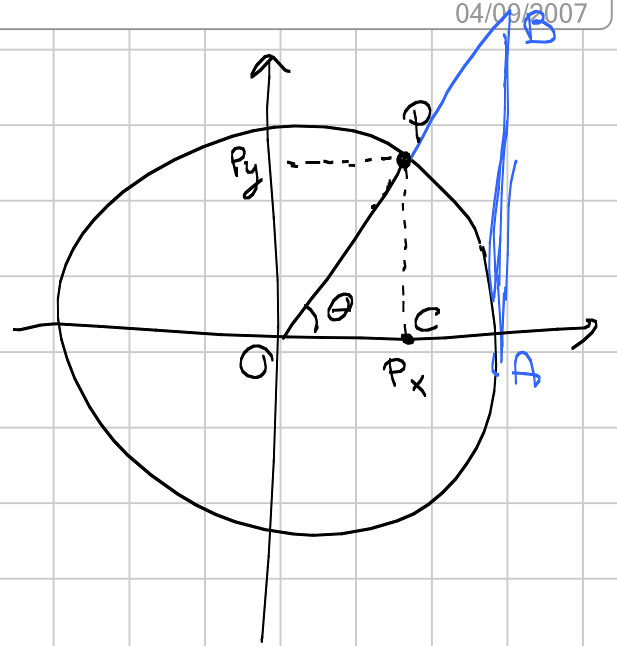
$$x^2+y^2=1$$



$$P(P_x, P_y)$$

$$360° : 2\pi = x° : x$$

$P_x = \cos\theta$ $\qquad$ $P_y = \sin\theta$

$$\sin^2\theta + \cos^2\theta = 1$$

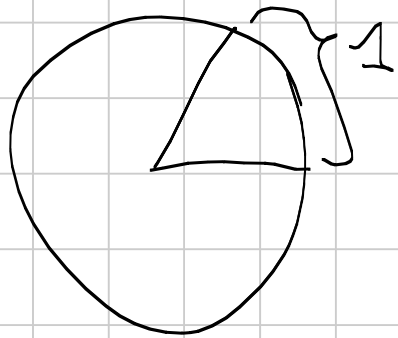
$$tg\,\theta = \frac{\sin\theta}{\cos\theta}$$

