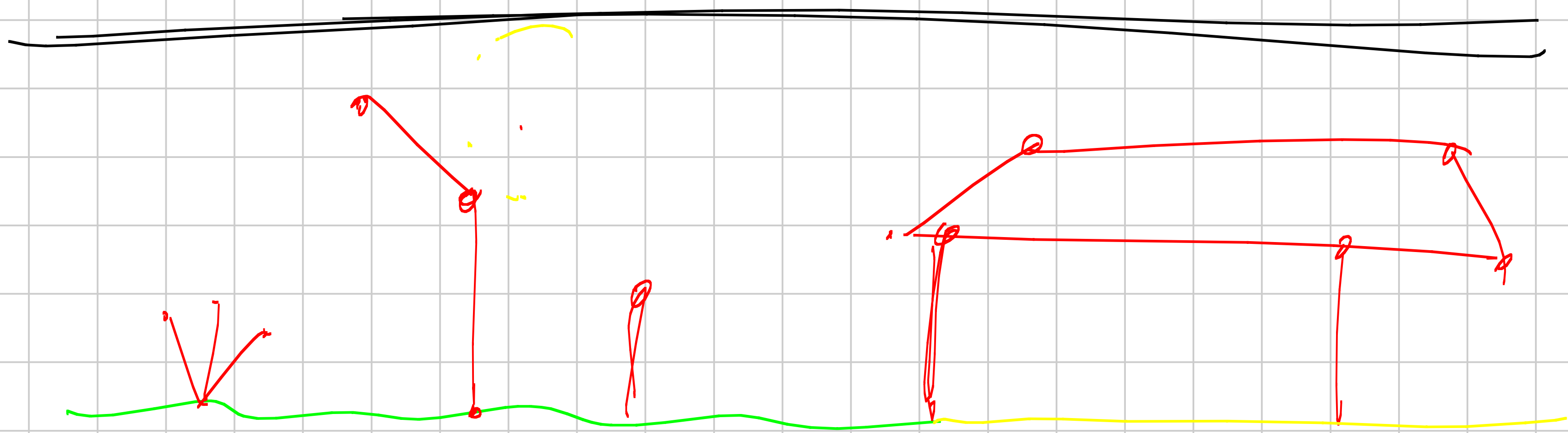
Titolo nota 07/09/2007

Def Gioco

un insieme (di giochi) (ereditariam. finito)



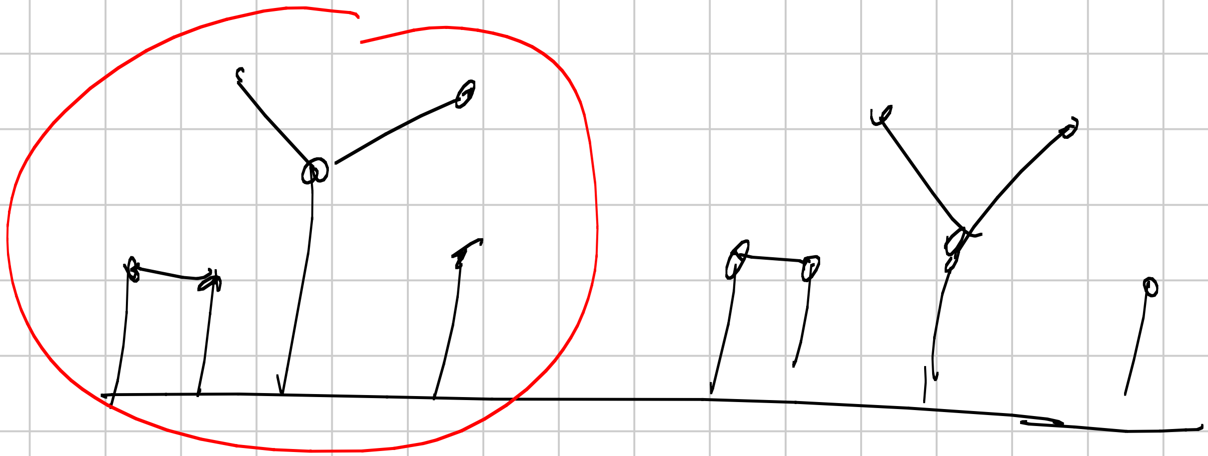
G è vinto per I sse

$\exists x \in G$: x è vinto per II

— — — — II sse

$\forall x \in G$: x — — — I

Def: $\oplus$



$$A \oplus B \equiv \{A \oplus x\}_{x \in B} \cup \{x \oplus B\}_{x \in A}$$

Def =

$A \approx B$ quando $A \oplus B$ è vinto per II

$$A = A$$

I $\quad A = B \Longrightarrow B = A \quad \Longleftarrow$

$$A = B \wedge B = C \Longrightarrow A = C$$

$$7 \equiv 0'$$

<u>Def.</u> $O = \emptyset$

$$A \oplus O \equiv \{x \oplus O\}_{x \in A} \cup \{A \oplus x\}_{x \in O} \equiv$$

$$\equiv \{x\}_{x \in A} \equiv A$$

$A = A \oplus O$ è unico per $\nexists \iff A = O$

- $A \oplus B \equiv B \oplus A$

  $A \oplus (B \oplus C) \equiv (A \oplus B) \oplus C$

- $A \oplus B \equiv \{X \oplus B\}_{X \in A} \vee \{A \oplus X\}_{X \in B} \equiv$

