Senior 2006 – Algebra 2 – esercizi

Titolo nota 14/09/2006

1, 3, 5, Hojoo Lee ...

$x_1, x_2, \dots, x_n$

$HM = 6$

$GM = 7$

$AM = 8$

$$X_i = \prod_{k \neq i} x_k = \frac{GM^n}{x_i} \qquad HM' = \frac{n}{\sum x_i \cdot \frac{1}{GM^n}} = \frac{GM^n}{AM}$$

$$GM' = \sqrt[n]{\frac{GM^{n^2}}{\prod x_i}} = \frac{GM^n}{GM} = GM^{n-1}$$

$$AM' = \frac{GM^n \cdot \sum \frac{1}{x_i}}{n} = \frac{GM^n}{HM}$$

$$\frac{1}{HM}$$

3

$N = a \cdot 10^8 \cdot b \cdot 10^7 \cdot c \cdot 10^6 + d \cdot 10^5 \ldots$

$q = 10^2 \cdot a + 10 \cdot b + c$

$r = 10^2 b + 10c + d$

$s = 10^2 c + 10d + e$

$z = 10^2 g + 10^2 g + g$

$q + r + s + t + u + v +$

$+ z = 100a + 110b + 111c + 111d + 111e + 111f + 11g + 1h$

$max = 1 + 1 + 2 + 100 \cdot 3 + 110 \cdot 4 + 111(5 + 6 + 7 + 8 + 9) =$

⑤

$$\frac{\underbrace{x_1\sqrt{y_1}+\ldots+x_n\sqrt{y_n}}_{\to SP}}{n}=9 \qquad =9n$$

$$\frac{y_1+\ldots+y_n}{n}=8 \qquad =8n$$

$(x_1, \ldots x_n)$ $(\sqrt{y_1} \ldots \sqrt{y_n})$

$$\underset{81n^2}{SP^2} \leq \underset{8n}{(y_1+\ldots+y_n)}\overset{A}{(x_1^2+\ldots+x_n^2)}$$

$$8nA \geq 81n^2 \qquad A \geq \frac{81n}{8}$$

$$\min QM = \sqrt[3]{\frac{81}{8}} = \frac{9}{2\sqrt{2}}$$

$$\frac{x}{y} + \frac{y}{z} + \frac{z}{x} \geqslant x+y+z \qquad \underline{xyz=1} \qquad x,y,z \in \mathbb{R}^+$$

$$x = \frac{a}{b} \quad y = \frac{b}{c} \quad z = \frac{c}{a}$$

$$\frac{ac}{b^2} + \frac{bc}{a^2} + \frac{ab}{c^2} \geqslant \frac{a}{b} + \frac{b}{c} + \frac{c}{a}$$

$$a^3c^3 + b^3c^3 + a^3b^3 \geqslant a^3bc^2 + \underline{a^2b^3c} + ab^2c^3$$

$$\begin{array}{cccccc} x_1 & x_2 & x_3 & y_1 & y_2 & y_3 \\ a^2b^2 & b^2c^2 & a^2c^2 & ab & bc & ac \end{array}$$

$xyz=1$

$$\underbrace{\frac{x}{y}+\frac{y}{z}}+\frac{z}{x}\geq x+y+z$$

$$\frac{x}{y}+\frac{x}{y}+\frac{y}{z}\geq 3\sqrt[3]{\frac{x^2}{yz}}=3\sqrt[3]{x^3}=3x$$

$$2\cdot\frac{x}{y}+\frac{y}{z}\geq 3x$$

$$3\left(\frac{x}{y}+\frac{y}{z}+\frac{z}{y}\right)\geq 3(x+y+z)$$

4 $\quad x+y+z=1 \qquad x^5yz$

$$1=x+y+z=5\frac{x}{5}+y+z=\frac{x}{5}+\frac{x}{5}+\frac{x}{5}+\frac{x}{5}+\frac{x}{5}+y+z\geq 7\sqrt[7]{\frac{x^5yz}{5^5}}$$

