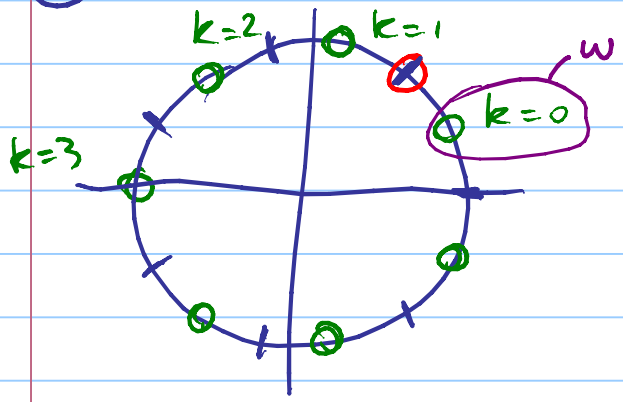
ALGEBRA 1 esercizi

Titolo nota 04/09/2007

(6) w radice settima non reale di -1



$w^7 = -1 \Rightarrow w^{14} = 1 \quad \pi = \mathrm{Arg}(-1) = \mathrm{Arg}(w^7) = 7\,\mathrm{Arg}(w) - 2k\pi$

$|w| = 1$ $\qquad \mathrm{Arg}\, w = \dfrac{\pi + 2k\pi}{7}$

w rdu 14-esima primitiva

$w^7 = -1 \Rightarrow w^{14} = 1$

$\rightarrow (w^2)^7 = w^{14} = 1$

$(w^{2k+1})^7 = (w^{14})^k \cdot w^7 = -1 \qquad \varphi(14) = 6$

$$\prod_{k=1}^{2002}\left(x-\omega^k\right)=\left[\prod_{k=1}^{14}\left(x-\omega^k\right)\right]^{143}=\left(x^{14}-1\right)^{143}$$

⑦ $A(x)=x(x-1)(x-2)(x-3)$

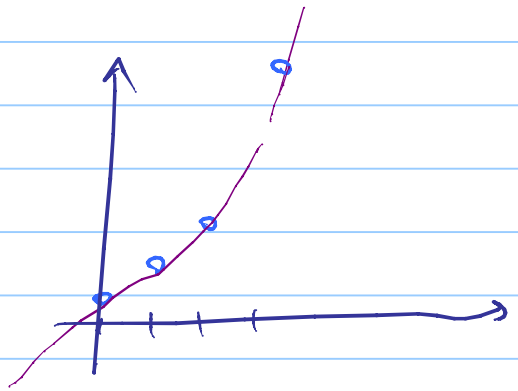
$P(x)=A(x)Q(x)+R(x)$

$\uparrow$ $\deg R(x)\le 3$

$A(x)=0 \quad x=0,1,2,3$

$P(x)=R(x) \quad x=0,1,2,3$

$R(0)=2 \quad R(1)=4 \quad R(2)=6 \quad R(3)=56$



$$R(x) = ax^3 + bx^2 + cx + d$$

$$\begin{cases} R(0) = a\cdot 0 + b\cdot 0 + c\cdot 0 + d = 2 \\ R(1) = a\cdot 1 + b\cdot 1 + c\cdot 1 + d = 4 \\ R(2) = a\cdot 8 \cdot b\cdot 4 + c\cdot 2 + d = 6 \\ R(3) = \ldots = 56 \end{cases}$$

$$R(x) = a\,x(x-1)(x-2) + b\,x(x-1)(x-3) + c\,x(x-2)(x-3)$$

$$+ d\,(x-1)(x-2)(x-3)$$

$$R(x) - 2 - 2x = B(x) = x(x-1)(x-2)\cdot 8$$

$P(0) = 2 \Rightarrow P(x) = 2 + x P_1(x)$

$P(1) = 4 \Rightarrow P(1) = 2 + 1 \cdot P_1(1) = 2 + P_1(1) \Rightarrow P_1(1) = 2$

$P_1(1) = 2 \Rightarrow P_1(x) = 2 + (x-1) P_2(x)$

$\ldots \quad P(x) = R(x) + A(x) Q(x)$

(9) Assurdo modulo 2.

