Titolo nota 05/09/2007

$\boxed{\text{Es. 3}}$

$$N = a_1 a_2 a_3 \dots a_8 a_9$$

$$\text{Somma} = 100 a_1 + 110 a_2 + 111 (a_3 + \dots + a_7) + 11 a_8 + a_9$$

	a_1	a_2	$a_3, \dots, a_7$	a_8	a_9
Max	3	4	5 – 9	2	1
min	7	6	1 – 5	8	9

$\boxed{\text{Es. 5}}$

$$9n = \sum x_i \sqrt{y_i} \le \left(\sum x_i^2\right)^{1/2} \left(\sum y_i\right)^{1/2}$$

$$= \left(\sum x_i^2\right)^{1/2} (8n)^{1/2}$$

$$\left(\sum x_i^2\right)^{1/2} \ge \frac{9 \cancel{n} \sqrt{n}}{\sqrt{8}\, \cancel{\sqrt{n}}}$$

$$\underbrace{\left(\frac{\sum x_i^2}{n}\right)^{1/2}}_{= Q(x_1, \dots, x_n)} \ge \frac{9}{\sqrt{8}}$$

Bisogna fare un esempio in cui vale l'uguaglianza

ad es. con $x_1 = \dots = x_n =$ valore opportuno

e idem $y_1 = \dots = y_n =$

$\boxed{\text{Es 10}}$ Disug. di NESBIT

$$\underbrace{\frac{a}{b+c}}_{x} + \underbrace{\frac{b}{c+a}}_{y} + \underbrace{\frac{c}{a+b}}_{z} \geq \frac{3}{2}$$

(Omog. di grado 0)

$\boxed{1^a \text{ dim.}}$

$x = b+c$

$y = c+a$

$z = a+b$

$$a = \frac{y+z-x}{2}$$

$$b = \frac{z+x-y}{2}$$

$$c = \frac{x+y-z}{2}$$

Sostituendo:

$$\cancel{\frac{1}{2}}\left\{ \frac{y}{x} + \frac{z}{x} - 1 + \frac{z}{y} + \frac{x}{y} - 1 + \frac{x}{z} + \frac{y}{z} - 1 \right\} \geq \frac{3}{\cancel{2}}$$

$$\boxed{\frac{y}{x}} + \boxed{\frac{z}{x}} + \boxed{\frac{z}{y}} + \boxed{\frac{x}{y}} + \boxed{\frac{x}{z}} + \boxed{\frac{y}{z}} \geq 6$$

$$\frac{y}{x} + \frac{x}{y} \geq 2$$

AM ≥ GM

$\boxed{2^a \text{ dim.}}$ C.S.

$$a+b+c = \boxed{\frac{\sqrt{a}}{\sqrt{b+c}}}\,\boxed{\sqrt{a}\sqrt{b+c}} + \boxed{\frac{\sqrt{b}}{\sqrt{c+a}}}\,\boxed{\sqrt{b}\sqrt{c+a}} + \boxed{\frac{\sqrt{c}}{\sqrt{a+b}}}\,\boxed{\sqrt{c}\sqrt{a+b}}$$

$$\overset{C.S.}{\leq} \Big(\underbrace{\frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b}}_{LHS}\Big)^{1/2} \left(ab+ac+bc+ba+ca+cb\right)^{1/2}$$

$$(LHS)^{\cancel{1/2}} \geq \frac{(a+b+c)^2}{\{\cancel{2}(ab+bc+ca)\}^{\cancel{1/2}}} \overset{\text{Hope}}{\geq} \frac{3}{\cancel{2}}$$

$$(a+b+c)^2 \underset{\text{Hope}}{\geq} 3(ab+bc+ca)$$

$$a^2+b^2+c^2 + \cancel{2(ab+bc+ca)} \geq \cancel{3}(ab+bc+ca)$$

CLASSICA

$\boxed{3^a \text{ SOLUZIONE}}$

$$a\cdot\frac{1}{b+c}+b\cdot\frac{1}{c+a}+c\cdot\frac{1}{a+b}$$

WLOG $\quad a\geq b\geq c \quad$ Without Loss of Generality

$$a+b\geq a+c\geq b+c$$

$$\frac{1}{b+c}\geq\frac{1}{a+c}\geq\frac{1}{a+b}$$

Media prodotti $\geq$ prodotto medie

$$\frac{LHS}{3}\geq\left(\frac{a+b+c}{3}\right)\cdot\underbrace{\overbrace{\left(\frac{\frac{1}{b+c}+\frac{1}{c+a}+\frac{1}{a+b}}{3}\right)}^{\boxed{\text{CMS}}}}_{\frac{1}{H(a+b,\,b+c,\,c+a)}}\overset{\text{Hope}}{\geq}\frac{1}{2}$$

