Stage senior 2007-08 - COMB 1

Titolo nota 03/09/2007

CONTEGGI

5A 7G 3E

1A 1G 1E

$5 \times 7 \times 3$ modi

A, B $\{(a, b)\}$ quante coppie?

$A \times B$

$|A \times B| = |A| \times |B|$

a $\quad d(a) = ?$ $\quad a = p_1^{\alpha_1} \cdots p_n^{\alpha_n}$

$(\alpha_1 + 1) \cdots (\alpha_n + 1) = d(a)$

MB
MA = 4 MB
MO = 7 MA
MP = 5 MO

quanti prezzi $\leq$ 1 MP ?
positivi

$4 \times 7 \times 5 - 1$

5 lettere dell'alfabeto (26 lettere)
prob di indovinare parola

$$\frac{1}{26^5}$$

- $f : A \longrightarrow B \quad |B^A| = |B|^{|A|}$

quanti numeri esistono di 3 cifre in base 8?

e se le lettere sono tutte diverse?

$$26 \cdot 25 \cdot 24 \cdot 23 \cdot 22$$

$f : A \longrightarrow B$ INIETTIVE

$$\frac{|B|!}{(|B|-|A|)!}$$

$n! = 1 \cdot 2 \cdot \ldots \cdot n$

42 stagisti premio 10 al test finale

$f : \{I \ldots X\} \longrightarrow \{1 \ldots 42\}$

INIETTIVE

$$42 \cdot 41 \cdot \ldots \cdot 33$$

ordinare 7 elementi? $7!$ modi

PERMUTAZIONI di n elementi $\longrightarrow n!$

$f: A \to A$ INIETTIVE ($\Rightarrow$ surgettive $\Rightarrow$ invertibili)

42 stagisti $\quad 2^{42}$ modi di prendere un sottinsieme

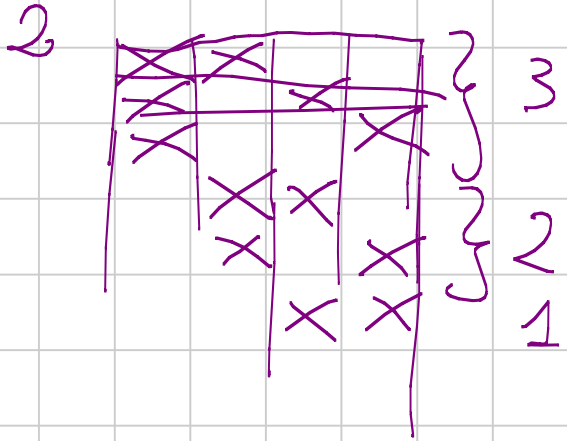
$A \quad \mathcal{P}(A) \quad |\mathcal{P}(A)| = 2^{|A|}$

$a_1 \text{ ---- } a_n \in A$

$0\ 1\ 0\ \ldots\ 1$

$2 \times 2 \ldots \times 2 = 2^n$

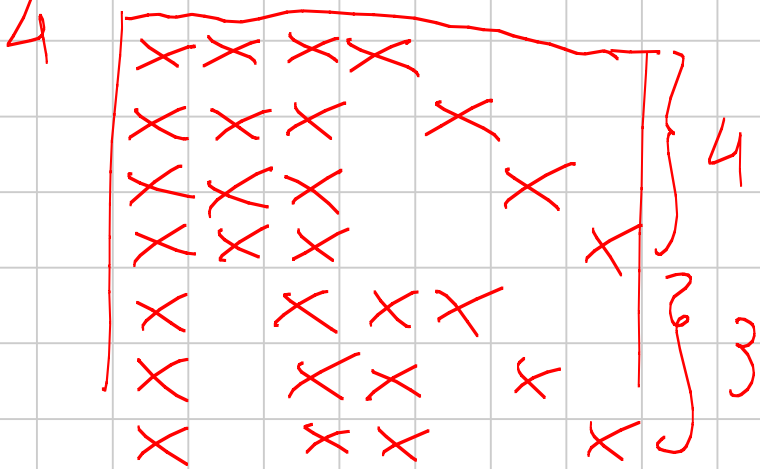
n elementi quanti sottinsiemi di K elementi?