  $\equiv \{B \oplus X\}_{X \in A} \vee \{X \oplus A\}_{X \in B} \equiv$

  $\neq \qquad \equiv B \oplus A$

[illegible]

[illegible]

$$A = A' \qquad B = B'$$

$$\Downarrow$$

$$A \oplus B = A' \oplus B'$$

$$0 = (A \oplus B) \oplus (A' \oplus B') =$$

$$= (A \oplus A') \oplus (B \oplus B') \overset{?}{=} 0$$

$A = 0 \qquad B = 0 \qquad 0 \oplus 0 \equiv 0$

$$\Downarrow$$

$A \oplus 0 = 0$

$0 = \emptyset$

$1 = \{\emptyset\}$

$2 = \{\emptyset, 1\}$

$3 = \{\emptyset, 1, 2\}$

$2 \oplus 1 = \{2 \oplus 0, 1 \oplus 1, 0 \oplus 1\} \equiv$

$\equiv \{2, =0, 1\} = 3$

$1 \oplus 1 = \{0 \oplus 1\} = \{1\}$

$\{2, =0, 1\} \oplus \{2, 0, 1\} = 0$

$0 \equiv \emptyset$

$n \equiv \{0, \dots, n-1\}$

$n+1 \equiv n \cup \{n\}$

$\{\pi^{-1}(x)\}_{x \in L}$

Teo Per ogni insieme A esiste $g \in \mathbb{N}$ tale che $A = g$

$$g = \min(\mathbb{N} \setminus A)$$

$$a = g$$

$$a \oplus g = 0 \qquad y \notin a$$

$$\overset{\in}{g' < y} \qquad a \oplus g' \ni a' \in a$$

$$a' = g'$$

$$a' \oplus g$$

$$a \supset a' = g' > y$$

- $a \oplus a = 0$
- $\sum 2^{a_i} = \bigoplus 2^{a_i}$

$$5 = 2^2 + 1 = 2^2 \oplus 1$$

$$7 \oplus 5 \oplus 2 = 4 \oplus 2 \oplus 1 \oplus 4 \oplus 1 \oplus 2 = 0$$

$2^a \oplus x = 2^a + x$

$x < 2^a$

$y < x$

$2^a \oplus y = 2^a + y$

111
101
10

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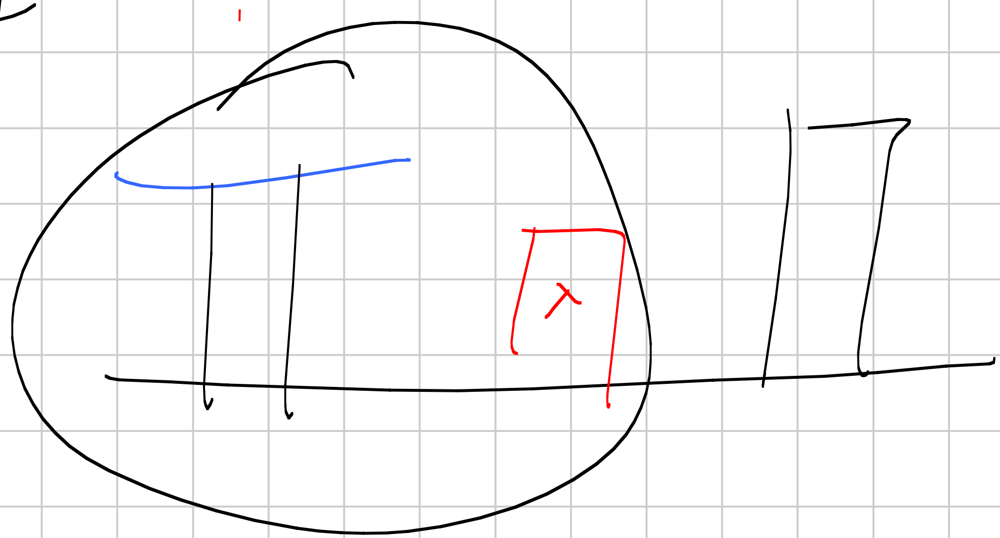


