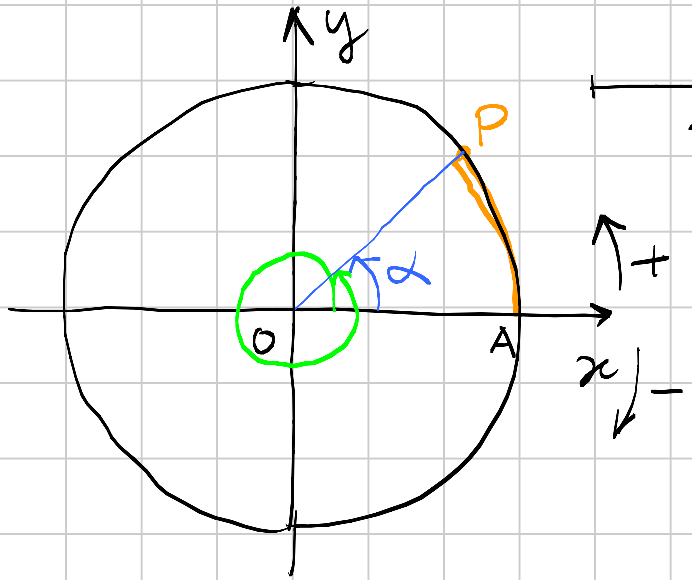
TRIGONOMETRIA

Titolo nota — 11/09/2006



1

$$x^2+y^2=1$$

Angolo giro $= 360^\circ$

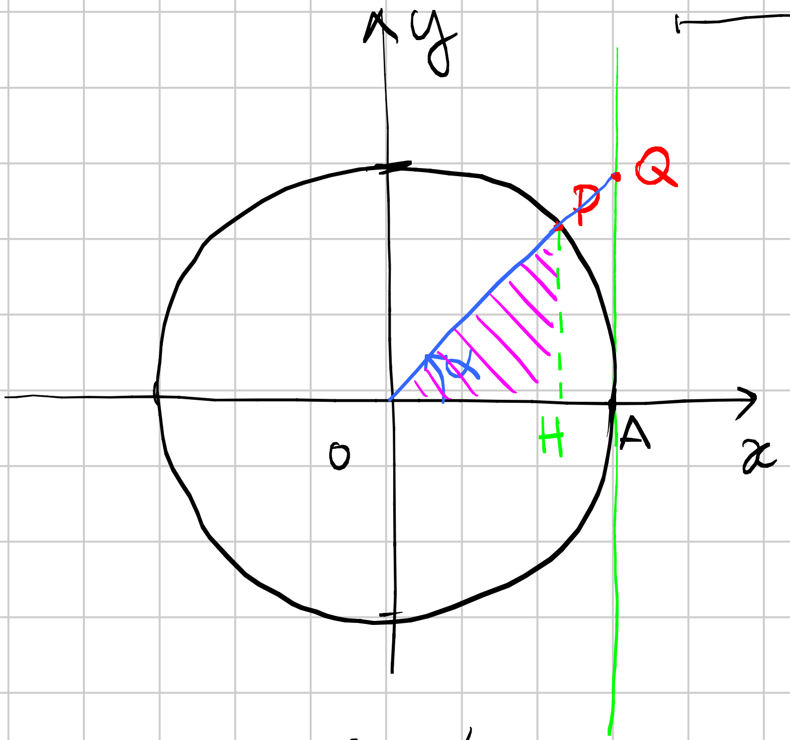
arco $\widehat{AP} \longleftrightarrow \alpha$