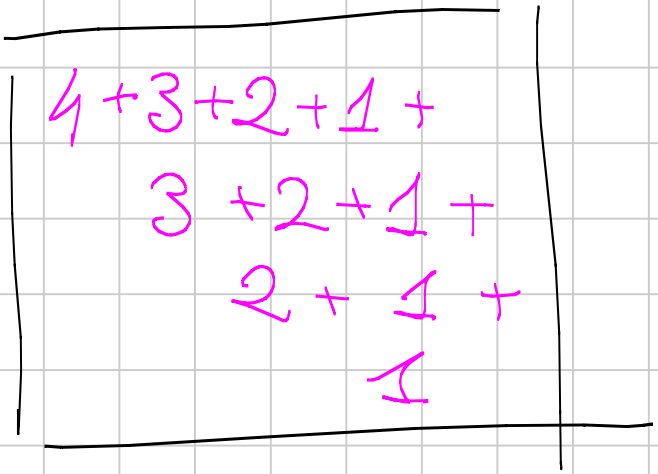


$3+2+1$

$1+2+\dots+n-1 = \frac{n(n-1)}{2}$

0	1
n	1
1	n
n-1	n
2	$\binom{n}{2}$
n-2	$\binom{n}{2}$





$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

BINOMIALE

$$\binom{80}{6} = \frac{80!}{6!\,74!} =$$

$$= \frac{75 \cdot \ldots \cdot 80}{6 \cdot 5 \cdot \ldots \cdot 2}$$

Binomio di Newton

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k}$$

$$= (x+y) \cdot \ldots \cdot (x+y)$$

$$\binom{n}{k} = \binom{n}{n-k}$$

$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$$

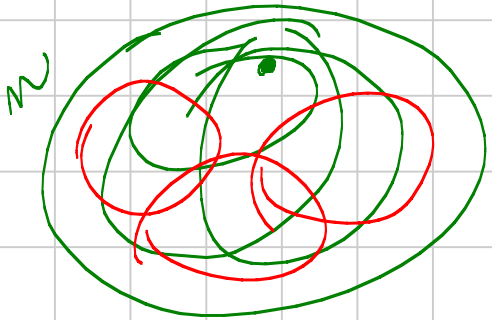
$$\frac{n!}{k!\,(n-k)!} \overset{?}{=} \frac{(n-1)!}{(k-1)!\,(n-k)!} + \frac{(n-1)!}{(n-1-k)!\,k!}$$

$$\frac{(n-1)!\,(n-k)}{(n-k)!\;k!} + \frac{(n-1)!\,k}{(n-k)!\;k!}$$

$$\overset{!!}{=} \frac{n!}{(n-k)!\;k!}$$

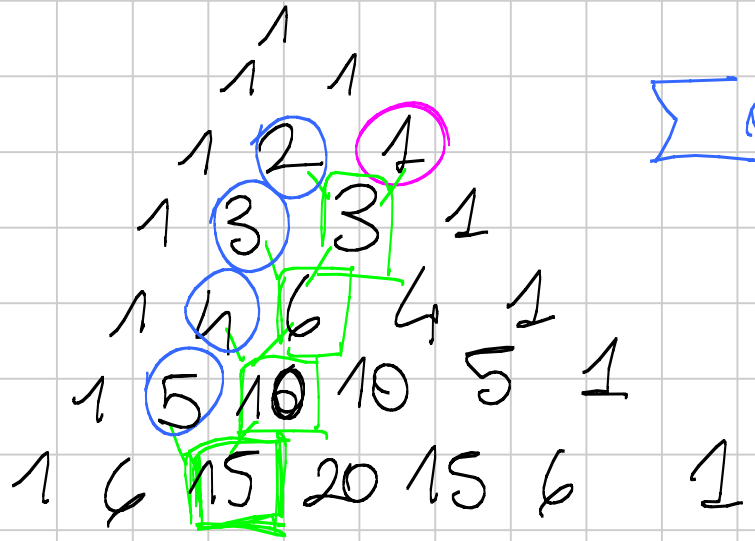
$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$$

↑
k-sottinsiemi di n insieme



TARTAGLIA

$\binom{n}{k}$



$$\sum \bigcirc = \square - \bigcirc$$

$$\sum_{n=a}^{b} \binom{n}{k} = \binom{b+1}{k+1} - \binom{a}{k+1}$$

$$a \leq b$$

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k}$$

$$2^n = \sum_{k=0}^{n} \binom{n}{k} \longrightarrow \sum_{k=0}^{[n/2]} \binom{n}{2k} =$$

$$(1-1)^n = 0 = \sum_{k=0}^{n} \binom{n}{k} (-1)^k$$

$$= \sum_{k=0}^{[n/2]} \binom{n}{2k+1} =$$

$$= 2^{n-1}$$

MULTINOMIALI

STAGISTI

8 lettere
2S, 2T, 2I

quanti anagrammi?

$$\frac{8!}{2!\,2!\,2!} = 7!$$

$$\binom{8}{2}\binom{8-2}{2}\binom{8-2-2}{2}\binom{8-2-2-2}{1} =$$

$$\binom{8}{2}\binom{6}{2}\binom{4}{2}\binom{2}{1}$$

$$(x+y+z)^n = (x+y+z)(\quad)\cdots(x+y+z) =$$

$$= \sum_{\substack{i,j \\ i+j\le n}} x^i y^j z^{n-i-j} \binom{n}{i}\binom{n-i}{j}$$