DEF. $(a \mid b) = 2^{n+1} - 1$

n max t.c. $a \equiv b \; (2^n)$

a	1 0 0 1 1 0
b	1 0 1 0
$(a \mid b)$	1 1 1

$$\sum_{x=0}^{\infty} \alpha^x = \frac{1}{1-\alpha} \qquad \sum_{x=0}^{\infty} 2^x = -1$$

$$x \oplus -1 = -x - 1$$

$$a \oplus -1 = -a - 1$$

$$-5 \oplus -2 = 4 \oplus -1 \oplus 1 \oplus -1 = 5$$

$$a \oplus (a \mid 0) = a - 1$$

$$a \oplus (a \mid -1) = a + 1$$

$$(a + k \mid b + k) = (a \mid b)$$

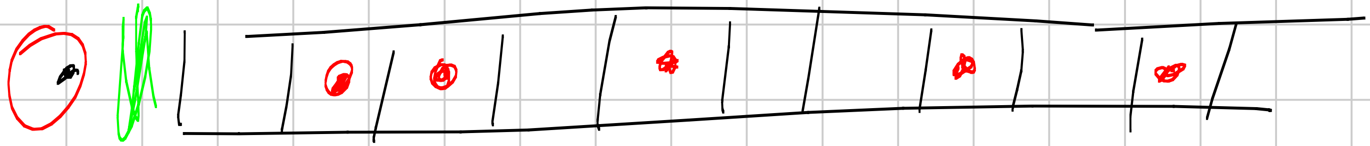