1 radiante $=$ α corrispondente ad $\widehat{AP}=1$

$$2\pi : 360^\circ = \alpha(\text{rad}) : \alpha^\circ$$

180°	$\simeq \pi$
90	$\pi/2$
45°	$\pi/4$
30°	$\pi/6$
60	$\pi/3$

FUNZIONI TRIGONOMETRICHE



$P \equiv (\cos\alpha, \operatorname{sen}\alpha)$

$-1 \leq \cos\alpha \leq 1$

$-1 \leq \operatorname{sen}\alpha \leq 1$

$\operatorname{sen}(2\pi + \alpha) = \operatorname{sen}\alpha$

$\cos(2\pi + \alpha) = \cos\alpha$

$\sin 2$

si intende

2 radianti

$\sin 2^\circ$

2 gradi

sessagesimali

$$tg\alpha = \frac{\operatorname{sen}\alpha}{\cos\alpha}$$

$\cos\alpha \neq 0 \qquad \alpha \neq \frac{\pi}{2} + k\pi$

$k \in \mathbb{Z}$

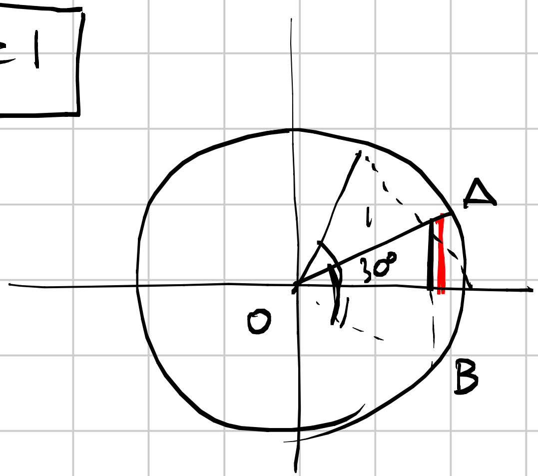
l'ordinata di Q è la tangente di α

tg α ha periodo π

$tg(\pi + \alpha) = tg\,\alpha$

$x^2 + y^2 = 1$

$$\boxed{\cos^2\alpha + \operatorname{sen}^2\alpha = 1}$$



$\cos 90° = 0$

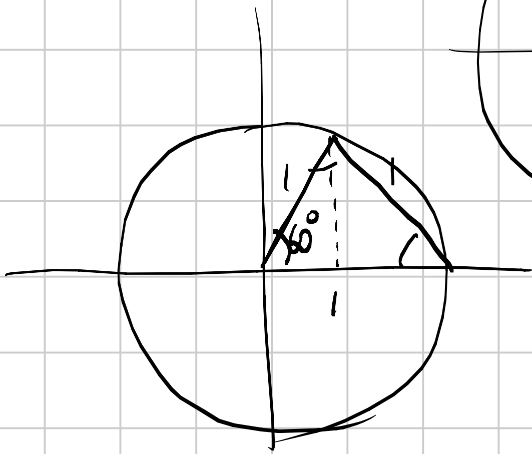
$\operatorname{sen} 90° = 1$

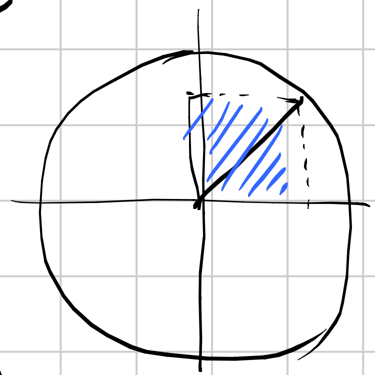
$\operatorname{sen} 0° = 0$

$\cos 0° = 1$

$\operatorname{sen} 30° = \frac{1}{2}$

$\cos 30° = \sqrt{3}/2$



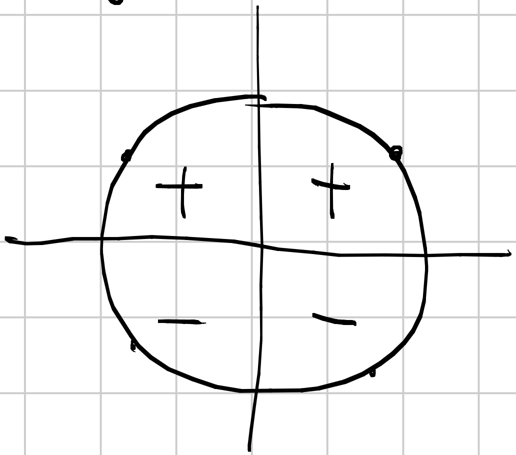


$\cos 60° = 1/2$

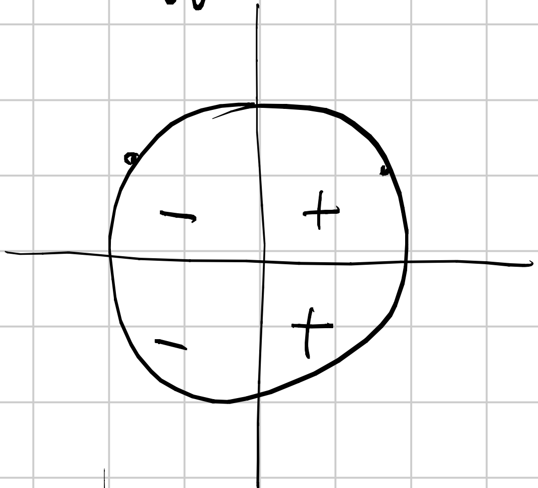
$\operatorname{sen} 60° = \sqrt{3}/2$

$\operatorname{sen} 45° = \cos 45° = \frac{1}{\sqrt{2}}$

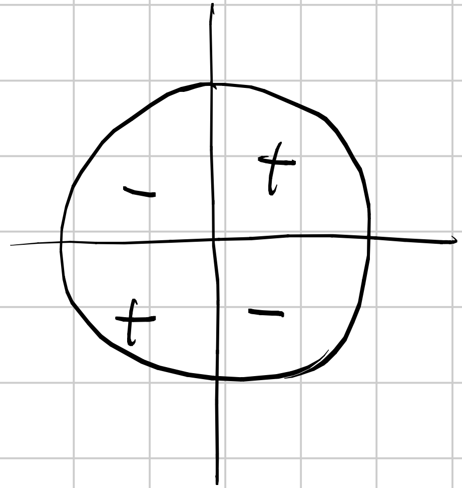
Segno di sen x



Segno di cos x



Segno di tg x



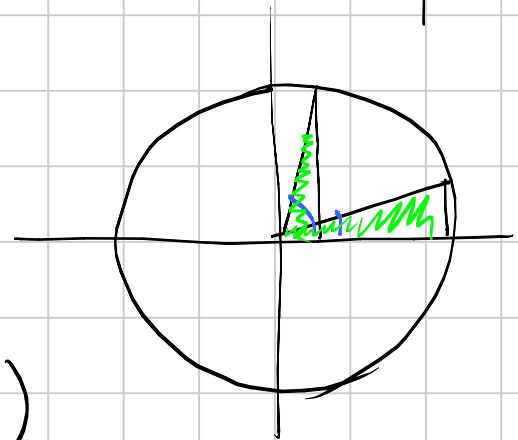
$$\alpha + \beta = \pi/2$$

$$\text{sen}\,\alpha = \cos\beta$$

$$\text{sen}\,\beta = \cos\alpha$$

$$\text{sen}\,\alpha = \cos\left(\frac{\pi}{2} - \alpha\right)$$

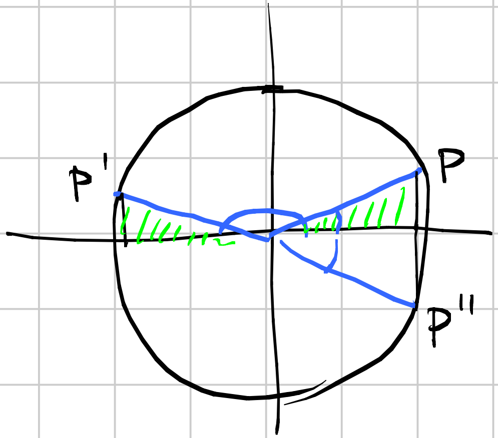
$$\cos\alpha = \text{sen}\left(\frac{\pi}{2} - \alpha\right)$$



$$\text{tg}\,\alpha = \text{ctg}\left(\frac{\pi}{2} - \alpha\right)$$

$$\frac{\cos\beta}{\text{sen}\,\beta} = \text{ctg}\,\beta$$

Angoli supplementari



$$\operatorname{sen}\alpha = \operatorname{sen}(\pi-\alpha)$$
$$\cos\alpha = -\cos(\pi-\alpha)$$

$$\cos(-\alpha) = \cos\alpha$$
$$\operatorname{sen}(-\alpha) = -\operatorname{sen}(\alpha)$$
$$\operatorname{tg}(-\alpha) = -\operatorname{tg}(\alpha)$$