Th: $AB = tg\,\theta$

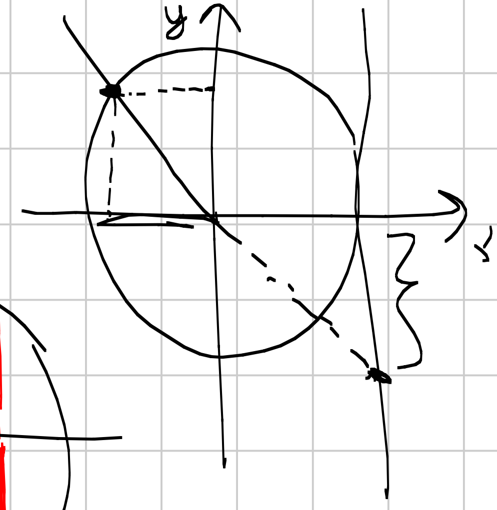
$\overset{\triangle}{OPC} \sim OAB$

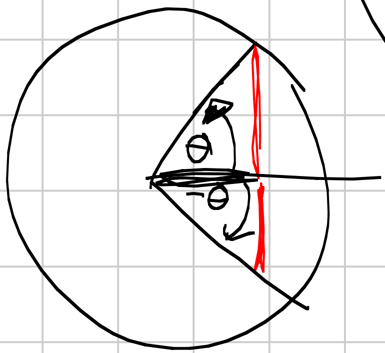
$$\frac{PC}{OC} = \frac{AB}{OA} = AB$$

$$AB = \frac{\sin\theta}{\cos\theta}$$

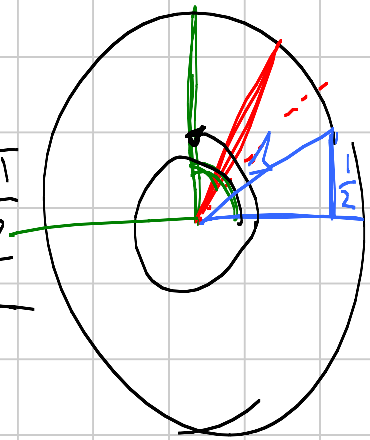


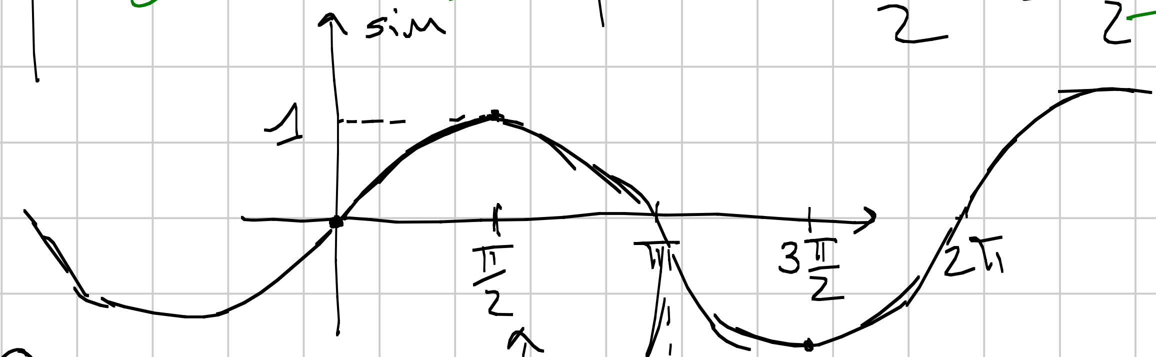
	rad	sin	cos	tg
0	0	0	1	0
30	$\pi/6$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
45	$\pi/4$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1
60	$\pi/3$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
90	$\pi/2$	1	0	
180	π	0	-1	0





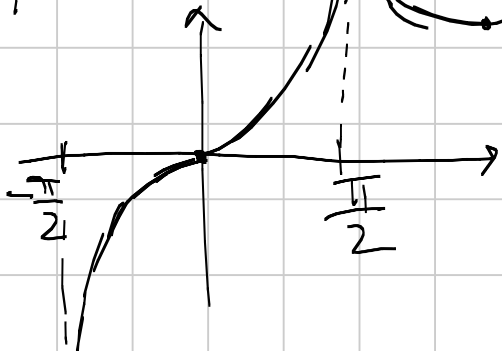
$$\frac{5}{2}\pi = 2\pi + \frac{\pi}{2}$$

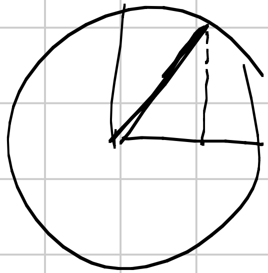




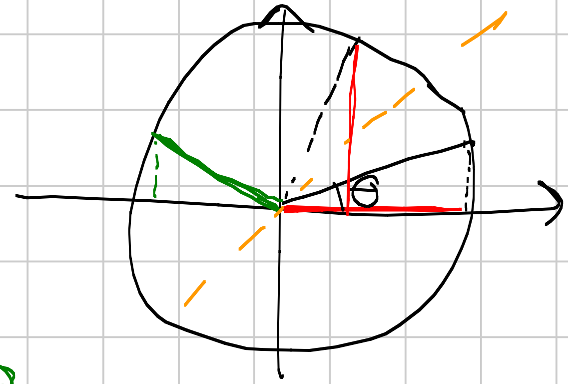
$$tg -\theta = -tg\,\theta$$

$$\frac{\sin -\theta}{\cos -\theta} = -\frac{\sin\theta}{\cos\theta} = -tg\,\theta$$





$\theta \qquad \frac{\pi}{2} - \theta \qquad \pi - \theta$



$$\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta \qquad \cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$$