Def. Funzioni lineari

$$f(x) = (((x + a_1) \oplus b_1) + a_2) \oplus b_2) \ldots$$

$$x + a$$

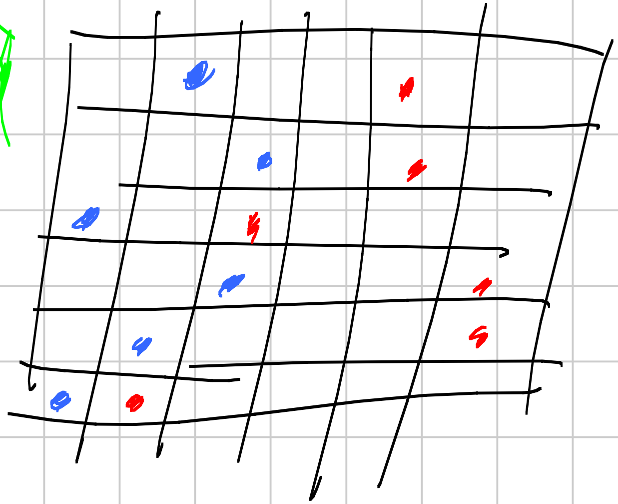
$$x \oplus b$$

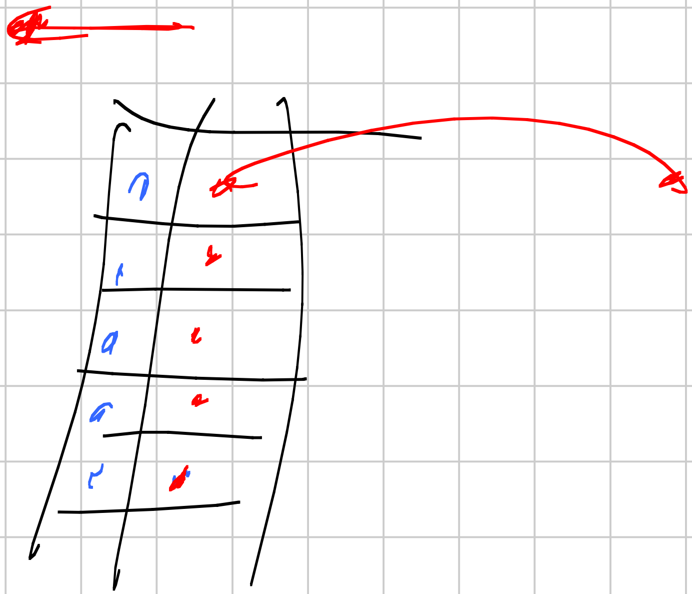
$$f(a \mid b) = (f(a) \mid f(b))$$



4 ORDINI SI PIU' SEZTERO

2 ORDINI





- $n \to \{k, n-k, n-2k\}$ $\quad 1 \leq k \leq \frac{n}{2}$

- $n \to n' < n$

  $\to a, b \quad a+b=n \quad a, b \geq 2$

- $n \to n-2$

  $\to a, b \quad a+b=n$