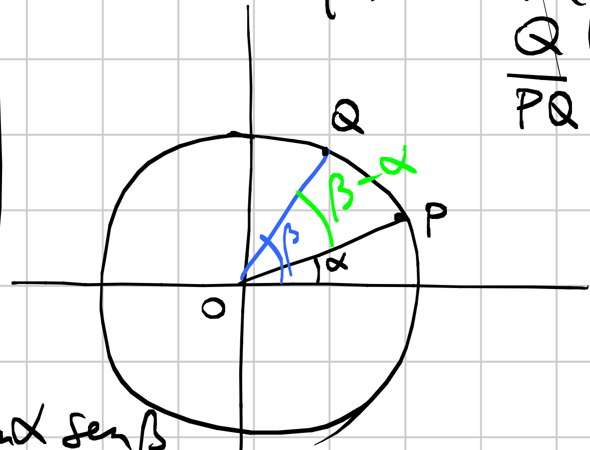
FORMULE DI SOMMA DIFFERENZA

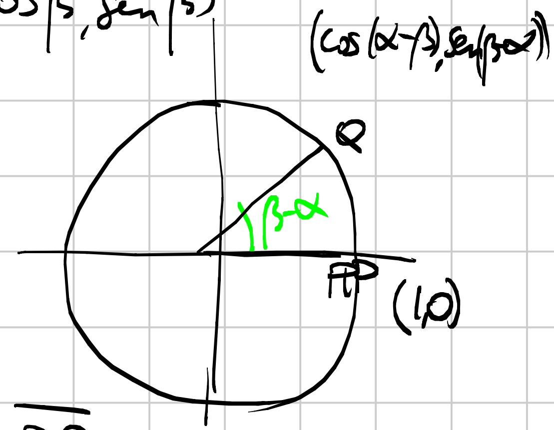
$\operatorname{sen}(\alpha+\beta)$

$P(\cos\alpha, \operatorname{sen}\alpha)$

$Q(\cos\beta, \operatorname{sen}\beta)$

$\overline{PQ}$





$\overline{PQ}$

$$\cos(\alpha-\beta) = \cos\alpha\cdot\cos\beta + \operatorname{sen}\alpha\operatorname{sen}\beta$$
$$\operatorname{sen}(\alpha-\beta) = \operatorname{sen}\alpha\cos\beta - \operatorname{sen}\beta\cos\alpha$$

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \operatorname{sen}\alpha\operatorname{sen}\beta$$

$$\operatorname{sen}(\alpha+\beta) = \operatorname{sen}\alpha\cos\beta + \cos\alpha\operatorname{sen}\beta$$

$$\operatorname{tg}(\alpha+\beta) = \frac{\operatorname{sen}(\alpha+\beta)}{\cos(\alpha+\beta)} = \frac{\operatorname{sen}\alpha\cos\beta + \cos\alpha\operatorname{sen}\beta}{\cos\alpha\cos\beta - \operatorname{sen}\alpha\operatorname{sen}\beta} =$$

$$= \frac{\operatorname{tg}\alpha + \operatorname{tg}\beta}{1 - \operatorname{tg}\alpha\operatorname{tg}\beta}$$

Somma di seno e coseno con lo stesso argomento

$$a\operatorname{sen}\alpha + b\cos\alpha =$$

$$\sqrt{a^2+b^2} > 0$$

$$= \sqrt{a^2+b^2}\left(\frac{a}{\sqrt{a^2+b^2}}\operatorname{sen}\alpha + \frac{b}{\sqrt{a^2+b^2}}\cos\alpha\right) =$$

$$-1 \le \underbrace{\frac{a}{\sqrt{a^2+b^2}}}_{\substack{\operatorname{sen}\varphi \\ \cos\varphi'}} \le 1 \qquad -1 \le \underbrace{\frac{b}{\sqrt{a^2+b^2}}}_{\substack{\cos\varphi \\ \operatorname{sen}\varphi'}} \le 1 \qquad \left(\frac{a}{\sqrt{a^2+b^2}}\right)^2 + \left(\frac{b}{\sqrt{a^2+b^2}}\right)^2 = 1$$

$= \sqrt{a^2+b^2}\,(\operatorname{sen}\varphi \operatorname{sen}\alpha + \cos\varphi \cos\alpha) =$

$\operatorname{sen}\alpha \cos\varphi' + \cos\alpha \operatorname{sen}\varphi'$

$\operatorname{sen}(\alpha+\varphi')$

$= \sqrt{a^2+b^2} \cdot \cos(\alpha-\varphi)$

IMO 1969

$a_1\ a_2\ a_3\ \dots\ a_n \in \mathbb{R}$

$y(x) = \cos(a_1+x) + \frac{1}{2}\cos(a_2+x) + \frac{1}{4}\cos(a_3+x) + \dots \frac{1}{2^{n-1}}\cos(a_n+x)$

$y(x_1) = 0$ allora $x_1 - x_2 = m\pi,\ m \in \mathbb{Z}$

$y(x_2) = 0$

$y(x) = \cos a_1 \cos x - \operatorname{sen} a_1 \operatorname{sen} x + \frac{1}{2}(\cos a_2 \cos x - \operatorname{sen} a_2 \operatorname{sen} x) + \dots$

$y(x) = \underline{A \cos x + B \operatorname{sen} x}$

$$A = \cos a_1 + \frac{1}{2}\cos a_2 + \frac{1}{4}\cos a_3 + \cdots + \frac{1}{2^{n-1}}\cos a_n$$

$$B = -\left(\operatorname{sen} a_1 + \frac{1}{2}\operatorname{sen} a_2 + \frac{1}{4}\operatorname{sen} a_3 + \cdots + \frac{1}{2^{n-1}}\operatorname{sen} a_n\right)$$

$$y(-a_1) = \underbrace{\cos(a_1 - a_1)}_{1} + \underbrace{\frac{1}{2}\cos(a_2 - a_1)}_{\geqslant -\frac{1}{2}} + \underbrace{\frac{1}{4}\cos(a_3 - a_1)}_{\geqslant -\frac{1}{4}} \cdots$$

$$y(-a_1) \geqslant 1 \underbrace{- \frac{1}{2} - \frac{1}{4} - \frac{1}{8} - \cdots - \frac{1}{2^{n-1}}} > 0$$

$$y(x) = \sqrt{A^2 + B^2} \cdot \operatorname{sen}(x + \varphi)$$

$$y(x_1) = y(x_2) = 0 \qquad \operatorname{sen}(x_1 + \varphi) = 0 \qquad \operatorname{sen}(x_2 + \varphi) = 0$$

$x_1 + \varphi = k_1\pi \quad k_1 \in \mathbb{Z} \qquad x_2 + \varphi = k_2\pi \quad k_2 \in \mathbb{Z}$

$$x_1 - x_2 = k_1\pi \cancel{-\varphi} - k_2\pi \cancel{+\varphi} = \underbrace{(k_1 - k_2)}_{\in \mathbb{Z}}\pi = m\pi$$

$$\text{sen } 2\alpha = \text{sen}(\alpha+\alpha) = \text{sen}\alpha\cos\alpha + \cos\alpha\,\text{sen}\alpha = 2\,\text{sen}\alpha\cos\alpha$$

$$P = \underbrace{\cos\alpha \cdot \cos 2\alpha \cdot \cos 4\alpha \cdot \ldots \cdot \cos(2^n\alpha)}_{n+1 \text{ fattori}} = ?$$

$$= \prod_{k=0}^{n} \cos(2^k\alpha)$$

$\alpha = k\pi$ banale

se $\alpha \neq k\pi$, allora $\text{sen}(k\pi) \neq 0$

$$\text{sen}\alpha \cdot P = \underbrace{\text{sen}\alpha \cdot \cos\alpha}_{\frac{1}{2}\text{sen }2\alpha} \cdot \cos 2\alpha \cdot \cos 4\alpha \cdot \ldots \cdot \cos(2^n\alpha)$$

