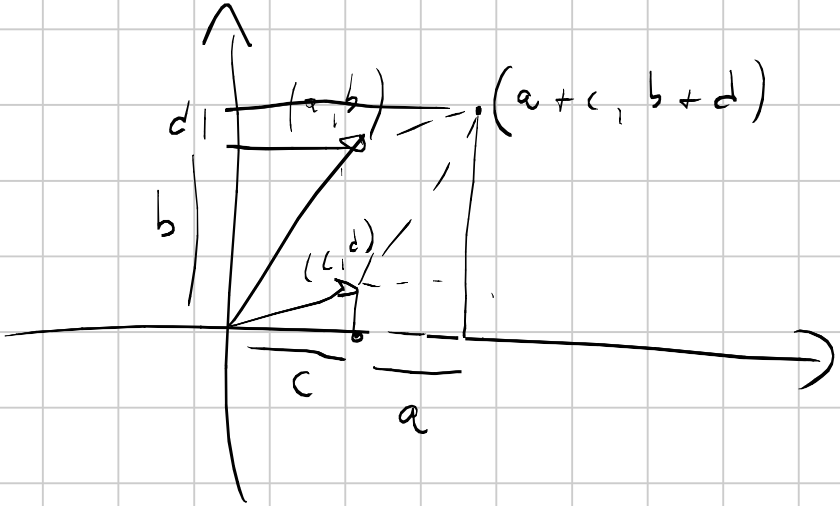
$(x_2, \; y_1)$

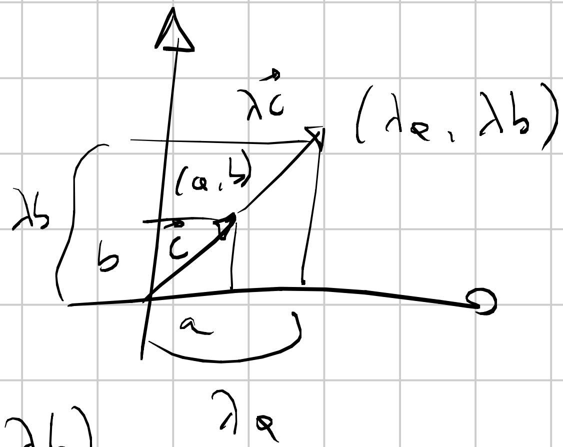
$x_1 \cdot \vec{u} + y_1 \cdot \vec{v}$

$$|\vec{w}| = |a\vec{u} + b\vec{v}| = \sqrt{a^2+b^2}$$

$$\vec{w} = (a, b) \longrightarrow |\vec{w}| = \sqrt{a^2+b^2}$$

$$(a, b) + (c, d) = (a+c, b+d)$$



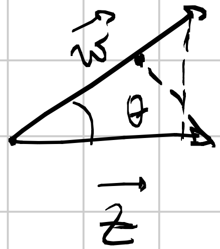


$$\lambda \cdot (a, b) = (\lambda a, \lambda b)$$

$\vec{w}$, $\vec{z}$

$$(\vec{w}, \vec{z}) = \langle \vec{w}, \vec{z} \rangle = \vec{w} \cdot \vec{z} = (a, b) \cdot (c, d) = ac + bd$$

prod. scalare



$$\vec{w} \cdot \vec{z} = |\vec{w}| \cdot |\vec{z}| \cdot \cos(\widehat{zw})$$

$$\vec{w} \cdot \vec{z} = 0 \iff \vec{z} \perp \vec{w}$$

$$(a, b, c) \cdot (d, e, f) = ad + be + cf \qquad \langle \vec{v}, \vec{v} \rangle = \vec{v} \cdot \vec{v} = \|\vec{v}\|^2$$

$$|(a, b)|^2 = a^2 + b^2 = (a, b) \cdot (a, b)$$

1) $\langle v, w \rangle = \langle w, v \rangle$

2) $\langle \lambda v, w \rangle = \lambda \langle v, w \rangle$

3) $\langle v + z, w \rangle = \langle v, w \rangle + \langle z, w \rangle$.

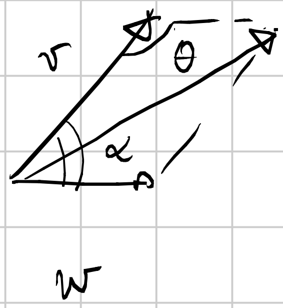
$$\langle a\vec{v} + b\vec{z}, \vec{w} \rangle = $$
$$= a \langle \vec{v}, \vec{w} \rangle + b \langle \vec{z}, \vec{w} \rangle$$

$$\|\vec{v}+\vec{w}\|^2 = \langle \vec{v}+\vec{w}, \vec{v}+\vec{w} \rangle = \langle \vec{v}, \vec{v}+\vec{w} \rangle + \langle \vec{w}, \vec{v}+\vec{w} \rangle =$$

$$= \langle \vec{v}, \vec{v} \rangle + \langle \vec{v}, \vec{w} \rangle + \langle \vec{w}, \vec{v} \rangle + \langle \vec{w}, \vec{w} \rangle =$$

$$= \|\vec{v}\|^2 + 2 \langle \vec{v}, \vec{w} \rangle + \|\vec{w}\|^2 =$$

$$= \|\vec{v}\|^2 + 2 \|\vec{v}\| \|\vec{w}\| \cdot \cos(\widehat{vw}) + \|\vec{w}\|^2$$



