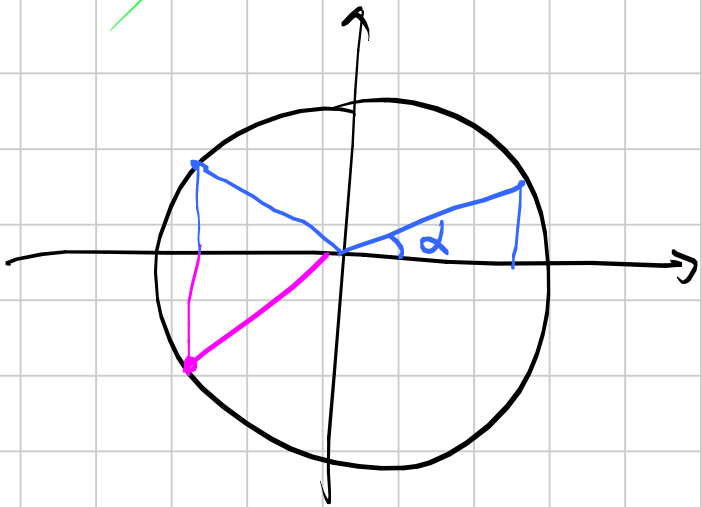
Senior 2006 - Algebra 1 - esercizi (3, 4, 7, 10, 3+)

Titolo nota 12/09/2006

(3) $\cos 15° + \cancel{\cos 35°} + \dots + \cos \cancel{355°}$



$15 \rightsquigarrow 165$

150

$15 \rightsquigarrow 195$

Dist : 180 OK

$\cos 15° + \cos 55° + \cos 95° + \ldots$

$i \sin 15° + i \sin 55° + \ldots$

$$\text{Testo} = \mathrm{Re}\left(e^{i15°} + e^{i55°} + \ldots + e^{i335°}\right)$$

$$= \mathrm{Re}\left(e^{0+i15°} + e^{i40°+i15°} + \ldots + e^{i320°+i15°}\right)$$

$$= \mathrm{Re}\ e^{i15°}\left(e^{i0°} + e^{i40°} + \ldots + e^{i320°}\right)$$

$$1 + \alpha + \alpha^2 + \ldots + \alpha^8 = \frac{\alpha^9 - 1}{\alpha - 1}$$

α = radice 9^a di 1

$$= \mathrm{Re}\ e^{i15°} \cdot 0 = 0$$

④

$$x^5 + x^4 + \cdots + 1 = \frac{x^6 - 1}{x - 1} =$$

$$= \frac{(x^3+1)(x^3-1)}{x-1} = \cdots$$

7

$$Q(x) = x(x-1)(x-2)(x-3)$$

$$P(x) = A(x)\cdot Q(x) + R(x)$$

$\boxed{R(x) \text{ ha grado } \leq 3}$

$x=0$ $\qquad 2 = P(0) = A(0)\cdot \underbrace{Q(0)}_{0} + R(0)$

$$\boxed{\begin{aligned} &2 = R(0) \\ &4 = R(1) \\ &6 = R(2) \\ &56 = R(3) \end{aligned}}$$

$x=1$

$x=2$

$x=3$

R(x) univoc. determinato

$2 = R(0) \Rightarrow R(x) = 2 + x\,R_1(x)$

$4 = R(1) \Rightarrow 4 = R(1) = 2 + 1 \cdot R_1(1)$

$R_1(1) = 2$

$R_1(x) = \boxed{2 + (x-1)\,R_2(x)}$

Perchè $R_1(x) - 2$ si annulla per $x = 1$

$$R(x) = 2 + x\left[2 + (x-1)\,R_2(x)\right]$$
$$= 2 + 2x + x(x-1)\,R_2(x) \rightsquigarrow$$

AVANTI ANALOG.

$$R(x) = A\,x\,(x-1)(x-2) + B\,x\,(x-1)(x-3) +$$
$$+ C\,x\,(x-2)(x-3) + D\,(x-1)(x-2)(x-3)$$

Cerco A. Pongo $x = 3$

$$56 = R(3) = A \cdot 3 \cdot (3-1)(3-2) + 0 + 0 + 0$$

Cerco C. Pongo $x = 1$

(10) $P(z)$ grado 2002 con radici distinte

Allora esistono $a_1, \dots, a_{2002}$ t.c.

$$P_1(z) = z - a_1$$
$$P_2(z) = [P_1(z)]^2 - a_2$$
$$P_3(z) = [P_2(z)]^2 - a_3$$
$$\vdots$$
$$P(z) \mid P_{2002}(z)$$

$$P(z) = A(x-\lambda_1) \dots (x-\lambda_n)$$

$P(z) \mid P_{2002}(z) \iff \lambda_1, \dots, \lambda_{2002}$ sono radici di $P_{2002}(z)$

Idea: costruire $P_k(z)$ in modo che abbia $\lambda_1, \ldots, \lambda_k$ come radici

$\boxed{k=1}$ $\quad a_1 = \lambda_1$

$\boxed{k \Rightarrow k+1}$ $\quad$ CLAIM: guadagnare λ_{k+1} come radice senza perdere $\lambda_1, \ldots, \lambda_k$

$P_k(z)$ ha come radici $\lambda_1, \ldots, \lambda_k$

$$P_{k+1}(z) = \left[P_k(z)\right]^2 - a_{k+1}$$

posso scegliere

$2001 \Rightarrow 2002$

$$P_{2002}(z) = \left[P_{2001}(z)\right]^2 - a_{2002}$$

Voglio che

$$P_{2002}(\lambda_{2002}) = 0$$

Se fosse $\boxed{P_{2001}(\lambda_1) = \dots = P_{2001}(\lambda_{2001})} = C$

$$[P_{2001}(\lambda_{2002})]^2 = a_{2002}$$

$$C^2 = a_{2002} \implies P_{2002}(\lambda_i) = 0$$

$$i = 1, \dots, 2001$$

Deve succedere che al passaggio k si deve avere

$$P_k(\lambda_1) = P_k(\lambda_2) = \dots = P_k(\lambda_k) = C$$

stesso C

$$P_k(\lambda_{k+1}) = -C$$

posso sceglierlo

$\boxed{k=1}$

$$p_1(\lambda_1) = c \qquad p_1(z) = z - a_1$$
$$p_1(\lambda_2) = -c$$

$$\lambda_1 - a_1 = -(\lambda_2 - a_1) = -\lambda_2 + a_1$$

$$2a_1 = \lambda_1 + \lambda_2$$

$\boxed{k=1 \Rightarrow k=2}$

$$\boxed{p_2(\lambda_1) = p_2(\lambda_2)} = -p_2(\lambda_3)$$

$$p_2(z) = [p_1(z)]^2 - a_2$$

è automatica perchè $p_1(\lambda_1) = -p_1(\lambda_2)$
indipendent. da a_2

Per scegliere a_2 basta imporre $p_2(\lambda_2) = -p_2(\lambda_3)$

$k=2 \Rightarrow k=3$

$$p_3(\lambda_1) = p_3(\lambda_2) = p_3(\lambda_3) = -p_3(\lambda_4)$$

autom.
indip.
da a_3

trovo a_3