$$\cos(\pi - \theta) = -\cos\theta \qquad \sin(\pi - \theta) = \sin\theta$$

Formule di addizione

$$\cos(x+y) = \cos x \cos y - \sin x \sin y \quad (1)$$

$$\cos(x-y) = \cos x \cos y - \sin x \sin(-y) \quad (2)$$

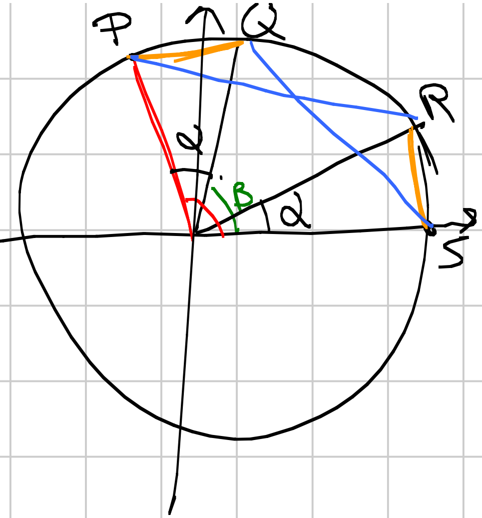
$$= \cos x \cos y + \sin x \sin y$$

$$\sin(x+y) = \sin x \cos y + \cos x \sin y \quad (3)$$

$$\sin(x-y) = \sin x \cos y - \cos x \sin y \quad (4)$$

$S(1,0)$ $\quad R(\cos\alpha, \sin\alpha)$

$Q(\cos\beta, \sin\beta)$ $\quad P(\cos\alpha+\beta, \sin\alpha+\beta)$



$PQ^2 = RS^2$

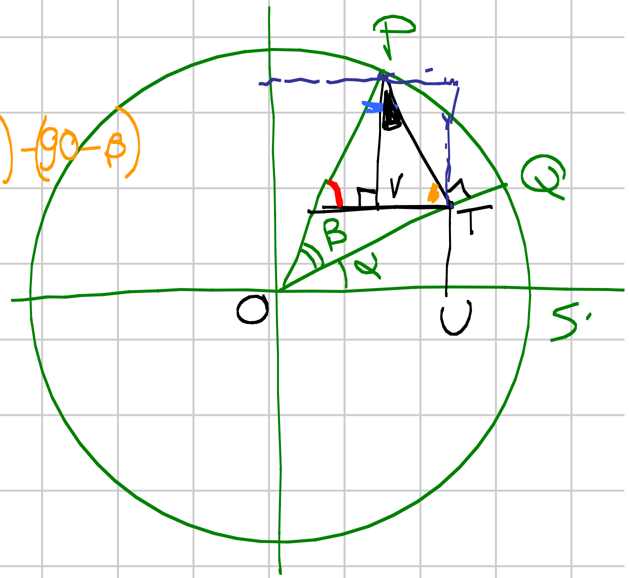
$PR^2 = QS^2$

$\bullet = \alpha+\beta$

$\bullet = 90-\beta$

$\bullet = 180-(\alpha+\beta)-(90-\beta)$
$= 90-\alpha$

$\bullet = \alpha$



$*\sin\alpha+\beta = PV + TU$

$\cos\alpha+\beta = OU - VT$

$PT = OP \cdot \sin\beta = \sin\beta$

$PV = PT \cdot \cos\alpha = \cos\alpha \sin\beta$

$OT = OP \cdot \cos\beta = \cos\beta$

$TU = OT \cdot \sin\alpha = \cos\beta \sin\alpha$

$\sin\alpha+\beta = PV + TU = \cos\alpha \sin\beta + \cos\beta \sin\alpha$

$$\cos(x+y) = \cos x \cos y - \sin x \sin y \qquad (1)$$

$$\cos(x-y) = \cos x \cos y - \sin x \sin(-y) \qquad (2)$$

$$= \cos x \cos y + \sin x \sin y$$

$$\sin(x+y) = \sin x \cos y + \cos x \sin y \qquad (3)$$

$$\sin(x-y) = \sin x \cos y - \cos x \sin y \qquad (4)$$

Formule product $\to$ sum

$$\cos x \cos y = \frac{1 + 2}{2} = \frac{\cos(x+y) + \cos(x-y)}{2}$$

$$\sin x \sin y = \frac{\cos(x-y) - \cos(x+y)}{2} = \frac{2 - 1}{2}$$

$$\sin x \cos y = \frac{3 + 4}{2} = \frac{\sin x+y + \sin x-y}{2}$$

Formule sum → product

$$\cos\alpha + \cos\beta = 2\cos x \cos y \qquad \begin{cases} \alpha = x+y \\ \beta = x-y \end{cases} \Rightarrow \quad x = \frac{\alpha+\beta}{2} \quad y = \frac{\alpha-\beta}{2}$$

$$= 2\cos\frac{\alpha+\beta}{2} \cdot \cos\frac{\alpha-\beta}{2}$$

$$\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2} \cdot \cos\frac{\alpha-\beta}{2}$$

Formule di duplicazione

$$\sin 2x = 2\sin x \cos x$$

$$\underline{\cos 2x} = \cos^2 x - \sin^2 x = (1-\sin^2 x) - \sin^2 x = \underline{1 - 2\sin^2 x}$$

$$= \underline{2\cos^2 x - 1}$$

$$\cos\frac{\alpha}{2} = \qquad \cos\alpha = 2\cos^2\frac{\alpha}{2} - 1 \Rightarrow \cos\frac{\alpha}{2} = \sqrt{\frac{1+\cos\alpha}{2}}$$

ESERCIZIO $\quad x \in [0, \frac{\pi}{2}]$

$$\frac{\sqrt{3}-1}{\sin x} + \frac{\sqrt{3}+1}{\cos x} = 4\sqrt{2}$$

$$\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)^2 + \left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right)^2 = 3+1-2\sqrt{3}+3+1+2\sqrt{3} = 8$$

$$= \left(\frac{2\sqrt{2}}{2\sqrt{2}}\right)^2$$

$$(\sqrt{3}-1)\cos x + (\sqrt{3}+1)\sin x = 2\sqrt{2} \cdot \underbrace{2\sin x \cos x}_{\sin 2x} =$$

$$= 2\sqrt{2} \quad \sin 2x$$

$$\underbrace{\frac{\sqrt{3}-1}{2\sqrt{2}}}_{a} \cdot \cos x + \underbrace{\frac{(\sqrt{3}+1)}{2\sqrt{2}}}_{b} \sin x = \sin 2x$$

$$a^2+b^2=1$$

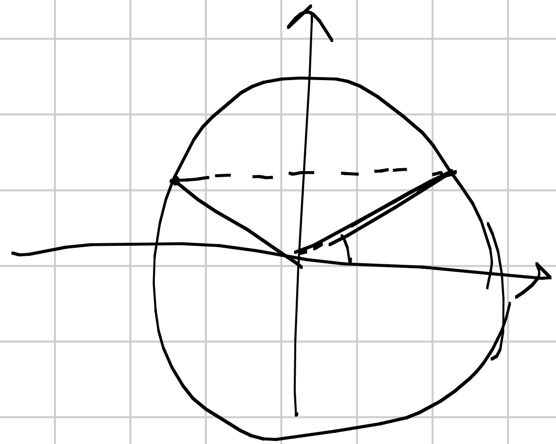
$$a = \sin\phi \qquad b = \cos\phi$$

$$\sin\phi \cos x + \cos\phi \sin x = \sin 2x$$

$$\sin x+\phi = \sin 2x$$

$$\begin{cases} x+\phi = 2x \\ x+\phi = \pi - 2x \end{cases} \Rightarrow x = \frac{\pi}{12} \quad \text{OK}$$