$$\operatorname{sen}\alpha \cdot P = \frac{1}{2}\underbrace{\operatorname{sen}2\alpha \cdot \cos 2\alpha}\cdot \cos 4\alpha \cdot \ldots \quad \cos(2^n\alpha)$$

$$\operatorname{sen}\alpha \cdot P = \frac{1}{2}\cdot\frac{1}{2}\cdot \operatorname{sen}4\alpha \cdot \cos 4\alpha \cdot \ldots$$

$$\operatorname{sen}\alpha \cdot P = \frac{1}{2^{n+1}}\cdot \operatorname{sen}(2^{n+1}\alpha)$$

$$P = \frac{\operatorname{sen}(2^{n+1}\alpha)}{2^{n+1}\operatorname{sen}\alpha}$$

$$P = \cos\frac{\pi}{15}\cdot\cos\frac{2\pi}{15}\cdot\cos\frac{3\pi}{15}\cdot\cos\frac{4\pi}{15}\cdot\cos\frac{5\pi}{15}\cdot\cos\frac{6\pi}{15}\cdot\cos\frac{7\pi}{15}$$

$\times$ $\times$ $\frac{1}{2}$ $-\cos\frac{8\pi}{15}$

$$\cos\frac{8\pi}{15}$$

$$\frac{8\pi}{15}+\frac{7\pi}{15}=\pi$$

$n=3 \quad n+1=4$

$$\cos\frac{\pi}{15}\cdot\cos\frac{2\pi}{15}\cdot\cos\frac{4\pi}{15}\cdot\cos\frac{7\pi}{15} =$$

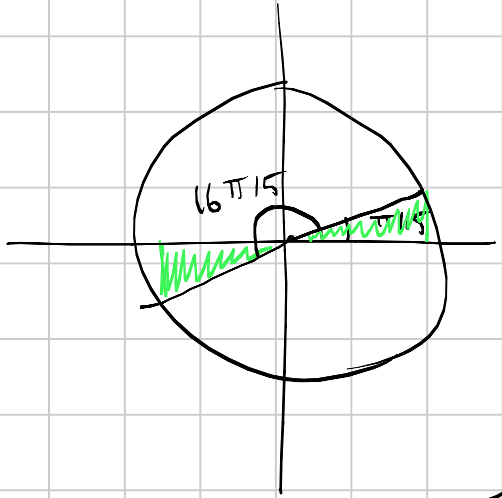
$$= -\cos\frac{\pi}{15}\cdot\cos\frac{2\pi}{15}\cdot\cos\frac{4\pi}{15}\cdot\cos\frac{8\pi}{15} = \frac{\operatorname{sen}\left(\frac{16\pi}{15}\right)}{2^4\cdot\operatorname{sen}\frac{\pi}{15}} =$$

$$= -\frac{-\cancel{\text{sen}\,\pi/15}}{2^4\,\cancel{\text{sen}\,\pi/15}} = \frac{1}{16}$$

$$\cos\frac{3\pi}{15}\cdot\cos\frac{6\pi}{15}$$

$$\frac{\text{sen}\left(\frac{12\pi}{15}\right)}{4\,\text{sen}\left(\frac{3\pi}{15}\right)} = \frac{1}{4}$$

$$P = \frac{1}{2}\cdot\frac{1}{4}\cdot\frac{1}{16}$$



$$\cos(2\alpha) = \cos(\alpha+\alpha) = \cos^2\alpha - \text{sen}^2\alpha$$

$$= 1 - 2\,\text{sen}^2\alpha$$

$$= 2\cos^2\alpha - 1$$

$$(x+y)(x-y) = x^2 - y^2$$

$$a\cos^2 x + b\cos x + c = 0$$

$$A\cos^2(2x) + B\cos(2x) + C = 0$$

$$2\cos^2\alpha = 1 + \cos(2\alpha)$$

chi sono A, B, C?

$$4\left(a\cos^2 x + c + b\cos x\right)\left(a\cos^2 x + c - b\cos x\right) = 0$$

$$4\left(a\cos^2 x + c\right)^2 - 4b^2\cos^2 x = 0$$

$$4a^2\cos^4 x + 2(4ac - 2b^2)\cos^2 x + 4c^2 = 0$$

$$a^2(1+\cos 2x)^2 + (4ac - 2b^2)(1+\cos 2x) + 4c^2$$

$$a^2\cos^2(2x) + (4ac - 2b^2 + 2a^2)\cos 2x + (a^2 + 4ac + 4c^2 - 2b^2) = 0$$

$$\boxed{\begin{aligned} a &= 4 \\ b &= 2 \\ c &= -1 \end{aligned}}$$

$$\operatorname{sen} 3\alpha = 3 \operatorname{sen}\alpha - 4 \sin^3\alpha \qquad 4\operatorname{sen}^3\alpha = 3\operatorname{sen}\alpha - \operatorname{sen}3\alpha$$

$$\cos 3\alpha = 4\cos^3\alpha - 3\cos\alpha$$

$$4\operatorname{sen}^3 x + 4\operatorname{sen}^3\left(\frac{2\pi}{3}+x\right) + 4\operatorname{sen}^3\left(\frac{4\pi}{3}+x\right) + 3\cos(2x) = 0$$

$$3\operatorname{sen}x - \operatorname{sen}3x + 3\operatorname{sen}\left(\frac{2\pi}{3}+x\right) - \operatorname{sen}\left(\cancel{2\pi}+3x\right)$$

$$+ 3\operatorname{sen}\left(\frac{4\pi}{3}+x\right) - \operatorname{sen}\left(\cancel{4\pi}+3x\right) + 3\cos(2x) = 0$$

$$-3\operatorname{sen}3x + 3\operatorname{sen}x + 3\,\overset{\sqrt{3}/2}{\operatorname{sen}\frac{2\pi}{3}}\cos x + 3\,\overset{-\frac{1}{2}}{\cos\frac{2\pi}{3}}\operatorname{sen}x$$

$$+ 3\,\underset{-\sqrt{3}/2}{\operatorname{sen}\frac{4\pi}{3}}\cos x + 3\,\underset{-\frac{1}{2}}{\cos\frac{4\pi}{3}}\operatorname{sen}x + 3\cos(2x) = 0$$

$$-\cancel{3}\operatorname{sen}3x + \cancel{3\operatorname{sen}x} - \cancel{\frac{3}{2}\operatorname{sen}x} - \cancel{\frac{3}{2}\operatorname{sen}x} + \cancel{3}\cos 2x = 0$$

$\cos 2x = \operatorname{sen} 3x$

$\alpha + \beta = \frac{\pi}{2} + 2\pi k \qquad \operatorname{sen}\alpha = \cos\beta$