$$x^5yz\leq\frac{5^5}{7^7}$$

9

$$\left(\sum \frac{x_i}{n}\right)\left(\sum \frac{y_i}{n}\right) \leq \left(\frac{\sum x_i y_i}{n}\right)$$

$$x_i = x_i$$

$$y_i = \left(\sqrt{1-x_i}\right)^{-1}$$

$$\left(\sum \frac{x_i}{n}\right)\left(\sum \frac{\frac{1}{\sqrt{1-x_i}}}{\cancel{n}}\right) \leq \left(\frac{\sum \frac{x_i}{\sqrt{1-x_i}}}{\cancel{n}}\right)$$

$$\left(\frac{1}{n}\right)\left(\sum \frac{1}{\sqrt{1-x_i}}\right) \leq LHS$$

$$\cancel{\frac{1}{n}}\left(\quad \ \right) \overset{?}{\geq} n\sqrt{\frac{n}{n-1}}$$

$$\frac{1}{HM}\cdot n \geq n\frac{1}{QM} = n\sqrt{\frac{n}{\sum 1-x_i}} = n\sqrt{\frac{n}{n-1}}$$

$$\sum \frac{x_i}{\sqrt{1-x_i}} \geq \sqrt{\frac{n}{n-1}}$$

$$\sum \sqrt{1-x_i} \leq n\sqrt{\frac{\sum 1-x_i}{n}}$$

$$= n\sqrt{\frac{n-1}{n}}$$

$$\sum \frac{1}{\sqrt{1-x_i}} - \sum \sqrt{1-x_i} \geq \text{RHS}$$

$$\sum \frac{1}{\sqrt{1-x_i}} \overset{?}{\geq} \sqrt{\frac{n}{n-1}} + n\sqrt{\frac{n-1}{n}}$$

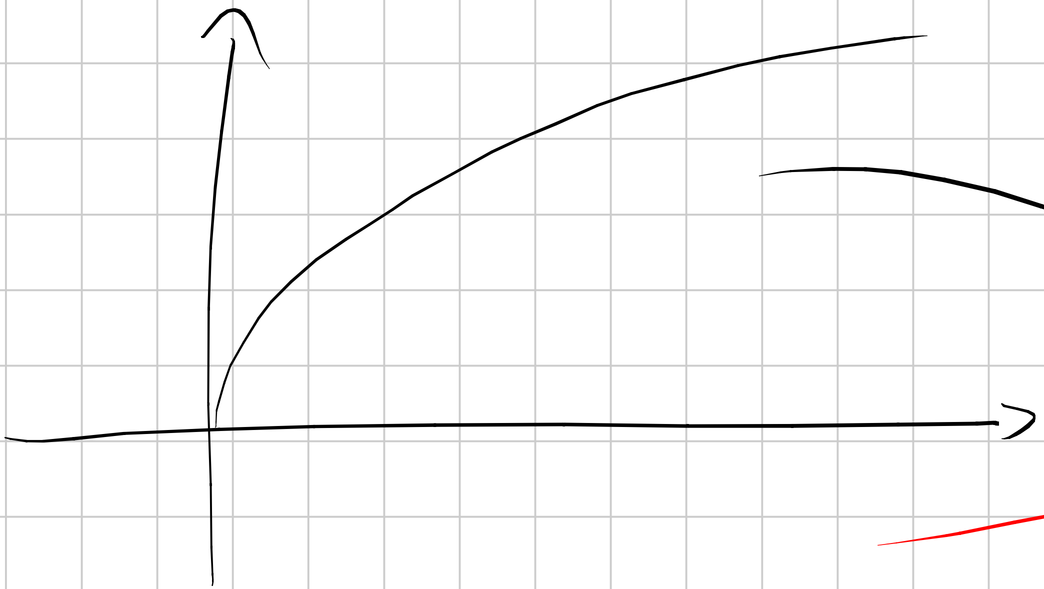
$$\text{LHS} \geq n\sqrt{\frac{n}{n-1}} \overset{?}{\geq} \sqrt{\frac{n}{n-1}} + n\sqrt{\frac{n-1}{n}}$$