$$x = \frac{11}{36}\pi$$



$$\frac{\sqrt{3}-1}{2\sqrt{2}} = \sin\phi$$

$$\phi = \frac{\frac{\pi}{6}}{2} = \frac{\pi}{12}$$

$$\cos 2\alpha = 1 - 2\sin^2\alpha$$

$$\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2} = 1 - 2\sin^2\frac{\pi}{12}$$

$$\sin\frac{\pi}{12} = \sqrt{\frac{-\frac{\sqrt{3}}{2}+1}{2}} = \sqrt{\frac{-\sqrt{3}+2}{4}} = \sqrt{\frac{-2\sqrt{3}+4}{8}} = \sqrt{\frac{3-2\sqrt{3}+1}{8}} = \sqrt{\frac{(\sqrt{3}-1)^2}{8}} = \frac{\sqrt{3}-1}{2\sqrt{2}}$$

$$\cos 3x = \cos(2x+x) = \cos 2x \cos x - \sin 2x \sin x$$
$$= 4\cos^3 x - 3\cos x$$

ESERCIZIO

$\{a_i\}$ $\quad a_1, a_2, \dots a_m \quad -2 \leq a_i \leq 2 \quad \underline{\sum a_i = 0}$

$$\left|\sum a_i^3\right| \leq 2m$$

$a_i = 2b_i \quad -1 \leq b_i \leq 1 \quad \sum b_i = 0 \quad \underline{b_i = \cos c_i}$

$$\left|\sum (2b_i)^3\right| = \cancel{8}^{4}\left|\sum b_i^3\right| \leq \cancel{2}m$$

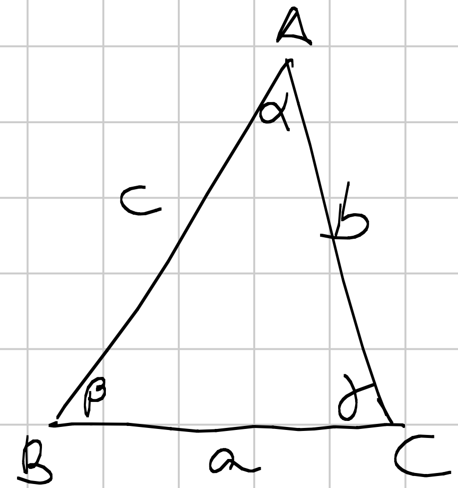
$$\left|4\sum b_i^3\right| = \left|4\sum b_i^3 - 3\sum b_i\right| = \left|\sum(4b_i^3 - 3b_i)\right| =$$

$$= \left|\sum(4\cos^3 c_i - 3\cos c_i)\right| = \left|\sum \cos 3c_i\right| \leq m$$