- $n \to n-2$

  $\quad -3$

  $\quad -5$

- $n \to -2$

  $\quad -3$

  $\quad -5$

  $\quad -17$

Controesempio

0	0	
1	1	0
2	2	1, 0
3	0	2, 3
4	2	0, 1
5	0	2, 3
6	2	
7	0	

1	1
2k	1
2k+1	0

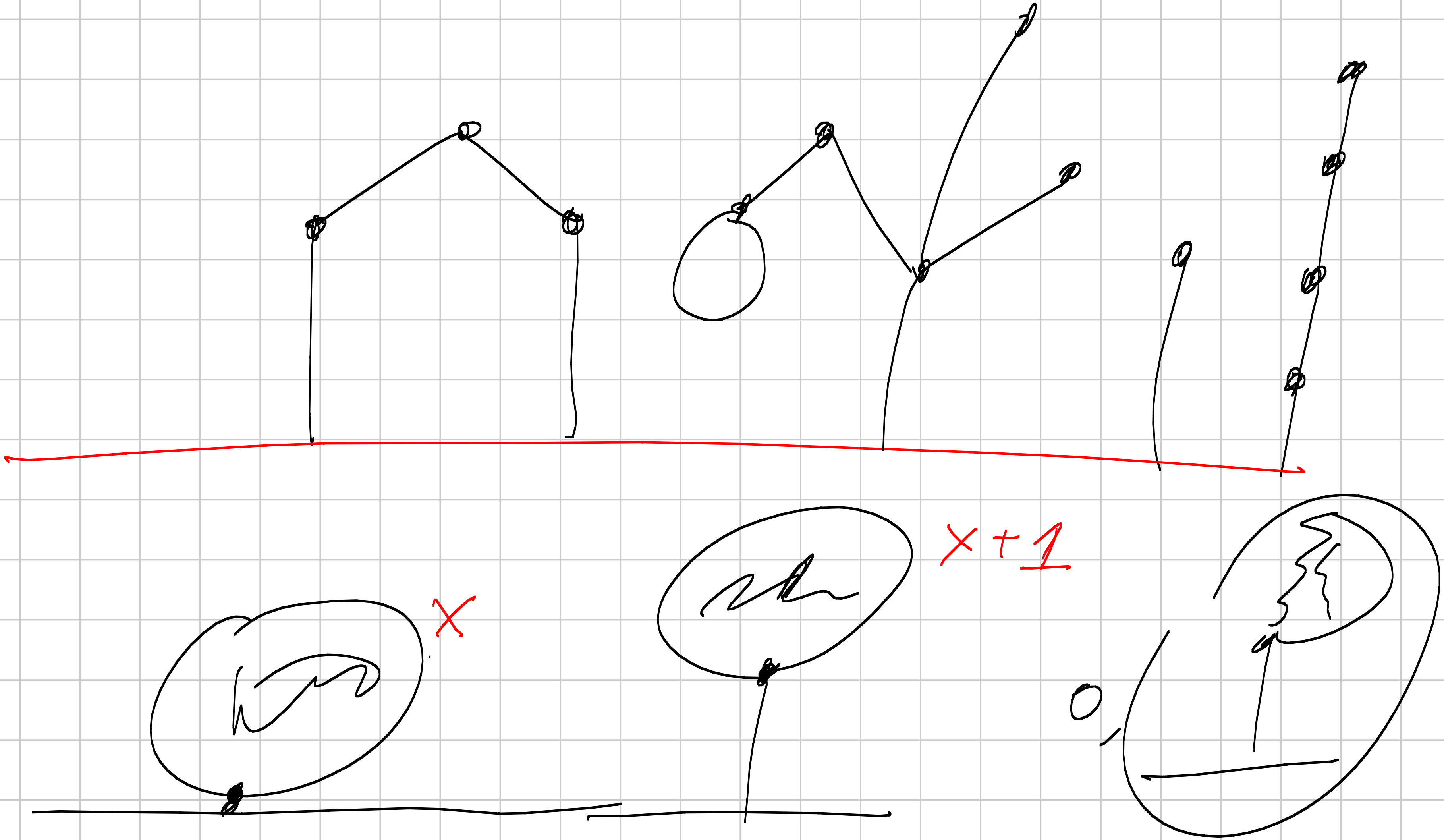
4k+1	4k+1
2	2
3	4
4	3

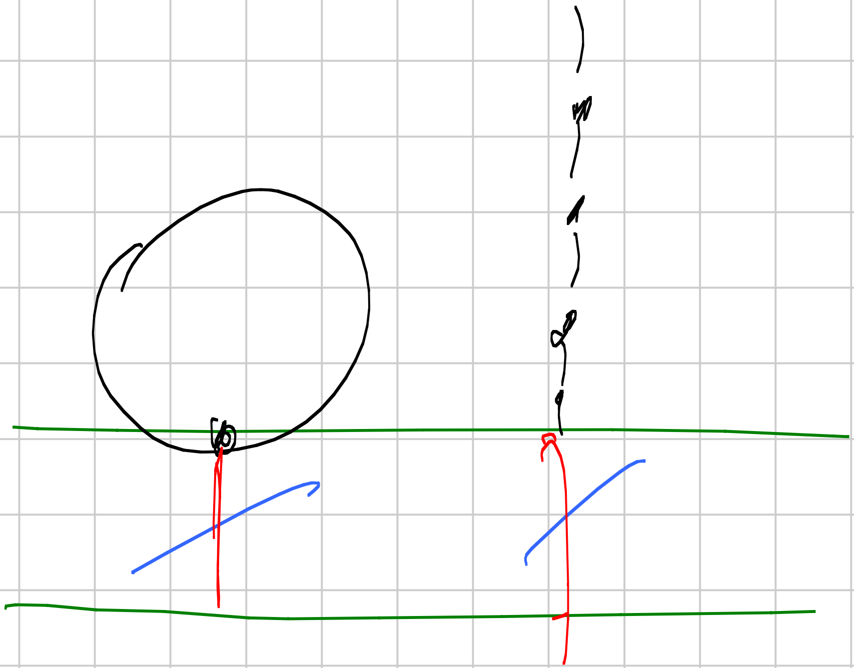
0 1 2 3 4 5 6 7

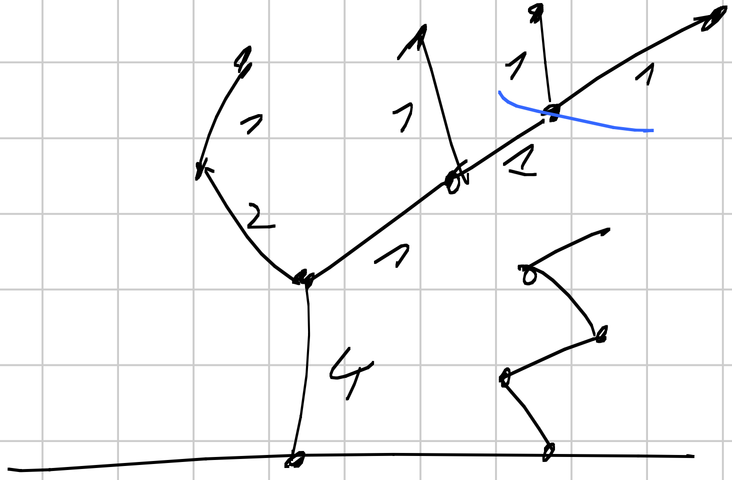
0 0 1 1 2 2 3 0

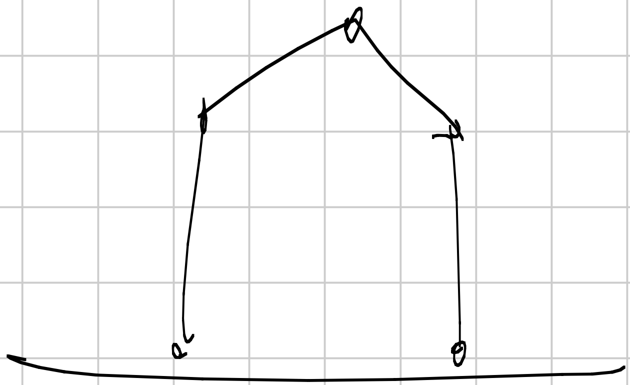
−2 −3 −5 −12