PARTIZIONI DI UN INTERO

n intero >0

quanti modi di scriverlo come somma di
k interi ≥ 1?

a meno dell'ordine? NO

$$\binom{n-1}{k-1}$$

e se dico ≥ 0?

spazi $n+k-1$
annerisco $k-1$ caselle
k "intervalli"

$$\binom{n+k-1}{k-1} =$$

"combinazioni" di $k-1$ elementi con ripetizioni fra $n+1$ elementi

DOUBLE - COUNTING

n righe $17n$ colonne

3 pedoni in ogni colonna

$\leq n$ pedoni in ogni riga $1 \dots n$

$n = ?$

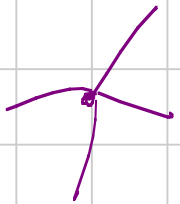
contiamo i pedoni!

$3 \cdot 17n = 51n = 1 + \dots + n = \frac{n(n+1)}{2}$

$102n = n^2 + n \Rightarrow$

$n(n-101) = 0 \Rightarrow n \neq 0 \quad n = 101$

un grafo ha un numero pari di nodi di grado dispari.



$$g_1 + \dots + g_n = 2E$$

(IMO '01) m dispari

m interi $n_1 \dots n_m$

$x = (x_1 \dots x_m)$ permutazione di $1 \dots m$

$f(x) = x_1 n_1 + \dots + x_m n_m$

$\exists\, a, b$ distinte t.c. $f(a) - f(b)$ è multiplo di $m!$

$\times$ assurdo tutte le $f(\text{perm})$ hanno resto diverso nella divisione per $m!$

$$\sum_x f(x)$$

$$\frac{(m!-1)\,m!}{2}$$

$$\sum_{i=1}^{m} n_i \cdot \sum_{j=1}^{m} j\,(m-1)! =$$

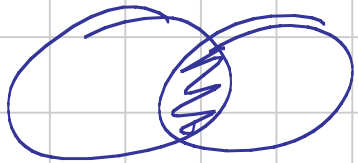
$$= \sum_{i=1}^{m} n_i \left| \frac{(m-1)!\; m(m+1)}{2} \right| =$$

$$= \frac{m!\,(m+1)}{2} \sum n_i$$

$$\boxed{m! \left| \sum_x f(x) \right|}$$

PIE

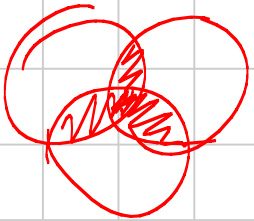
$$|A \cup B| \qquad |A| + |B| - |A \cap B|$$



$$|A \cup B \cup C| \qquad |A| + |B| + |C|$$

$$- |A \cap B| - |B \cap C| - |C \cap A| +$$

$$|A \cap B \cap C|$$



$$\left| \bigcup_{i=1}^{n} A_i \right| = \sum_{i=1}^{n} |A_i| - \sum_{i,j \,:\, 1 \le i < j \le n} |A_i \cap A_j| + \sum_{i,j,k \,:\, 1 \le i < j < k \le n} |A_i \cap A_j \cap A_k|$$

A''

$$- \dots \pm |A_1 \cap \dots \cap A_n|$$

$K(a)$ = quantità di insiemi che contengono a

$$\sum_{a \in A} \left[\binom{K(a)}{1} - \binom{K(a)}{2} + \binom{K(a)}{3} + \dots \pm \binom{K(a)}{n} \right]$$

$$= 1$$

$$\| $$

$$\sum_{a \in A} 1 = \left| \bigcup A_i \right| = |A|$$

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permutazioni di n elementi senza punti fissi?

$$A_i = \{ \sigma \in S_n \text{ che tengono fisso } i \}$$

$$|S_n| - \left| \bigcup_i A_i \right| = n! - n(n-1)! + \binom{n}{2}(n-2)! - \dots + |A_1| + \dots + |A_n|$$

$$\binom{n}{3}(n-3)! \; - \; - \; - \; \pm 1 = \boxed{n! \sum_{k=0}^{n} (-1)^k \frac{1}{k!}}$$

$f : A \to B$ suriettive come si contano ??

$|A| = a \quad |B| = b$

$$\sum_{k} \binom{b}{k} (b-k)^a (-1)^k$$

$$\binom{2n}{n} = \sum_{i=0}^{n} \binom{n}{i}\binom{n}{n-i}$$

$\max a \text{ t.c. } p^a \mid n!$

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