Formule parametriche: $t = tg\frac{\theta}{2}$

$\sin\theta = \frac{2t}{1+t^2}$ $\cos\theta = \frac{1-t^2}{1+t^2}$

TRIANGOLI

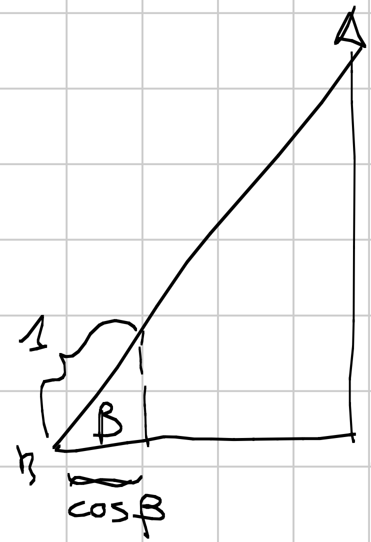


$BC = AB \cdot \cos\beta$

$AC = AB \sin\beta$

$\frac{1}{AB} = \frac{\cos\beta}{BC}$

$\frac{AC}{BC} = tg\beta$



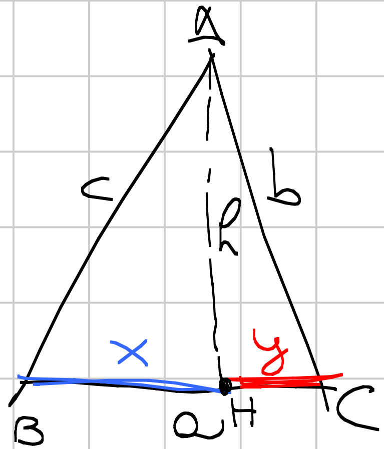
1) Erone

2) Formula per l'area

3) Carnot

4) Th. dei seni

1) $$[ABC] = \sqrt{p(p-a)(p-b)(p-c)} \qquad p = \frac{a+b+c}{2}$$



$$[ABC] = \frac{ah}{2}$$

$$x+y=a \qquad x^2+h^2=c^2 \qquad y^2+h^2=b^2$$

$$x^2-y^2 = c^2-b^2 = (x-y)(x+y) = a(x-y)$$

$$x-y = \frac{c^2-b^2}{a} \qquad y = \frac{a^2+b^2-c^2}{2a}$$

$$h^2 = b^2-y^2 \qquad h = \sqrt{\frac{(4a^2b^2) - (a^2+b^2-c^2)^2}{4a^2}}$$

$$[ABC] = \frac{ah}{2} = \sqrt{\frac{4a^2b^2-(a^2+b^2-c^2)^2}{16}} =$$

$$= \sqrt{\frac{(2ab+a^2+b^2-c^2)(2ab-a^2-b^2+c^2)}{16}} =$$

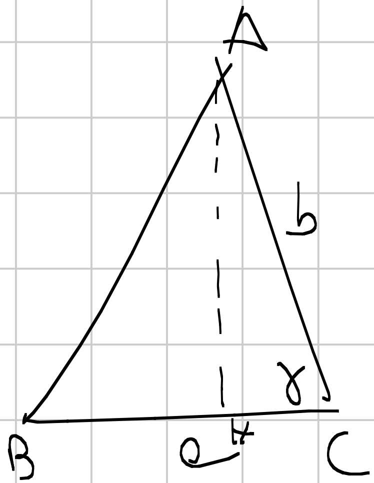
$x^2 - y^2 =$

$= (x-y)(x+y)$

$$= \sqrt{\frac{[(a+b)^2-c^2][c^2-(a-b)^2]}{16}} =$$

$$= \sqrt{\frac{(a+b-c)(a+b+c)(c-a+b)(c+a-b)}{16}} =$$

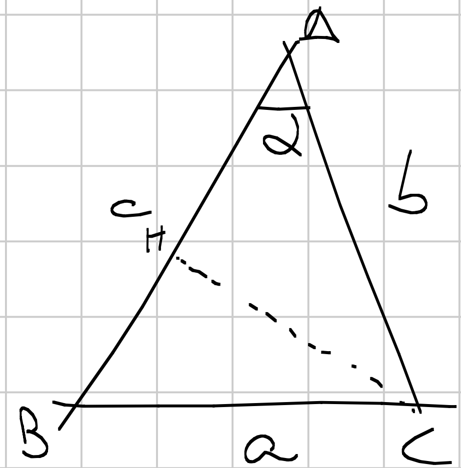
$$= \sqrt{(p-c)p(p-a)(p-b)}$$



2) $[ABC] = \frac{1}{2} ab \sin\gamma$

$$[ABC] = \frac{a \cdot AH}{2} = \frac{a \cdot b \sin\gamma}{2}$$

3) Carnot b, c, α



$$a^2 = b^2 + c^2 - 2bc\cos\alpha$$

$AH = b\cos\alpha$ $CH = b\sin\alpha$

$BH = c - b\cos\alpha$

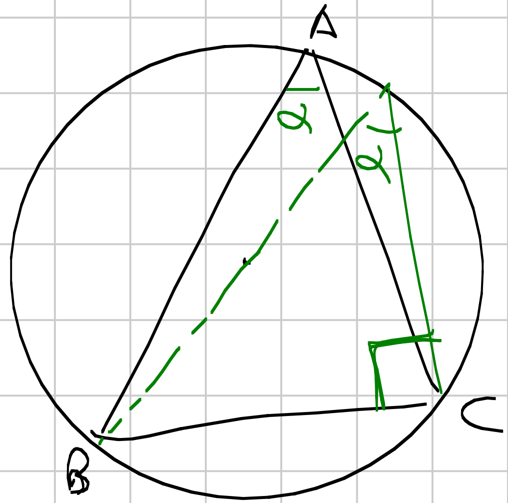
Pitagora $\overset{\triangle}{BHC}$

$$a^2 = BH^2 + CH^2 = (c - b\cos\alpha)^2 + (b\sin\alpha)^2 =$$

$$= c^2 - 2bc\cos\alpha + \underline{b^2\cos^2\alpha} + \underline{b^2\sin^2\alpha}$$

$$= b^2 + c^2 - 2bc\cos\alpha$$

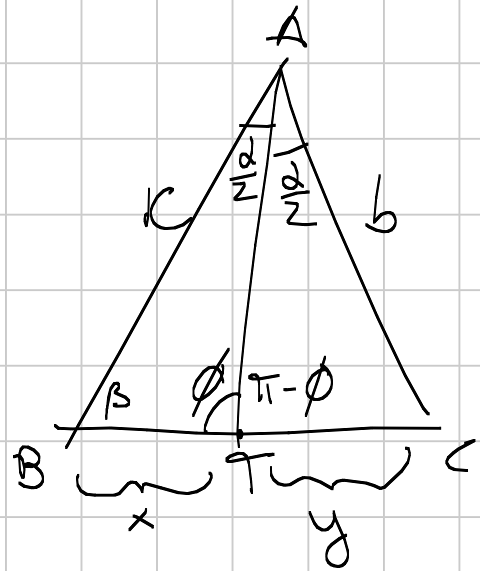
4) $\dfrac{a}{\sin\alpha} = \dfrac{b}{\sin\beta} = \dfrac{c}{\sin\gamma} = 2R$