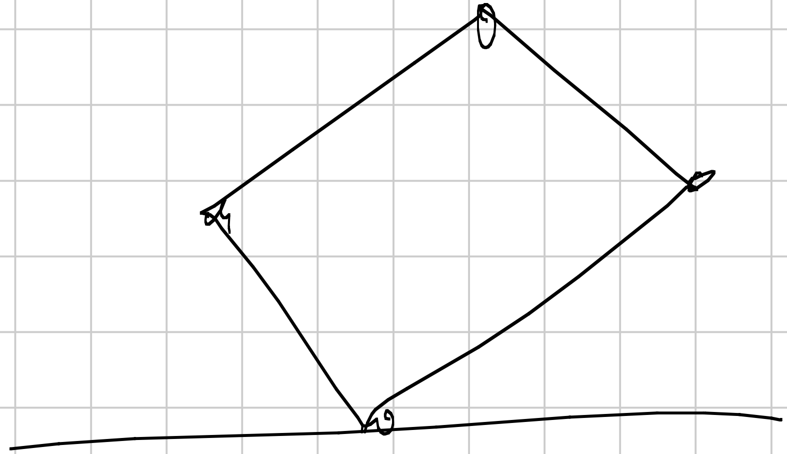
$i < j$

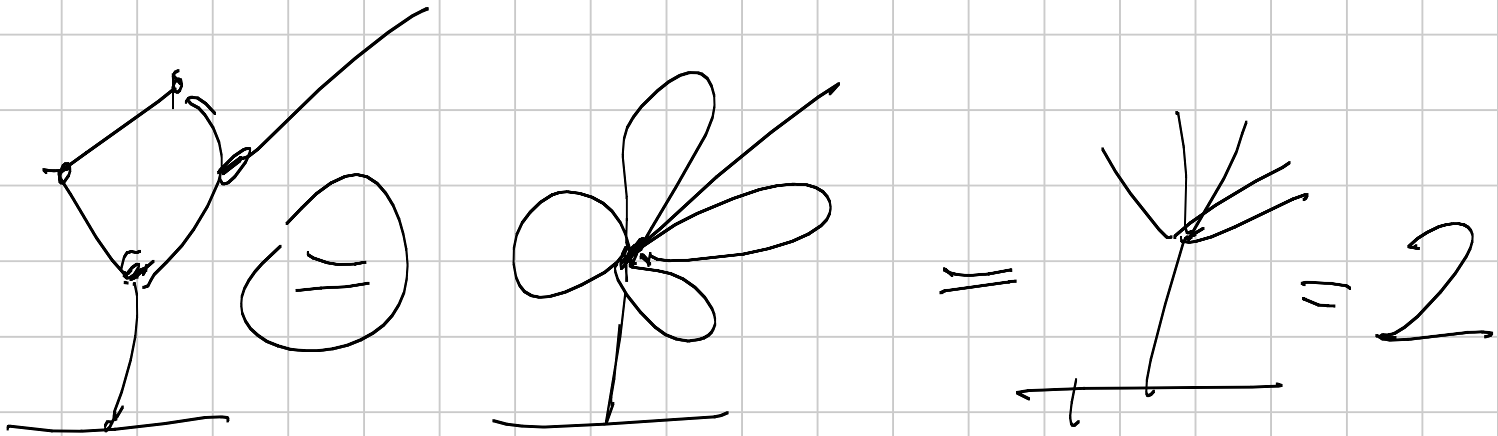

x
x+1
0,

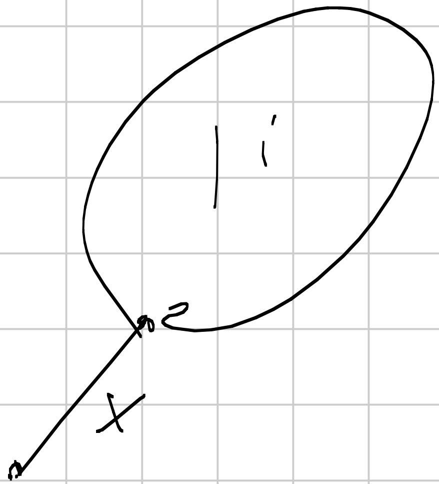










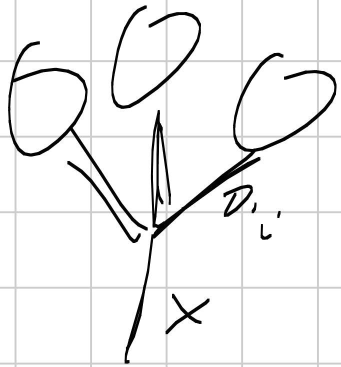


$w(x)$

$\sigma(x) = (w(x) \mid 0)$

Crtnz

$$w(x) = \bigoplus_{i > x} \sigma(i)$$



$$w(x) = 1 + \bigoplus w(a_i) = 1 + \bigoplus_{i > x} \sigma(i)$$

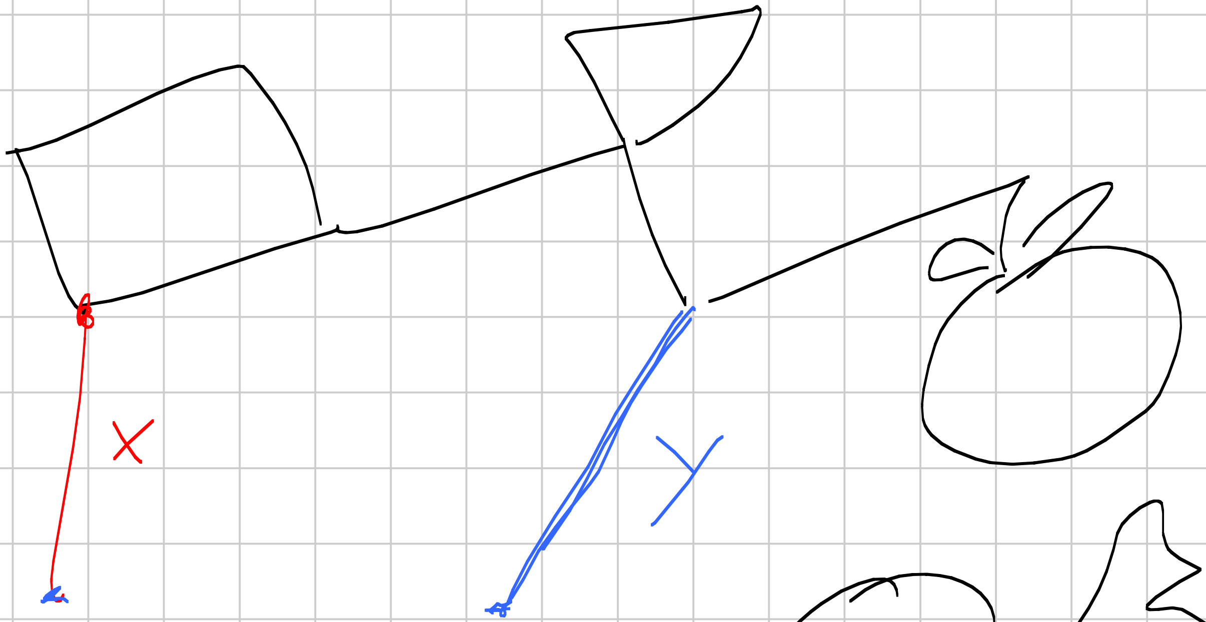
$$\sigma(x) = \left(1 + \bigoplus_{i > x} \sigma(i) \mid 0\right)$$

