Senior 2006 - Algebra 3 - esercizi (4,5,9,10)

Titolo nota 15/09/2006

$f : \mathbb{R} \longrightarrow \mathbb{R}$ MONOTONA

$$\boxed{f(x+f(y)) = f(x) + \underline{y}}$$

$f(100) = ?$

ASSUME TUTTI I VALORI IN $\mathbb{R}$

<u>f È INIETTIVA E SURIETTIVA</u>

$f(y) = f(y')$

$$\underbrace{f(x+f(y))}_{f(x)+y} = \underbrace{f(x+f(y'))}_{f(x)+y'}$$

$y = y'$

$x=0$ $\quad$ $f(f(y)) = f(0)+y$

$f(k)=0$ (f SURIETTIVA)

$y=k$ $\quad$ $f(x) = f(x)+k$ $\quad$ $k=0$

$f(0)=0$

$f(f(y)) = y \quad (*)$ $\qquad$ $f(x+f(y)) = f(x)+y$

$y=f(z)$ $\qquad$ $\boxed{f(x+z)=f(x)+f(z)}$

$\boxed{f(x)=\alpha x}$ SE ESISTE UN INTERVALLO $[a,b]$ E UN $L \geq 0$

TALE CHE $|f(x)| \leq L$ PER $x\in[a,b]$

NEL NOSTRO CASO $x\in[a,b]$ $\qquad$ $f(a) \leq f(x) \leq f(b)$

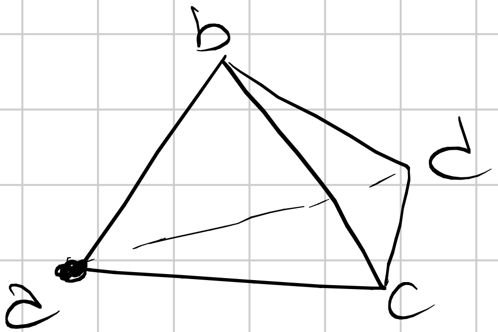
$ax + a^2y = a(x + ay) = ax + y$ $\qquad a = \pm 1$

$f(x) = x$ OPPURE

$f(x) = -x$

---

5)



a_k = PROB. CHE LA PULCE SIA IN a DOPO k SALTI

b_k, c_k, d_k — —

$a_{k+1} = \frac{1}{3}(b_k + c_k + d_k)$

$$\begin{cases} b_{k+1} = \frac{1}{3}(a_k + c_k + d_k) \\ c_{k+1} \\ d_{k+1} \end{cases}$$

$b_k + c_k + d_k = e_k$

$$\begin{cases} e_{k+1} = a_k + \frac{2}{3} e_k = \frac{2}{3} e_k + \frac{1}{3} e_{k-1} \\ a_{k+1} = \frac{1}{3} e_k \longrightarrow a_k = \frac{1}{3} e_{k-1} \end{cases}$$

$$x^2 - \frac{2}{3} x - \frac{1}{3} = 0$$

$1, \; -\frac{1}{3}$

$$e_k = \lambda + \mu \left(-\frac{1}{3}\right)^k$$

$e_0 = 0$
$e_1 = 1$

$\lambda + \mu = 0$
$\lambda - \frac{1}{3}\mu = 1$

$\lambda = \frac{3}{4} \quad \mu = -\frac{3}{4}$

$$e_{2002} = \frac{3}{4} - \frac{3}{4}\left(-\frac{1}{3}\right)^{2002}$$

$$\boxed{a_{k+1} = \frac{1}{3}(1 - a_k)}$$

g)

$$x_{n+2} = x_{n+1} + x_n$$

$$\boxed{\begin{array}{l} x_0 = a \\ x_1 = 1 \end{array}}$$

$4091 \mid x_{4091} - 1$

$$y_{n+2} = y_{n+1} + y_n$$

$y_0 = 1$

$y_1 = 0$

$y_2 = 1$

$y_n = F_{n-1}$

$$\boxed{x_n = a\,\underline{y_n} + \underline{F_n}}$$

$X_n = 2F_{n-1} + F_n$

$4091 \mid \underbrace{2F_{4090}} + \underbrace{F_{4091} - 1}$

FIBONACCI MODULO 4091

$4091 + 5 = 4096 = 64^2$

$5 \equiv b^2 \pmod{4091}$

$x^2 - x - 1 \equiv 0 \pmod{4091}$

$\left(\frac{1+\sqrt{5}}{2} \right) \quad \frac{-1+\sqrt{5}}{2}$

$(1+b) \cdot 2^{-1}$

$(1-b)\, 2^{-1}$

$F_n = \frac{1}{\sqrt{5}} \left(\frac{1+\sqrt{5}}{2} \right)^n - \frac{1}{\sqrt{5}} \left(\frac{1+\sqrt{5}}{2} \right)^n$

$$F_{4090} \equiv \frac{1}{\sqrt5}\left(\frac{1+\sqrt5}{2}\right)^{4090} - \frac{1}{\sqrt5}\left(\frac{-1+\sqrt5}{2}\right)^{4090} \equiv 0$$

$\equiv 1 \mod 4091$ (4091 è primo)

$$F_{4091} \equiv \frac{1}{\sqrt5}\left(\frac{1+\sqrt5}{2}\right) - \frac{1}{\sqrt5}\left(\frac{-1+\sqrt5}{2}\right) \equiv 1$$

---

10) $f(x\,f(x)+f(y)) = f(x)^2 + y \qquad f:\mathbb{R}\to\mathbb{R}$

f è iniettiva e surjettiva $\qquad f(k)=0$

$x=y=k \qquad f(0)=k$

$x=0 \quad y=k \qquad f(0) = f(0)^2 + k = f(0)^2 + \cancel{f(0)}$

$f(0)=0$

$x=0 \qquad \boxed{f(f(y)) = y}$

$y=0 \qquad f(x\,f(x)) = f(x)^2$

$$\underbrace{f(x\,f(x))}_{\parallel}$$

$$f\big(f(f(x))\,f(x)\big) = f(f(x))^2 = x^2$$

$f(x)^2 = x^2 \qquad f(x) = \pm x$ PER OGNI X

$\boxed{f(1) = 1} \qquad x=1$

$$\left.\begin{matrix} 1 + f(y) \\ -1 - f(y) \end{matrix}\right\rangle = f(1 + f(y)) = 1 + y$$

$1 + f(y) = 1+y \implies f(y) = y$

$1+y = -1 - f(y)$

$2 = -y - f(y)$

$\underline{f(y) = y}$ $\quad 2 = 0$ no!

$f(y) = -y$ $\quad y = -1$

$$f(\underline{1+f(y)}) = \underline{1+y}$$

$\cancel{1} + 2y + \cancel{y^2} = (1+y)^2 = (1+f(y))^2 = \cancel{1} + 2f(y) + \cancel{f(y)^2}$

$\underline{f(y) = y}$

$f(1) = -1 \implies f(x) = -x$ PER OGNI x