$$\frac{1}{H(a+b,\,b+c,\,c+a)}\geq\frac{1}{A(a+b,\,b+c,\,c+a)}$$

$$\text{CHS} \geq \frac{\cancel{a+b+c}}{3} \cdot \frac{1}{2\left(\cancel{\frac{a+b+c}{3}}\right)} = \frac{1}{2}$$

Per omogeneità possiamo supporre $a+b+c=1$

$$\text{LHS} = \frac{a}{1-a} + \frac{b}{1-b} + \frac{c}{1-c} \qquad f(x) = \frac{x}{1-x}$$

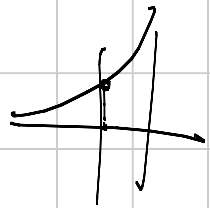
$$3\,\frac{f(a)+f(b)+f(c)}{3} \underset{\substack{\text{CONV.}\\ \text{(Hope)}}}{\geq} 3\, f\left(\frac{a+b+c}{3}\right)$$

$$= 3\, f\left(\frac{1}{3}\right)$$

$$= 3 \cdot \frac{1}{2}$$

Se f è convessa tutto ok

$$\frac{x}{1-x} = \frac{\boxed{x}+1\boxed{-1}}{1-x} = -1 + \frac{1}{1-x} \longrightarrow$$



OCCHIO: Basta convessa in $[0,1]$.

$$\frac{a}{a+b}+\frac{b}{b+c}+\frac{c}{c+a} > \frac{a}{a+b+c}+\frac{b}{a+b+c}+\frac{c}{a+b+c} = 1$$

Si potrebbe far vedere che può essere vicino a 1 quanto voglio

a enorme $\qquad c = 1 \qquad b$ piccolissimo

$\sim 1 \qquad \sim 0 \qquad \sim 0$

$$\frac{a}{a+b} = \boxed{\frac{a}{a+b}} + 1 \boxed{-1} = \frac{\cancel{a}-\cancel{a}-b}{a+b} + 1 = 1 - \frac{b}{a+b}$$

$$CHS = 3 \overset{A}{\boxed{- \frac{b}{a+b} - \frac{c}{b+c} - \frac{a}{c+a}}} \geq k$$

$$1 < \frac{b}{a+b}+\frac{c}{b+c}+\frac{a}{c+a} \leq \underbrace{3-k}$$

$3-k > 1$

$k < 2$

$CHS = 3 - A < 3 - 1 = 2 \qquad A > 1$

$-A < -1$

Come prima ci si avvicina a 2 quanto si vuole.

— o — o —

$\boxed{\text{Es. 9}}$

$$\sum \frac{x_i}{\sqrt{1-x_i}} = \underbrace{\sum \frac{1}{\sqrt{1-x_i}}}_{1} - \underbrace{\sum \sqrt{1-x_i}}_{2} \geq$$

$$\frac{\sum \frac{1}{\sqrt{1-x_i}}}{n} \geq \frac{1}{A} = \frac{n}{\sqrt{1-x_1} + \dots + \sqrt{1-x_n}}$$

$$\left[H\left(\sqrt{1-x_1}, \dots, \sqrt{1-x_n}\right) \right]^{-1} = \frac{1}{A}$$

$$\frac{1}{H} \geq \frac{1}{Q} = \sqrt{\frac{n}{(1-x_1)+\dots+(1-x_n)}} = \sqrt{\frac{n}{n-1}}$$

$$① \geq n\sqrt{\frac{n}{n-1}}$$

$$② = \underbrace{\frac{\sum \sqrt{1-x_i}}{n}}_{\text{Aritm}} \cdot n \leq \underbrace{\sqrt{\frac{(1-x_1)+\dots(1-x_n)}{n}}}_{\text{Quadr}} \cdot n$$

$$= \sqrt{\frac{n-1}{n}} \cdot n$$

$$-② \geq -n\sqrt{\frac{n-1}{n}}$$