$2x + 3x = \frac{\pi}{2} + 2\pi k \qquad x = \frac{\pi}{10} + \frac{2k\pi}{5} \quad k \in \mathbb{Z}$

$2 \operatorname{sen}\alpha \cos\beta = \operatorname{sen}(\alpha+\beta) + \operatorname{sen}(\alpha-\beta)$

$2 \cos\alpha \cos\beta = \cos(\alpha+\beta) + \cos(\alpha-\beta)$

$2 \operatorname{sen}\alpha \operatorname{sen}\beta = -\cos(\alpha+\beta) + \cos(\alpha-\beta)$

$\operatorname{sen}\alpha + \operatorname{sen}\beta = 2 \operatorname{sen}\frac{\alpha+\beta}{2} \cos\frac{\alpha-\beta}{2}$

$\cos^2 x + \cos^2(2x) + \cos^2(3x) = 1$

$\cos(2x) = 2\cos^2 x - 1$

$$\cos^2\alpha = \frac{1+\cos 2\alpha}{2} \qquad \frac{1+\cos 2x}{2} + \frac{1+\cos(4x)}{2} + \cos^2(3x) = 1$$

$$\cancel{\frac{1}{2}} + \cancel{\frac{1}{2}} + \frac{1}{2}\cos(2x) + \frac{1}{2}\cos(4x) + \cos^2(3x) = \cancel{1}$$

$$\underbrace{\cos(2x) + \cos(4x)} + 2\cos^2(3x) = 0$$

$$\cancel{2}\cos\left(\frac{2x+4x}{2}\right)\cdot\cos\left(\frac{4x-2x}{2}\right) + \cancel{2}\cos^2(3x) = 0$$

$$\cos(3x)\cdot\cos x + \cos^2(3x) = 0$$

$$\cos(3x)\left[\cos x + \cos(3x)\right] = 0$$

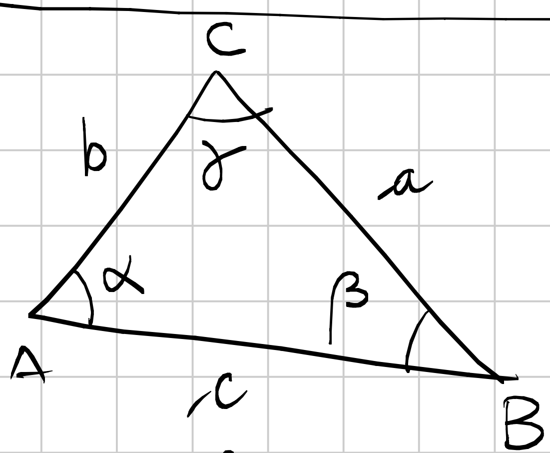
$$\cos(3x)\cdot\frac{1}{2}\cos\left(\frac{x+3x}{2}\right)\cdot\cos\left(\frac{3x-x}{2}\right) = 0$$

$$\cos(3x)\cdot\cos(2x)\cdot\cos x = 0$$

$$\cos\alpha = 0 \qquad \alpha = \pm\frac{\pi}{2} + 2k\pi$$

$$x = \pm\frac{\pi}{6} + \frac{2k\pi}{3} \qquad x = \frac{\pi}{4} + k\pi \qquad x = \pm\frac{\pi}{2} + 2k\pi$$

$k \in \mathbb{Z}$



$$\frac{\operatorname{sen}^2\alpha + \operatorname{sen}^2\beta + \operatorname{sen}^2\gamma}{\cos^2\alpha + \cos^2\beta + \cos^2\gamma} = 2 \Rightarrow$$ ABC è un triangolo rettangolo

$$\operatorname{sen}^2\alpha + \operatorname{sen}^2\beta + \operatorname{sen}^2\gamma = 2\cos^2\alpha + 2\cos^2\beta + 2\cos^2\gamma$$

$$\underbrace{\operatorname{sen}^2\alpha + \cos^2\alpha} + \underbrace{\operatorname{sen}^2\beta + \cos^2\beta} + \underbrace{\operatorname{sen}^2\gamma + \cos^2\gamma} =$$

$$= 3\left(\cos^2\alpha + \cos^2\beta + \cos^2\gamma\right)$$

$$3 = 3\left(\cos^2\alpha + \cos^2\beta + \cos^2\gamma\right)$$

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

$$\cos^2\alpha = \frac{1 + \cos 2\alpha}{2} \qquad \alpha + \beta + \gamma = 180^\circ$$

$$\cos^2\beta = \frac{1 + \cos 2\beta}{2}$$

$$\cos^2\gamma = \cos^2\left(\pi - (\alpha + \beta)\right) = + \cos^2(\alpha + \beta)$$

$$\cancel{\frac{1}{2}} + \frac{1}{2}\cos 2\alpha + \cancel{\frac{1}{2}} + \frac{1}{2}\cos 2\beta + \cos^2(\overset{\gamma}{\alpha + \beta}) = \cancel{1}$$

$$\frac{1}{2}\left(\cos(2\alpha) + \cos(2\beta)\right) + \cos^2\gamma = 0$$

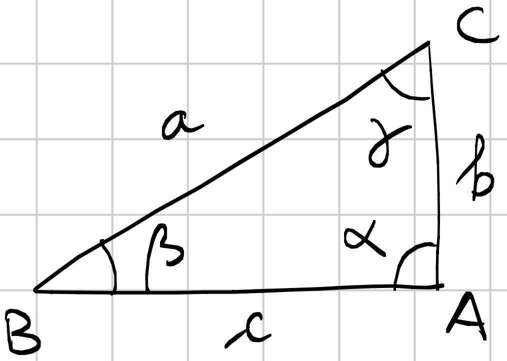
$$\cos\left(\frac{2\alpha + 2\beta}{2}\right) \cdot \cos\left(\frac{2\alpha - 2\beta}{2}\right) + \cos^2(\alpha + \beta) = 0$$

$$\cos(\alpha + \beta)\cos(\alpha - \beta) + \cos^2(\alpha + \beta) = 0$$

$$2\cos(\alpha+\beta)\cdot[\cos(\alpha-\beta)+\cos(\alpha+\beta)]=0$$

$\alpha+\beta=\frac{\pi}{2}$ $\qquad$ $2\cos\alpha\cos\beta$

$\gamma=\frac{\pi}{2}$ $\qquad$ $\alpha=\frac{\pi}{2}$ $\quad$ $\beta=\frac{\pi}{2}$