$BC = 2R \cdot \sin\alpha = a$

ES: Th. della bisettrice

$$\frac{x}{y} = \frac{c}{b}$$



$$\frac{x}{\sin\frac{\alpha}{2}} = \frac{c}{\sin\phi} \qquad \frac{y}{\sin\frac{\alpha}{2}} = \frac{b}{\sin(\pi-\phi)} = \frac{b}{\sin\phi}$$

$$\frac{\dfrac{x}{\cancel{\sin\frac{\alpha}{2}}}}{\dfrac{y}{\cancel{\sin\frac{\alpha}{2}}}} = \frac{\dfrac{c}{\cancel{\sin\phi}}}{\dfrac{b}{\cancel{\sin\phi}}} \Longrightarrow \frac{x}{y} = \frac{c}{b}$$

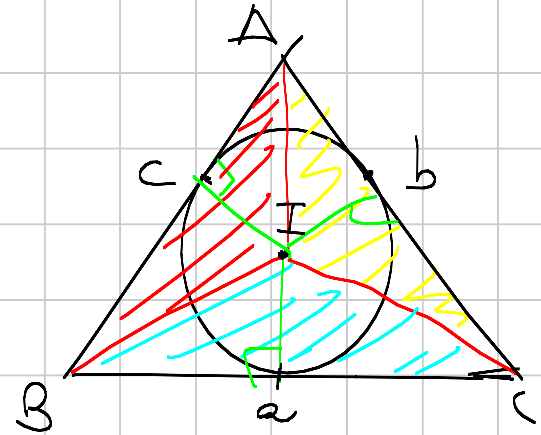
ESERCIZI: $[ABC] = \dfrac{abc}{4R}$ $\qquad [ABC] = rp$

$$[ABC] = \frac{1}{2}ab\sin\gamma = \frac{1}{2}\,\frac{abc}{2R} = \frac{abc}{4R}$$

$$\frac{c}{\sin\gamma} = 2R \qquad \sin\gamma = \frac{c}{2R}$$

$$[ABC] = \ \ + \ \ + \ \ = \frac{cr}{2} + \frac{br}{2} + \frac{ar}{2} =$$

$$= \frac{a+b+c}{2}\, r = p r$$



ES ABC

$$\mathrm{tg}^{x} A + \mathrm{tg}^{y} B + \mathrm{tg}^{z} C = \mathrm{tg}A\, \mathrm{tg}B\, \mathrm{tg}C \geq 3\sqrt{3}$$

$$\sin A \cos B \cos C + \sin B \cos C \cos A + \sin C \cos A \cos B \overset{?}{=}$$
$$= \sin A \sin B \sin C$$

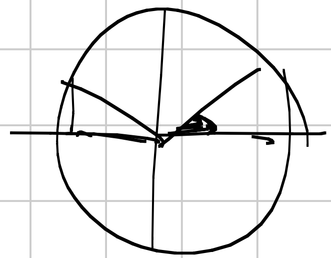
$$\cos C\,(\sin A \cos B + \sin B \cos A) \overset{?}{=} \sin C\,(\sin A \sin B - \cos A \cos B)$$

$$\cos C\,(\sin (A+B)) \overset{?}{=} \sin C\,(-\cos (A+B))$$

$$\hat{A} + \hat{B} + \hat{C} = \pi$$

$$\cos C \cdot (\sin (\pi - C)) \overset{?}{=} \sin C\,(-\cos (\pi - C))$$

$$\cos C \,\sin C = \sin C \cos C$$



$$x+y+z=xyz$$

$$\frac{x+y+z}{3} \geq \sqrt[3]{xyz}$$

$$\frac{xyz}{3} \geq \sqrt[3]{xyz} \Rightarrow \frac{(xyz)^{\cancel{3}2}}{3^3} \geq \cancel{xyz} \Rightarrow xyz \geq 3^{\frac{3}{2}}$$

$$= 3\sqrt{3}$$

ES: formule di Briggs

$$\sin\frac{A}{2} = \sqrt{\frac{(p-b)(p-c)}{bc}}$$

$$a^2 = b^2 + c^2 - 2bc\cos\alpha$$

$$\cos\alpha = \frac{b^2+c^2-a^2}{2bc}$$

$$\sin^2\frac{A}{2} = \frac{1-\cos A}{2} =$$

$$= \frac{2bc - b^2 - c^2 + a^2}{4bc} = \frac{a^2-(b-c)^2}{4bc} = \frac{(a+b-c)(a-b+c)}{4bc}$$

$$= \frac{(p-c)(p-b)}{bc}$$

GS: $\cos A + \cos B + \cos C = 1 + \frac{r}{R}$

Lemma:

$$\frac{r}{R} = 4 \sin\frac{A}{2} \sin\frac{B}{2} \sin\frac{C}{2}$$

Dim:

$$\frac{r}{R} = \frac{\frac{[ABC]}{p}}{\frac{abc}{4[ABC]}} = \frac{4[ABC]^2}{p\,abc} = \frac{4\cancel{p}(p-a)(p-b)(p-c)}{\cancel{p}\,abc}$$

$$[ABC] = \frac{abc}{4R} \qquad [ABC] = rp$$

$$= 4 \frac{\sqrt{p-a}\sqrt{p-b}}{\sqrt{ab}} \cdot \frac{\sqrt{p-a}\sqrt{p-c}}{\sqrt{ac}} \cdot \frac{\sqrt{p-b}\sqrt{p-c}}{\sqrt{bc}} =$$

$$= 4 \sin\frac{A}{2} \sin\frac{B}{2} \sin\frac{C}{2}$$

$$\cos A + \cos B + \cos C - 1 = 4 \sin\frac{A}{2} \sin\frac{B}{2} \sin\frac{C}{2}$$

$$\cos A + \cos B = 2 \cos \overbrace{\frac{A+B}{2}}^{\frac{\pi}{2}-\frac{C}{2}} \cos\frac{A-B}{2} = \qquad A+B+C=\pi$$

$$\cos C - 1 = -2 \sin^2\frac{C}{2} \qquad \cos 2x = 1 - 2\sin^2 x$$

$$\cos A + \cos B + \cos C - 1 = 2 \sin\frac{C}{2} \cos\frac{A-B}{2} - 2\sin^2\frac{C}{2}$$

$$2 \sin\frac{C}{2}\left(\cos\frac{A-B}{2} - \sin\frac{C}{2}\right) \overset{?}{=} \text{Hope}$$

$$= 4 \sin\frac{A}{2} \sin\frac{B}{2} \sin\frac{C}{2}$$

$$\cos\frac{A-B}{2} - \sin\frac{C}{2} \overset{?}{=} 2 \sin\frac{A}{2} \sin\frac{B}{2} \qquad \sin\frac{C}{2} = \sin\left(\frac{\pi}{2} - \frac{A+B}{2}\right) = \cos\frac{A+B}{2}$$

$$\cos \underbrace{\frac{A-B}{2}}_{x} - \cos \overbrace{\frac{A+B}{2}}^{y} = 2 \sin\frac{A}{2} \sin\frac{B}{2}$$

$$\cos x - \cos y = 2 \sin\frac{x+y}{2} \sin\frac{y-x}{2}$$

AB = CD

AD = BC

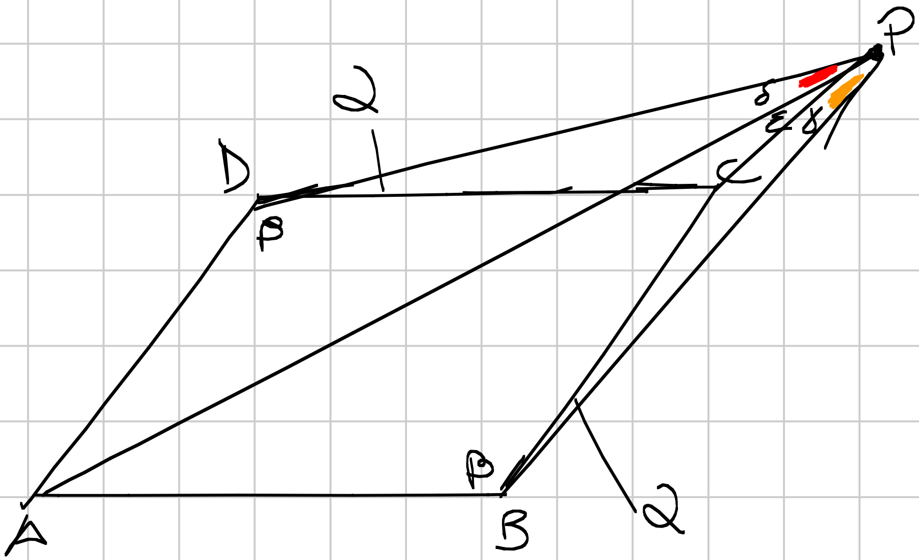
$P\hat{D}C = P\hat{B}C$

Th: $D\hat{P}A \overset{?}{=} C\hat{P}B$

$\delta \overset{?}{=} \gamma$

$= \delta$

$= \gamma$



$\overset{\triangle}{PCD}$ $\quad$ $\overset{\triangle}{PAD}$ $\quad$ $\overset{\triangle}{ABP}$ $\quad$ $\overset{\triangle}{PBC}$

$$\frac{CD}{\sin \delta+\varepsilon} \cancel{= \frac{PC}{\sin \alpha}} = \frac{BC}{\sin \gamma} \cancel{= \frac{PC}{\sin \alpha}}$$

$$\frac{AD}{\sin \delta} \cancel{= \frac{AP}{\sin \alpha+\beta}} = \frac{AB}{\sin \gamma+\varepsilon} \cancel{= \frac{AP}{\sin \alpha+\beta}}$$

$$\frac{CD}{BC} = \frac{\sin \delta+\varepsilon}{\sin \gamma} \qquad \frac{AD}{AB} = \frac{\sin \delta}{\sin \gamma+\varepsilon}$$