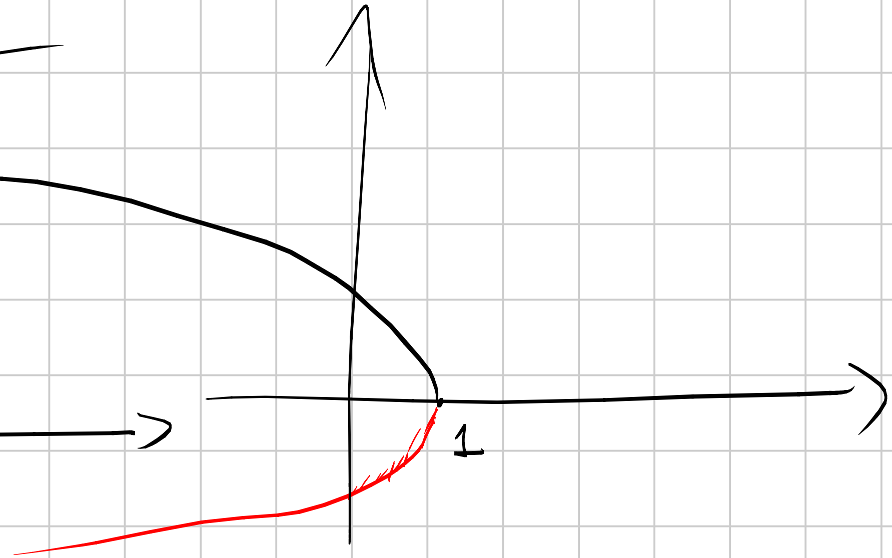
$$(n-1)\sqrt{\frac{n}{n-1}} \geq n\sqrt{\frac{n-1}{n}}$$

$$\sqrt{n(n-1)}$$

$$\frac{x_1}{\sqrt{1-x_1}} = \frac{1}{\sqrt{1-x_1}} - \sqrt{1-x_1}$$

$$x^{-\frac{1}{2}}$$





$$f(x) = \frac{1}{\sqrt{x}}$$

$$f(1-x)$$

$$\sum \frac{x_i}{\sqrt{1-x_i}} \geq n f\left(\frac{x_1+x_2 \dots x_n}{n}\right) = n \frac{\frac{1}{n}}{\sqrt{1-\frac{1}{n}}} = \sqrt{\frac{n}{n-1}}$$

$$\parallel$$

$$\frac{1}{n}$$

$\sqrt{x} + \sqrt{y} + \sqrt{z} \geq xy + yz + zx$ $\quad x+y+z=3$

$x, y, z \geq 0$

$\frac{3}{2}$ $\quad \frac{1}{2}$ $\quad 2$

$\left(\frac{x+y+z}{3}\right)^3$. $\quad LHS^2 \geq RHS^2$

$$xy+yz+zx = \frac{3^2 - x^2 - y^2 - z^2}{2} \qquad x+y+z=3$$

$$2\sqrt{x} + 2\sqrt{y} + 2\sqrt{z} + x^2 + y^2 + z^2 \geq 9$$

$$2\sqrt{x} + x^2 = \sqrt{x} + \sqrt{x} + x^2 \geq 3\sqrt[3]{\underbrace{\sqrt{x}\sqrt{x}x^2}_{x^3}} = 3x$$

$$LHS \geq 3x + 3y + 3z = 9$$

$$\frac{a}{b} + \frac{b}{c} + \frac{c}{a} \geq \frac{a+b}{b+c} + \frac{c+b}{a+b} + 1$$

SI FA CON CONTI

C.V.D.