$b = a\,\text{sen}\,\beta$ $\qquad$ $c = a\cos\beta$

$b = a\cos\gamma$ $\qquad$ $c = a\,\text{sen}\,\gamma$

$\frac{b}{c} = \text{tg}\,\beta$ $\qquad$ $b = c\,\text{tg}\,\beta$

$c = \frac{b}{\text{tg}\,\beta}$

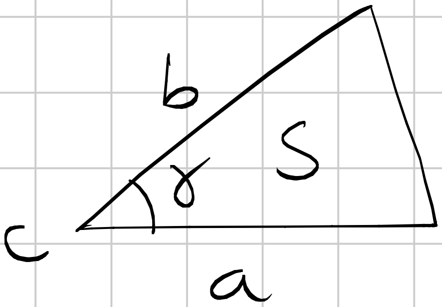
$\frac{c}{b} = \text{tg}\,\gamma$ $\qquad$ $c = b\,\text{tg}\,\gamma$

$b = \frac{c}{\text{tg}\,\gamma}$

FORMULA DI ERONE

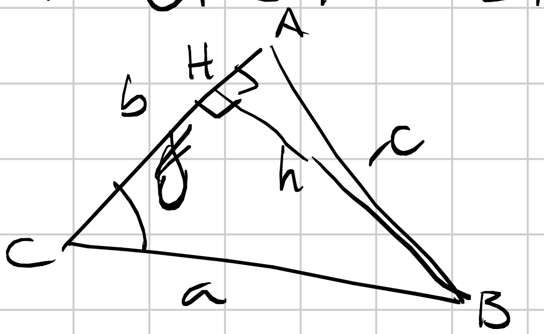
$$S = \sqrt{p(p-a)(p-b)(p-c)} \qquad p = \frac{a+b+c}{2}$$

$$S = \frac{1}{4}\sqrt{(a^2+b^2+c^2)^2 - 2(a^4+b^4+c^4)}$$



$$S = \frac{1}{2} ab \,\text{sen}\,\gamma$$

TEOREMA DI CARNOT



$$c^2 = a^2 + b^2 - 2ab\cos\gamma$$

$$h = a\,\text{sen}\,\gamma$$

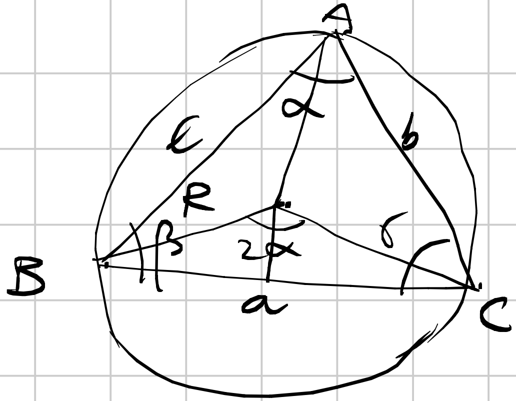
$$HA = b - a\cos\gamma$$

$$c^2 = \overline{HA}^2 + h^2 =$$

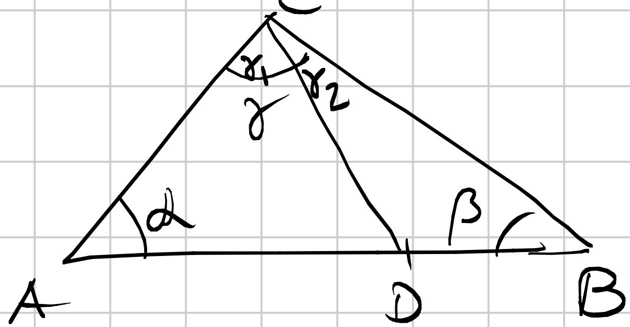
$$= b^2 + a^2\cos^2\gamma - 2ab\cos\gamma + a^2\,\text{sen}^2\gamma$$

$$= a^2 + b^2 - 2ab\cos\gamma$$

$$\frac{a}{\operatorname{sen}\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma} = 2R$$



$$a = 2R \operatorname{sen}\alpha$$
$$b = 2R \operatorname{sen}\beta$$
$$c = 2R \operatorname{sen}\gamma$$



$\gamma_1 = D\hat{C}A$

$\gamma_2 = D\hat{C}B$

Dato ABC

$\exists D$ su AB tale che $CD = \sqrt{AD \cdot DB}$ $\iff$ $\sqrt{\operatorname{sen}\alpha \operatorname{sen}\beta} \le \operatorname{sen}\frac{\gamma}{2}$

$$f(D) = \frac{CD^2}{AD \cdot BD} = \frac{CD}{AD} \cdot \frac{CD}{BD}$$

$$\frac{AD}{\operatorname{sen}\gamma_1} = \frac{CD}{\operatorname{sen}\alpha} \qquad \frac{CD}{AD} = \frac{\operatorname{sen}\alpha}{\operatorname{sen}\gamma_1} \qquad \frac{CD}{BD} = \frac{\operatorname{sen}\beta}{\operatorname{sen}\gamma_2}$$

$$f(D) = \frac{\operatorname{sen}\alpha \cdot \operatorname{sen}\beta}{\operatorname{sen}\gamma_1 \cdot \operatorname{sen}\gamma_2} = \frac{\operatorname{sen}\alpha \cdot \operatorname{sen}\beta}{\frac{1}{2}\left[\cos(\gamma_1-\gamma_2) - \cos(\underbrace{\gamma_1+\gamma_2}_{\uparrow\gamma})\right]} =$$

$$= \frac{\operatorname{sen}\alpha \operatorname{sen}\beta}{\frac{1}{2}\left[\cos(\gamma_1-\gamma_2) - \cos\gamma\right]} \geqslant \frac{\operatorname{sen}\alpha \operatorname{sen}\beta}{\underbrace{\frac{1}{2}(1-\cos\gamma)}_{\nwarrow \operatorname{sen}^2\frac{\gamma}{2}}}$$

$$\boxed{f(D) \geqslant \frac{\operatorname{sen}\alpha \operatorname{sen}\beta}{\operatorname{sen}^2\gamma/2}}$$

$$\frac{\operatorname{sen}\alpha \operatorname{sen}\beta}{\operatorname{sen}^2\gamma/2} \leq 1 \qquad \sqrt{\operatorname{sen}\alpha \operatorname{sen}\beta} \leq \operatorname{sen}\frac{\gamma}{2}$$

$$f(D) = 1 \qquad \frac{CD^2}{AD \cdot DB} = 1 \qquad CD = \sqrt{AD \cdot DB}$$

$f(D)$: $AB \longrightarrow \left[\frac{\operatorname{sen}\alpha \operatorname{sen}\beta}{\operatorname{sen}^2 \gamma/2}, +\infty \right)$

$1 \in$

$$\frac{\operatorname{sen}\alpha \operatorname{sen}\beta}{\operatorname{sen}^2 \gamma/2} \leq 1$$