$P(0) \equiv 1 \pmod 2$ se n è pari $P(n) \equiv P(0) \equiv 1 \pmod 2 \Rightarrow P(n) \neq 0$

$x \equiv y \pmod n \Rightarrow P(x) \equiv P(y) \pmod n$

$P(x) = \sum_{n=0}^{k} a_n x^n \equiv \sum_{n=0}^{k} a_n y^n = P(y) \pmod n$

$P(13) \equiv 1 \pmod 2$ se n è dispari $P(n) \equiv P(13) \equiv 1$ ...

⑧ $x^4 - x^3 - \frac{1}{2}x^2 - \frac{1}{6}x + \frac{1}{24}$

$\frac{1}{a} + \frac{1}{b} = \frac{1}{2007} \iff \frac{a+b}{ab} = \frac{1}{2007} \Rightarrow ab = 2007(a+b)$

$\Updownarrow$

$ab - 2007(a+b) = 0$

$\Updownarrow$

$(a-2007)(b-2007) = 2007^2$

$= 3^4 \cdot 233^2$

$\uparrow 5 \quad \uparrow 3^2 \quad 15 \quad 30 \quad 29$

(a,b) a,b interi positivi : $\boxed{a+b > ab}$

$ab - a - b < 0$

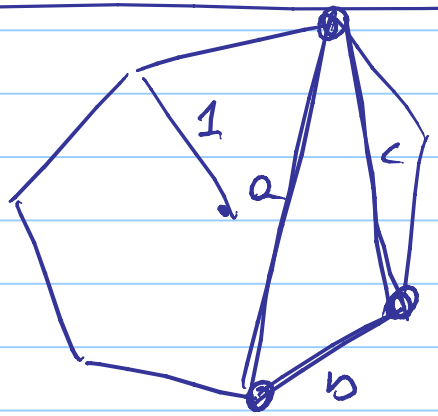
$ab - a - b + 1 < 1$ | $a = 1$ o $b = 1$.

$(a-1)(b-1) < 1$

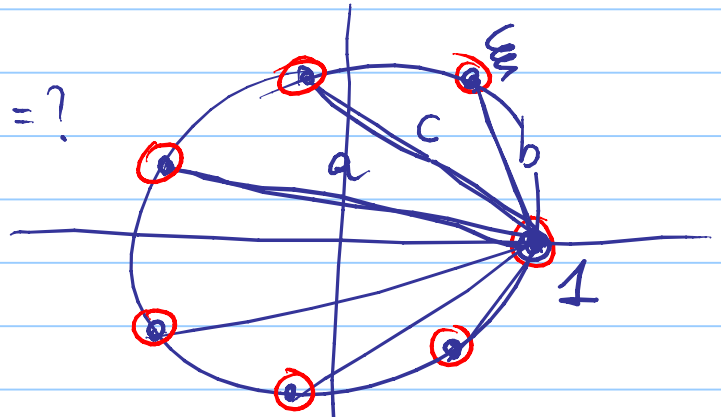
$$|ab(a^2-b^2)+bc(b^2-c^2)+ca(c^2-a^2)| \le M(a^2+b^2+c^2)^2$$

$a=b \Rightarrow LHS = 0$

$$LHS = |(a-b)(b-c)(a-c)(a+b+c)|$$



$abc = ?$



$$(abc)^2 = |(1-\xi)(1-\xi^2)\dots(1-\xi^6)|$$

$$(abc)^2 \overset{x\to 1}{=} |(x-\xi)(x-\xi^2)\dots(x-\xi^6)| = |1+x+\dots+x^6| \overset{x=1}{=} 7$$

$$x^7 - 1 = (x-\xi)(x-\xi^2) \dots (x-\xi^6)(x-\underbrace{1}_{\xi^7})$$

$$abc = \sqrt{7}$$