Senior 2006 – Combinatoria 1 – 1, 5, 7, 10

Titolo nota 13/09/2006

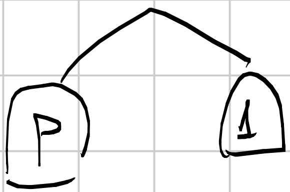
1)

× 5 × × × × ×

$\binom{4}{1} \, 5 \, \binom{15}{5}$

$$\frac{4\binom{15}{5}}{\binom{20}{7}}$$

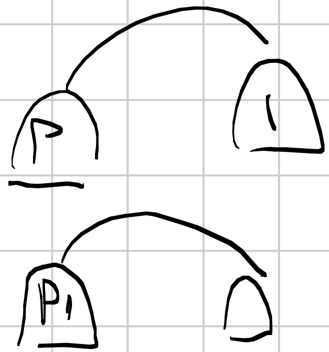
5)



$P \leq 3^n$

$P = 3P' + j$

$j = 0, 1, 2$



1 10 100

$$\frac{3^n - 1}{2}$$

2 9 3 1

$$\begin{array}{ccccccc} & & & & 9 & 3 & 1 \\ & & & & \downarrow & \downarrow & \downarrow \\ 1 & 0 & 2 & 1 & 1 & 2 & 0 \end{array}$$

0, 1, 2 (base 4)

$$\sum_{k=0}^{n-1} 3^k a_k = \sum_{k=0}^{n-1} 3^k b_k$$

$a_k, b_k = 0, 1, 2$

Non vi sono 2 configurazioni diverse che danno lo stesso peso

R.P.A.

$$p_1\, p_2 \cdots p_k \;>\; q_1\, q_2 \cdots q_l$$

$$+ \qquad\qquad +$$

$$q'_1\, q'_2 \cdots q'_s \;<\; p'_1\, p'_2 \cdots p'_h$$

$$p \;\; q' \qquad\qquad q \;\; p'$$

$$0, 1, 2 \;=\; 0, 1, 2$$

$1, 2, \dots, 2^{n-1} \longrightarrow 2^n - 1$ peso max

$2^n - \cancel{2}\,1$

$\frac{3^n - 1}{2}$

$1+2+ \dots + 2^{n-1} = 2^n - 1$

vero!

7) $F = \{(A_1, \dots, A_n) \mid A_i \subseteq H\}$

$H = \{1, 2, \dots, 1998\}$

$|H| = 1998$

$|\mathcal{P}(H)| = 2^{1998}$

$|F| = 2^{1998n}$

$$\sum_{(A_1,\dots A_n)\in F} |A_1 \cap \dots \cap A_n| = \sum_{(\ )\in F} \sum_{a \in A_1 \cap \dots \cap A_n} 1$$

$$= \sum_{a\in H} \sum_{\substack{(A_1 \cdots A_n)\in F \\ a \in A_1 \cap \dots \cap A_n}} 1 = \sum_{a \in H} 2^{1997n} = 1998 \cdot 2^{1997n}.$$

$$\sum_{(A_1 \dots A_n) \in F} \sim\sim = \sum_{A_1 \subseteq H} \sum_{A_2 \subseteq H} \dots \sum_{A_n \subseteq H} \sim\sim$$

$$\sum_{A_1 \subseteq H} \sum_{A_2 \subseteq H} \dots \sum_{A_n \subseteq H} |A_1 \cup A_2 \cup \dots \cup A_n|$$

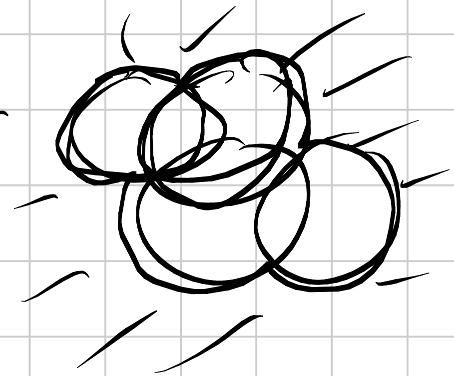
$$= \sum \sum \dots \sum \sum_{a \in A_1 \cup A_2 \dots A_n} 1 = \sum_{(A_1, \dots A_n) \in F} \sum_{a \in \bigcup A_i} 1$$

$$= \sum_{a \in H} \sum_{\substack{(A_1 \dots A_n) \in F \\ a \in \bigcup A_i}} 1$$

risp H

$$|A_1 \cup \dots \cup A_n| = |(A_1^c \cap A_2^c \cap \dots \cap A_n^c)^c|$$

$$= |H| - |A_1^c \cap \dots \cap A_n^c|$$

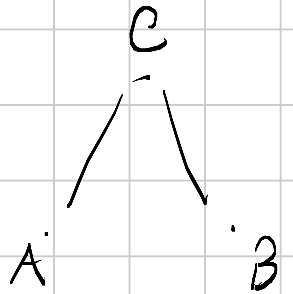


$$\sum_{(A_1 \dots A_n)\in F} |\cup A_i| = \sum_{(\ \ )\in F} |H| - \sum_{(\ \ )\in F} |A_1^c \cap \dots \cap A_n^c|$$

$$= 2^{1998n} \cdot 1998 - 1998 \cdot 2^{1997n}$$

$$= \underbrace{1998 \cdot 2^{1997n}} \left(2^n - 1\right)$$

(10)



$$12K \binom{3K+6}{2} = \binom{12K}{2} N$$

$$(3K+6)(3K+5) = (12K-1)N$$

$$12K-1 \mid 9K^2 + 33K + 30$$

$$12K-1 \mid 36K^2 + 132K + 120$$

$$12K^2 + 44K + 40$$

$$(12k-1)k + 45k + 40$$

$$(12k-1)(k+4) - 3k + 44$$

$$12k-1 \mid 3k-44$$

$$12k - 176$$

$$12k-1 \mid 175$$

$$35 \qquad k = 3$$

$$N = \frac{9k^2+33k+30}{12k-1} = ak$$

$$\frac{5k^2-3k+1}{k-1} = \frac{5k(k-1)}{k-1} + \frac{2k+1}{k-1}$$

$$= 5k + \frac{2(k-1)}{k-1} + \frac{3}{k-1}$$

$$\Longleftrightarrow \quad = 5k + 2 + \frac{3}{k-1} \qquad k = -2, 0, 2, 4$$

$$5k^2 - 3k + 1 =$$

$$(k-1)(5k+2) + 3$$

3i $\rightarrow \dfrac{9k^2+33k+30}{12k-1} \iff \dfrac{3k^2+11k+10}{12k-1}$

$\iff \dfrac{12k^2+44k+40}{12k-1} = k + \dfrac{45k+40}{12k-1}$

$\iff \dfrac{180k+160}{12k-1} = 15 + \dfrac{175}{12k-1}$

$\iff \dfrac{175}{12k-1}$

$35 = 12k-1$ $\quad k=3$

$k=0$!

$k=-2$

$\boxed{36}$