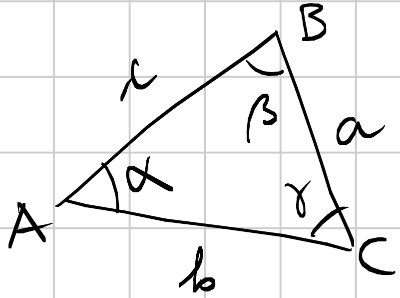
$$\operatorname{sen}\alpha \operatorname{sen}\beta \leq \operatorname{sen}^2 \frac{\gamma}{2}$$

$$\sqrt{\operatorname{sen}\alpha \operatorname{sen}\beta} < \operatorname{sen} \gamma/2$$

TRIGONOMETRIA 2

TEOREMA DI CARNOT
TEOREMA DEI SENI

1) Noti 2 lati e l'angolo compreso



$b \quad c \quad \alpha$ noti

$$a = \sqrt{b^2 + c^2 - 2bc\cos\alpha} \qquad \text{CARNOT}$$

$\left.\begin{matrix}\beta \\ \gamma\end{matrix}\right\}$ teor. seni $\qquad \dfrac{b}{\operatorname{sen}\beta} = \dfrac{a}{\operatorname{sen}\alpha}$

$$\operatorname{sen}\beta = \frac{b \operatorname{sen}\alpha}{a}$$

2) Noto 1 lato e gli angoli ad esso adiacenti

$b \quad \alpha, \quad \gamma$

$$\beta = \pi - (\alpha + \gamma)$$

$$\frac{a}{\operatorname{sen}\alpha} = \frac{b}{\operatorname{sen}(\alpha+\gamma)} \qquad a = \frac{b \operatorname{sen}\alpha}{\operatorname{sen}(\alpha+\gamma)}$$

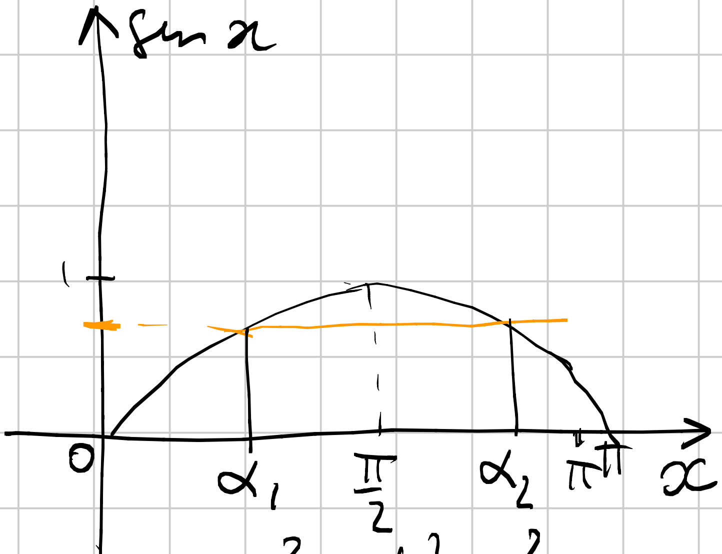
$$S = \frac{ab \operatorname{sen}\gamma}{2} = \frac{b^2 \operatorname{sen}\alpha \operatorname{sen}\gamma}{2 \operatorname{sen}(\alpha+\gamma)}.$$

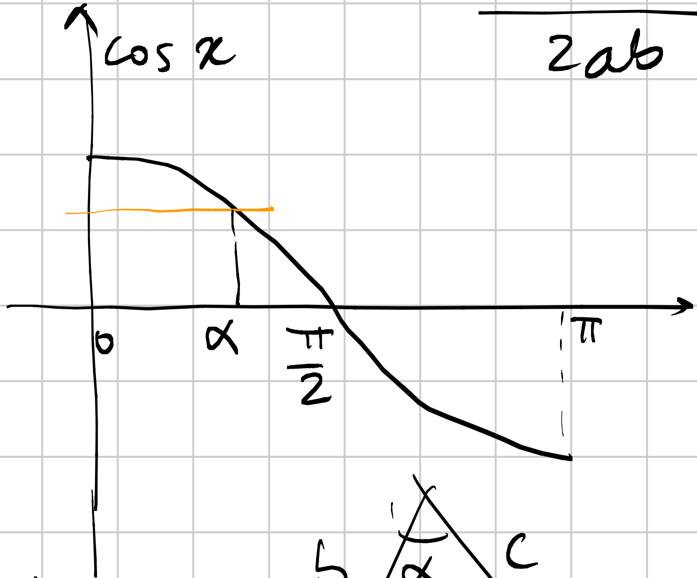
3) Noti i 3 lati a, b, c

$$S = \sqrt{p(p-a)(p-b)(p-c)}$$

$$\frac{ab}{2} \operatorname{sen}\gamma = S \qquad \operatorname{sen}\gamma = \frac{2\sqrt{}}{ab}$$

$$c^2 = a^2 + b^2 - 2ab\cos\gamma \qquad \cos\gamma = \frac{a^2+b^2-c^2}{2ab}$$

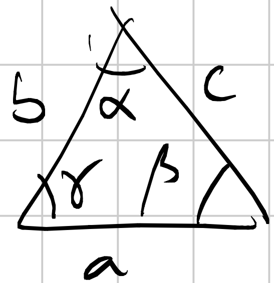




se $a^2 < b^2 + c^2$, α è acuto

se $a^2 = b^2 + c^2$, $\alpha = 90°$

se $a^2 > b^2 + c^2$ α è ottuso



$$a^2+b^2+c^2 \geqslant 4S\sqrt{3}$$

$$S = \frac{bc \operatorname{sen}\alpha}{2}$$

$$a^2 = b^2+c^2-2bc\cos\alpha$$

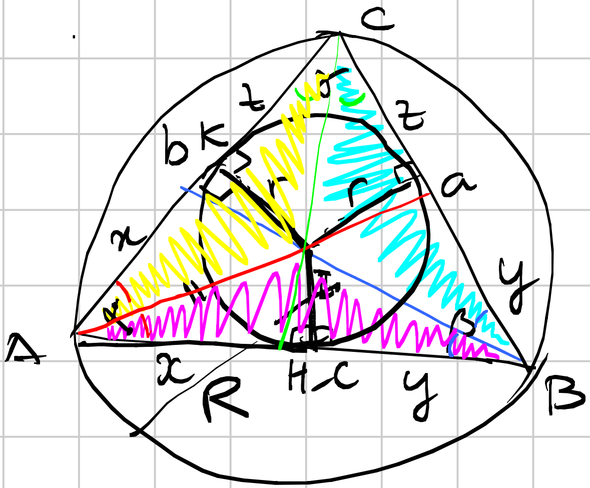
$$\cancel{2}b^2+\cancel{2}c^2-\cancel{2}bc\cos\alpha \overset{?}{\geqslant} \cancel{2}\sqrt{3}\, bc \operatorname{sen}\alpha$$

$$b^2+c^2-bc\left(\sqrt{3}\operatorname{sen}\alpha+\cos\alpha\right) \overset{?}{\geqslant} 0 \qquad \sqrt{\sqrt{3}^2+1^2}=2$$

$$\underbrace{b^2+c^2-2bc}+2bc-2bc\left(\operatorname{sen}\alpha\cdot\overbrace{\frac{\sqrt{3}}{2}}^{\operatorname{sen}60^\circ}+\underbrace{\frac{1}{2}}_{\cos 60^\circ}\cos\alpha\right)$$

$$\to \underbrace{(b-c)^2}_{\geqslant 0}+2bc\left(1-\underbrace{\cos(\alpha-60^\circ)}_{\leqslant 1}\right)\geqslant 0$$