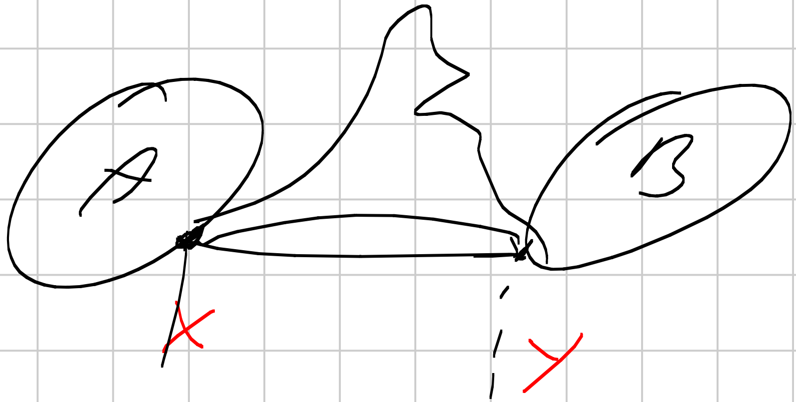
$$\boxed{\bigoplus_{i>x} \sigma(i)} \oplus \left(1 + \bigoplus_{i>x} \sigma(i) \mid 0\right) =$$

$$\alpha \oplus (\alpha \mid -1) = \alpha + 1$$

$$(\alpha \mid -1) = (\alpha + 1 \mid 0).$$

$$\bigoplus_{i>x} \sigma(i) + 1 = w(x)$$





$$w(x) = \big((B+2) \ominus A\big) + 2$$

$$\big(B \ominus (A+2)\big) + 2 = w(y)$$

$$\sigma(x) = \left(\left((B + \underline{1}) \oplus A \right) + \underline{2} \;\middle|\; 0 \right) =$$

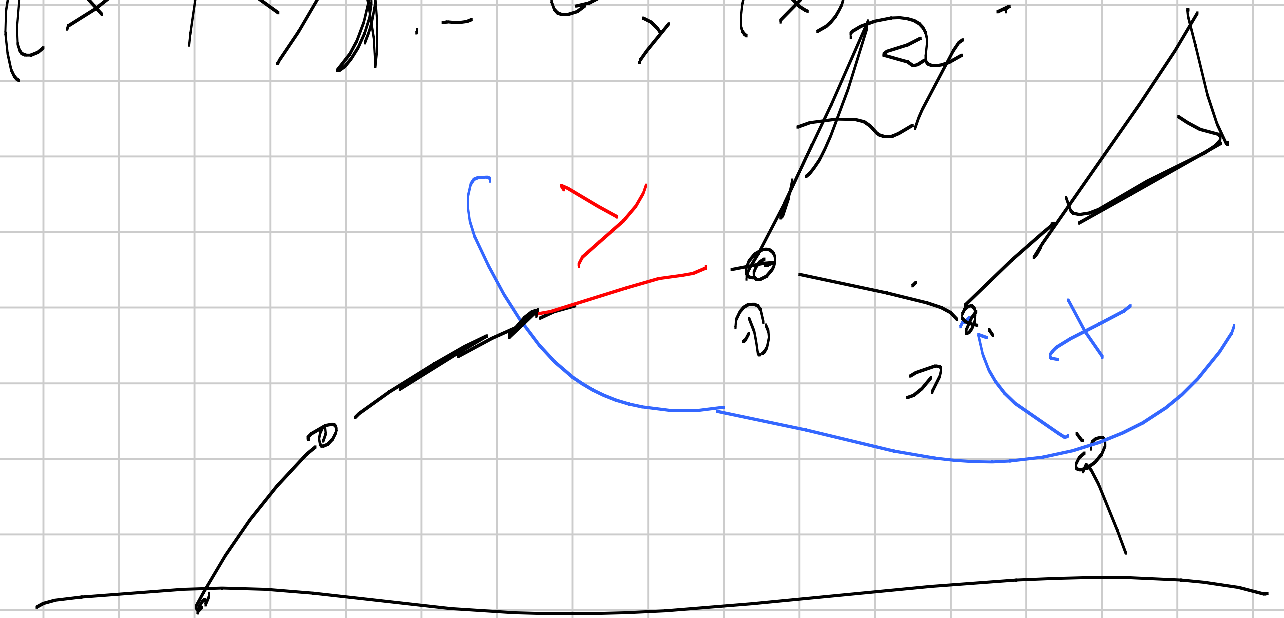
$$= \left((B + 1) \oplus A \;\middle|\; -\underline{1} \right) =$$