$$\frac{CD}{BC} = \frac{AB}{AD} \qquad \frac{\sin \delta+\varepsilon}{\sin\gamma} = \frac{\sin\gamma+\varepsilon}{\sin\delta}$$

$$\sin\delta \ \sin\delta+\varepsilon = \sin\gamma \cdot \sin\gamma+\varepsilon$$

$$f(x) = \sin x \ \sin x+\varepsilon \qquad f(x) = f(y) \Rightarrow x=y$$

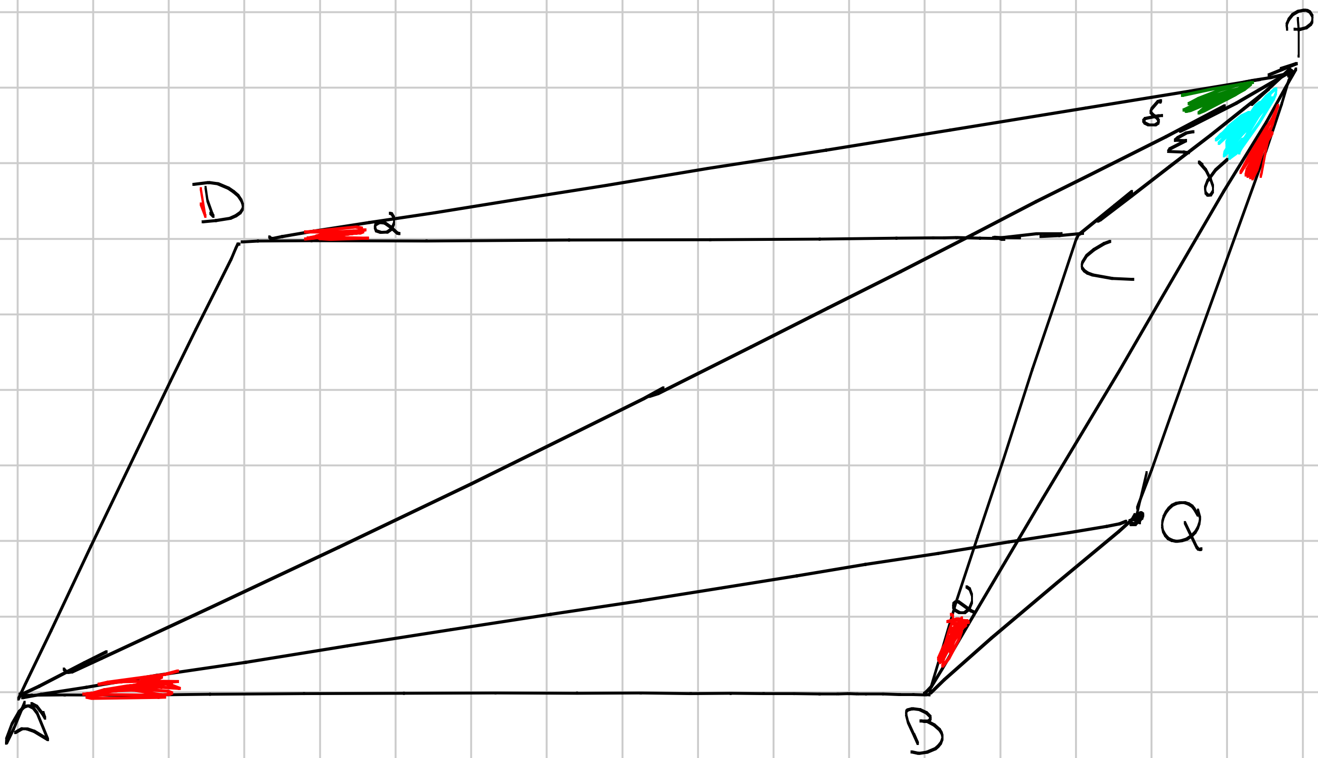
$$\sin x \sin y = \frac{\cos(x-y) - \cos(x+y)}{2}$$

$$\sin\delta \cdot \sin\delta+\varepsilon = \frac{\cancel{\cos(-\varepsilon)} - \cos 2\delta+\varepsilon}{\cancel{2}}$$

$$\|$$

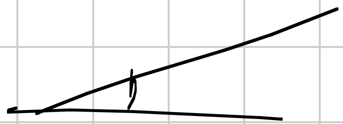
$$\sin\gamma \cdot \sin\gamma+\varepsilon = \frac{\cancel{\cos(-\varepsilon)} - \cos 2\gamma+\varepsilon}{\cancel{2}}$$

$$\cos(2\delta+\varepsilon) = \cos(2\gamma+\varepsilon) \iff \delta = \gamma$$



DP DC

AQ AB



BCPQ = parallelogr $PQ \parallel BC$ $PQ = BC$

ADPQ = parallelogr.

$Q\hat{A}B = B\hat{P}Q \Longrightarrow$ ABQP è ciclico

$A\hat{P}B = \varepsilon + \gamma = A\hat{Q}B = \qquad \triangle PCD \cong \triangle ABQ$

$= D\hat{P}C = \varepsilon + \delta \Longrightarrow \gamma = \delta$

P