$b=c \qquad \cos(\alpha-60)=1 \Rightarrow \alpha=60^\circ \qquad \to$ ABC è equilatero



$$R = \frac{a}{2\operatorname{sen}\alpha} = \frac{b}{2\operatorname{sen}\beta} = \frac{c}{2\operatorname{sen}\gamma}$$

$$S = \frac{bc\cdot\operatorname{sen}\alpha}{2}$$

$$\operatorname{sen}\alpha = \frac{2S}{bc} \qquad R = \frac{a}{2\cdot\frac{2S}{bc}} = \frac{abc}{4S}$$

$$S = \frac{a\cdot r}{2} + \frac{b\cdot r}{2} + \frac{c\cdot r}{2} = \underbrace{\frac{a+b+c}{2}}_{p}\cdot r$$

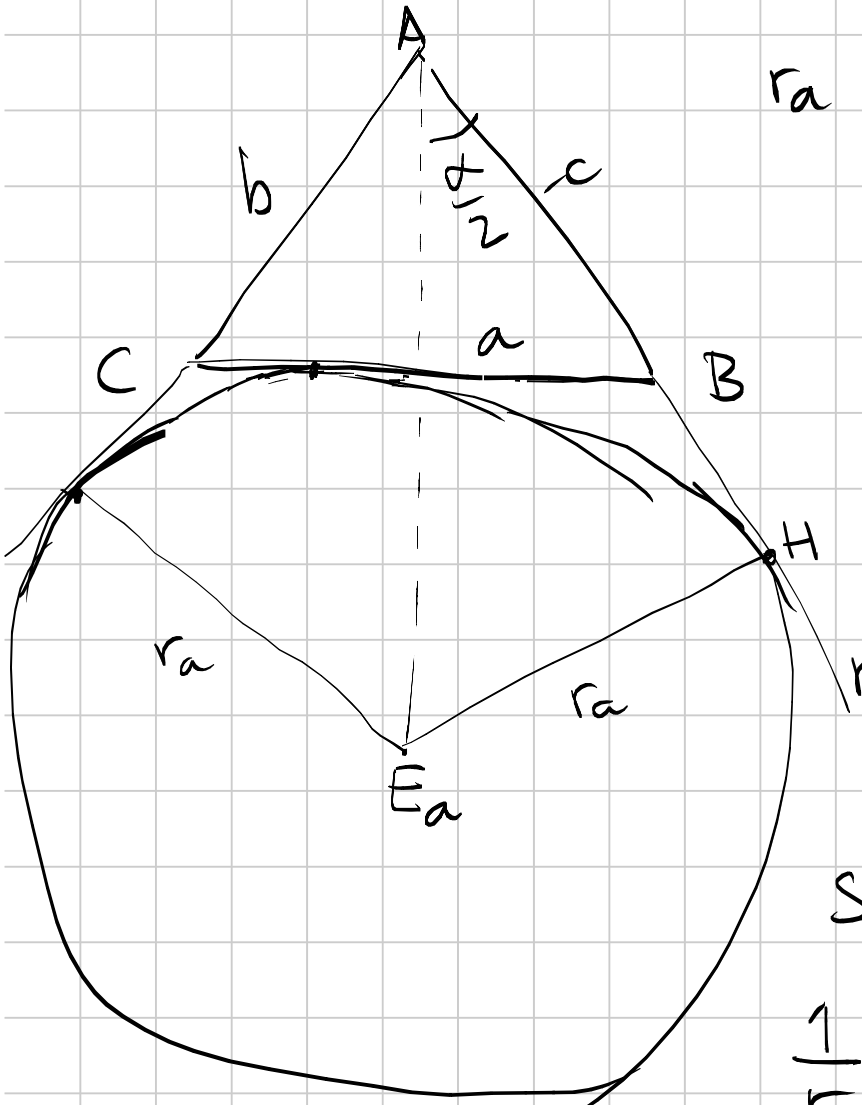
$$S = p\cdot r \qquad r = \frac{S}{p}$$

$$\begin{cases} x+z=b \\ y+z=a \\ x+y=c \end{cases} \qquad \begin{aligned} x &= p-a \\ y &= p-b \\ z &= p-c \end{aligned}$$

$$r = \underbrace{AH}_{x}\cdot \operatorname{tg}\frac{\alpha}{2} \qquad \triangle AIH = \triangle AIK$$

$$AK = AH$$

$$r = (p-a)\operatorname{tg}\frac{\alpha}{2} = (p-b)\operatorname{tg}\frac{\beta}{2} = (p-c)\operatorname{tg}\frac{\gamma}{2}$$



$$r_a = p \cdot tg\frac{\alpha}{2} = \frac{S}{p-a}$$

$$r_b = p \cdot tg\frac{\beta}{2} = \frac{S}{p-b}$$

$$r_c = p \cdot tg\frac{\gamma}{2} = \frac{S}{p-c}$$

$$S = \sqrt{r\, r_a\, r_b\, r_c}$$

$$r \cdot r_a \cdot r_b \cdot r_c = \frac{S^3 \cdot S}{(p-a)(p-b)(p-c)\,p} = \frac{S^4}{S^2} = S^2$$

$$S = \sqrt{r \cdot r_a \cdot r_b \cdot r_c}$$

$$\frac{1}{r} = \frac{1}{r_a} + \frac{1}{r_b} + \frac{1}{r_c}$$

$$\frac{1}{r_a} = \frac{p-a}{S}$$ e analoghe

$$\frac{1}{r_a}+\frac{1}{r_b}+\frac{1}{r_c}=\frac{p-a+p-b+p-c}{S}=\frac{3p-(a+b+c)}{S}=$$

$$=\frac{3p-2p}{S}=\frac{p}{S}=\frac{1}{r}$$

FORMULE DI BRIGGS $\operatorname{sen}\frac{\alpha}{2}=\sqrt{\frac{(p-b)(p-c)}{bc}}$

$\cos\frac{\alpha}{2}=\sqrt{\frac{p(p-a)}{bc}}$

TEOREMA DI NEPERO

$$\frac{b+c}{b-c}=\frac{\tan\frac{\beta+\gamma}{2}}{\tan\frac{\beta-\gamma}{2}}$$