TdN - esercizi

Titolo nota 11/09/2006

$x^2 + px - 444p = 0$

$\Delta = p^2 + 2^2 \cdot 3 \cdot 37 \cdot 4 \cdot p = p(p + 2^4 \cdot 3 \cdot 37)$

2 3 37

$\alpha^2 + 2\alpha - 888$

Δ

$\alpha \quad \beta$

$p = -\alpha - \beta$

$-444p = \alpha\beta$

$p \mid \alpha\beta \Rightarrow p \mid \alpha$

$p \mid \beta$

$\frac{\alpha}{p} \qquad \frac{\beta}{p}$

$x^2 - \left(\frac{\alpha+\beta}{p}\right) x + \frac{\alpha\beta}{p^2}$

$x^2 + x - \frac{444}{p} = 0$

$p \mid \alpha \qquad p \mid \beta$

$p^2 \mid \alpha\beta = -444p$

$p \mid 444$

$p=2$ $\quad \frac{444}{2} = \underbrace{2 \cdot 3}\cdot \boxed{37} \quad 37 \cdot 6$

$p=3$ $\quad \frac{444}{3} = 2^2 \cdot 37 \quad 37 \cdot 4$

$p=37$ $\quad 2^2 \cdot 3 \quad 4 \cdot 3$

$x^2 + x \cancel{+} - 12 \quad$ 3 $\quad 3 \cdot 37$

$+4x$

$-3x \quad -4 \quad -4 \cdot 37$

$(x-3)(x+4)$

$(x-\alpha)(x-\beta) = x^2 - \alpha x - \beta x + \alpha\beta$

$x^2 - 5x \quad + p$

Eisenstein

p primo

$p(x)$

- monico [$p \nmid$ coeff. di testa]
- tutti i coeff (tranne il primo) sono divisibili per p
- termine noto non div. per p^2

è irriducibile su $\mathbb{Q}$

$\ast$

$a \mid b^3, \; b \mid c^3, \; c \mid a^3$ $\qquad p \mid a \Rightarrow p \mid b \wedge p \mid c$

$(a,b,c) > 1$ $\qquad p \parallel c \qquad b \mid c^3 \to b = p^3 \to a = p^9$

$\Downarrow$

$abc = p^{13} \qquad (a+b+c)^n = p^n(1+p^2+p^8)^n$

$\qquad\qquad\qquad\qquad\qquad \equiv 1 \quad (p)$

$p \parallel a+b+c$

$p^n \parallel (a+b+c)^n$

$p^{13} \mid$

$n \geq 13$

$\ast \Rightarrow abc \mid (a+b+c)^{13}$

$p^{\alpha} \parallel a \qquad p^{\beta} \parallel b \quad p^{\gamma} \parallel c \qquad p^{\alpha+\beta+\gamma} \parallel abc$

$p^{3\beta} \parallel b^3$

$a \mid b^3$

$\alpha \leq 3\beta$

- $\alpha \leq 3\beta$ $\qquad \beta \leq 3\gamma$ $\qquad \gamma \leq 3\alpha$ $\qquad \alpha$ sia il minimo

- $\beta \leq 3\gamma \leq 9\alpha$

- $\gamma \leq 3\alpha$

$$\alpha+\beta+\gamma \leq 13\alpha$$

α $\qquad$ $p \nmid$

$$p^{13\alpha} \mid (a+b+c)^{13}$$

$$p^{\alpha}\cdot q_1 + p^{\beta} q_2 + p^{\gamma} q_3 = p^{\alpha} \underbrace{\left(q_1 + p[\dots]\right)}_{p^{\alpha} \nmid}$$

$$(100+n^2, \quad 100 + (n+1)^2) = \star$$

$$\quad\quad\quad\quad\quad\quad\quad\quad\quad \| $$

$$100 + n^2 + 2n + 1$$

$$100 + n^2 + 2n + 1 = (100 + n^2) \cdot 1 + (2n+1)$$

$$\star = (100 + n^2, 2n+1)$$

$$\|$$

$$(200 + 2n^2, 2n+1)$$

$$\cancel{2n^2} - \cancel{2n^2} - n + 2n^2 + n + 200 - =$$

$$200 + 2n^2 = (2n+1)q + \underline{200 - n}$$

$d = (2n+1,\ n-200) = (401,\ 2n+1)$

primo

$\cancel{2n}+1 - \cancel{2n} + 400 + \underline{2n - 400}$

$2n+1 = (n-200)q + 401$

$n = 200$

$100 + n^2 = 40100$

$100 + (n+1)^2 = 40501$