$$\|v\|^2 + \|w\|^2 - 2 \|v\| \|w\| \cos(\theta) =$$

$$= \|v\|^2 + \|w\|^2 + 2 \|v\| \|w\| \cos(\alpha)$$

G bari   H orto   O circo

$$\vec{G} = \frac{\vec{A} + \vec{B} + \vec{C}}{3}$$

O, G, H allineati e $GH = 2\,GO$

O = circo. = orig

$\Downarrow$

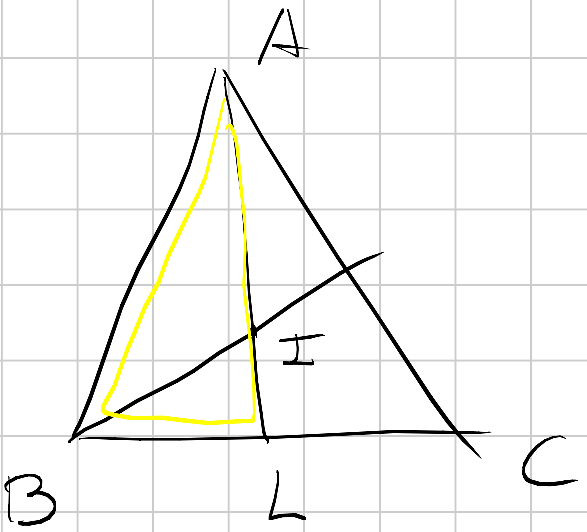
$$|\vec{A}| = |\vec{B}| = |\vec{C}| = R$$

$$\vec{H} = \vec{A} + \vec{B} + \vec{C}$$

$P_1 \; \ldots \; P_n$ da O a $P_i = \vec{P_i}$

$$\overrightarrow{\text{bari}} = \frac{\vec{P_1} + \ldots + \vec{P_n}}{n}$$

$$\lambda \vec{A} + (1-\lambda)\vec{B} = \vec{P_\lambda} \quad \text{t.c.} \quad \frac{|P_\lambda A|}{|P_\lambda B|} = \left|\frac{1-\lambda}{\lambda}\right|$$



$$\frac{BL}{LC} = \frac{AB}{AC} = \frac{\frac{AB}{AB+AC}}{\frac{AC}{AB+AC}}$$

$$\vec{B}\left(\frac{AC}{AB+AC}\right) + \vec{C}\left(\frac{AB}{AB+AC}\right) = \vec{L}$$

$$\frac{LI}{IA} = \frac{LB}{BA} = \frac{\frac{BC \cdot AB}{AB+AC}}{BA}$$

$$\vec{I} = \vec{L}\left(\frac{\cancel{BA}\ 1}{\cancel{AB}_{1} + \frac{\cancel{AB} \cdot BC}{AB+AC}}\right) + \vec{A}\left(\frac{\frac{BC \cdot \cancel{AB}}{\cancel{AB+AC}}}{AB+BC+CA}\right) =$$

$$\begin{cases} LB + LC = BC \\ \frac{LB}{LC} = \frac{AB}{AC} \end{cases}$$

$$LB = \frac{BC}{AB+AC} \cdot AB$$

$$= \left(\vec{B}\,\frac{AC}{\cancel{AB+AC}} + \vec{C}\,\frac{AB}{\cancel{AB+AC}}\right)\left(\frac{\cancel{AB+AC}}{AB+BC+CA}\right) +$$

$$BA + \frac{AB \cdot BC}{AB+AC} = \frac{AB^2 + AB\,AC + AB \cdot BC}{AB+AC}$$

$$\frac{\vec{B}\cdot AC + \vec{C}\cdot AB + A\cdot BC}{AB + BC + CA}$$

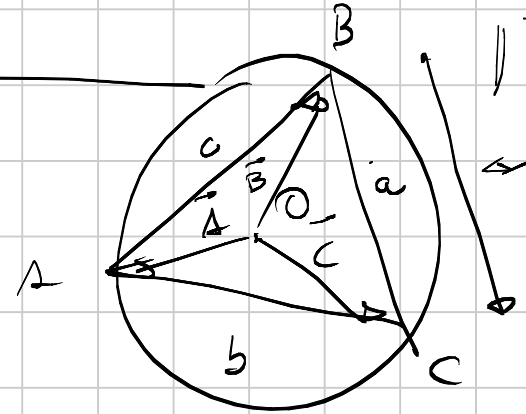
$AB = c$

$BC = a$

$AC = b$

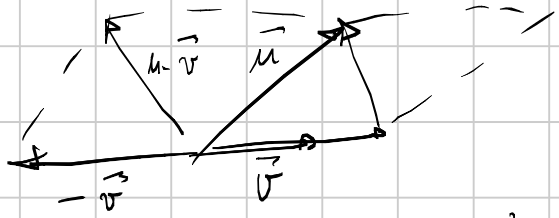
$$\frac{a\vec{A} + b\vec{B} + c\vec{C}}{a+b+c} = \vec{I}$$

$\|\vec{C} - \vec{B}\| = a$



$$\|\vec{GO}\|^2 = \|\vec{G}\|^2 = \left\| \frac{\vec{A}+\vec{B}+\vec{C}}{3} \right\|^2 =$$

$$= \frac{1}{9}\,\|\vec{A}+\vec{B}+\vec{C}\|^2 =$$



$a =$

$$= \frac{1}{9}\left(\|A\|^2 + \|B\|^2 + \|C\|^2 + 2\langle A, B\rangle + 2\langle B, C\rangle + 2\langle C, A\rangle \right) =$$

$$= \frac{1}{9}\left(3R^2 + 2R^2 - c^2 + 2R^2 - a^2 + 2R^2 - b^2 \right) = \frac{1}{9}\left(9R^2 - a^2 - b^2 - c^2\right) =$$

$$a^2 = \| \vec{C} - \vec{B} \|^2 = \|C\|^2 + \|B\|^2 - 2\langle B, C\rangle \qquad 2\langle B, C\rangle = 2R^2 - a^2$$

$$= R^2 - \frac{a^2 + b^2 + c^2}{9}$$