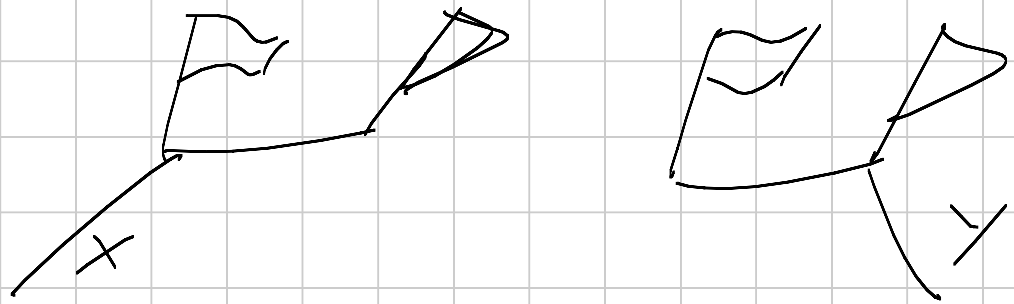
$$= \left(B + \underline{1} \;\middle|\; A \oplus -\underline{1} \right) =$$

$$= \left(B + \underline{1} \;\middle|\; -A - \underline{1} \right) = \left(-B - 1 \;\middle|\; A + 1 \right)$$

$$-(A + 1) \,)$$

def $(x \mid y) := \sigma_y(x)$



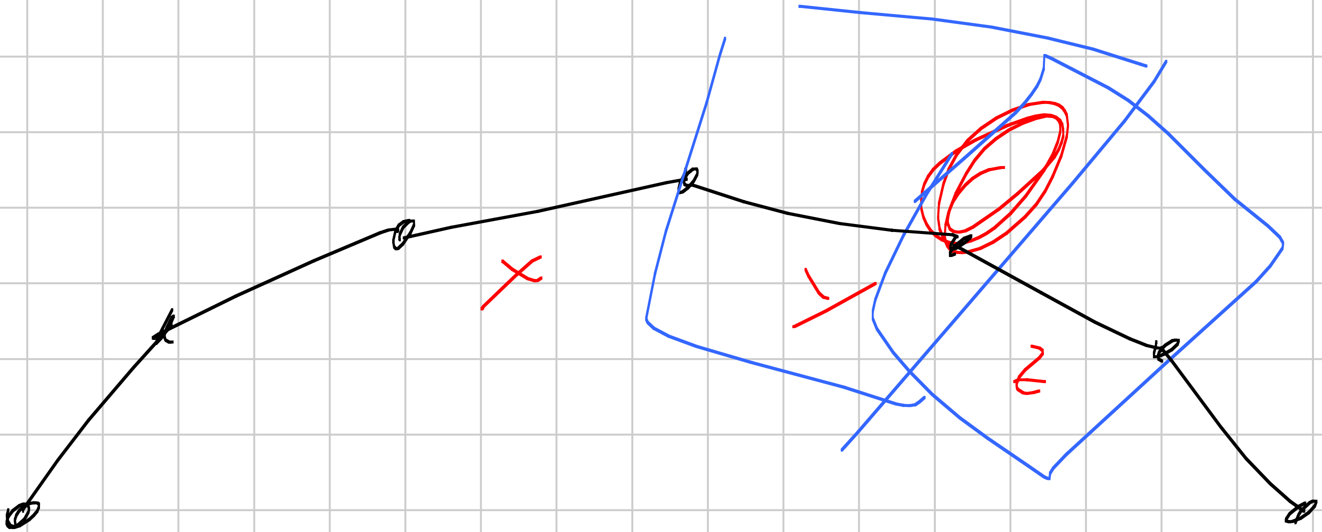


$(x \mid y) = (y \mid x)$

$(\sigma_x(y) \mid 0)$

$x \quad y \quad z$

$(x/y) \qquad (x/z) \qquad (y/z)$



(x/h)

$\sigma_y(z)$

$(x \mid i)$