Titolo nota 04/09/2007

Esercizi: 1 - 4 - 8 - 9

1) $S = \sum_{n=0^\circ}^{90^\circ} \sin^2 n$

$\sin(90^\circ - n) = \cos n$

$\sin^2(90^\circ - n) = \cos^2 n = 1 - \sin^2 n$

$\sin^2(90^\circ - n) + \sin^2 n = 1 - \cancel{\sin^2 n} + \cancel{\sin^2 n} = 1$

$$\sum_{n=0^\circ}^{44^\circ} \sin^2 n + \sin^2(90^\circ - n) = \sum_{n=0^\circ}^{44^\circ} 1 = 45$$

$\sin^2 45^\circ = \left(\frac{\sqrt{2}}{2}\right)^2 = \frac{1}{2}$ $\qquad S = 45 + \frac{1}{2} = \frac{91}{2}$

4

$R\hat{P}Q = \frac{\alpha}{2}$

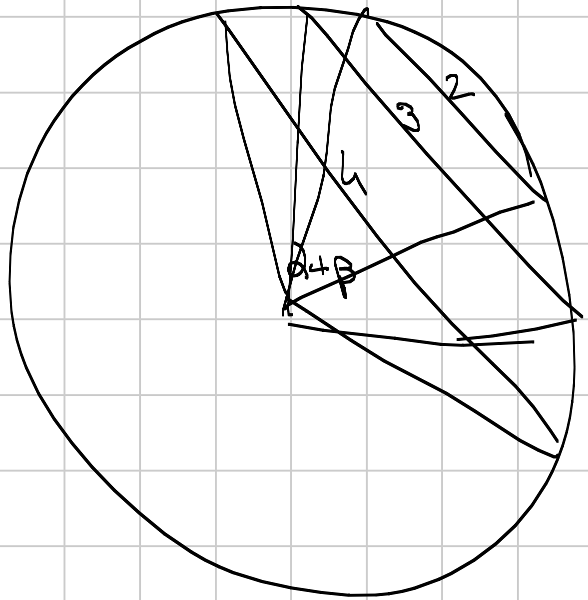
$\triangle PQR$

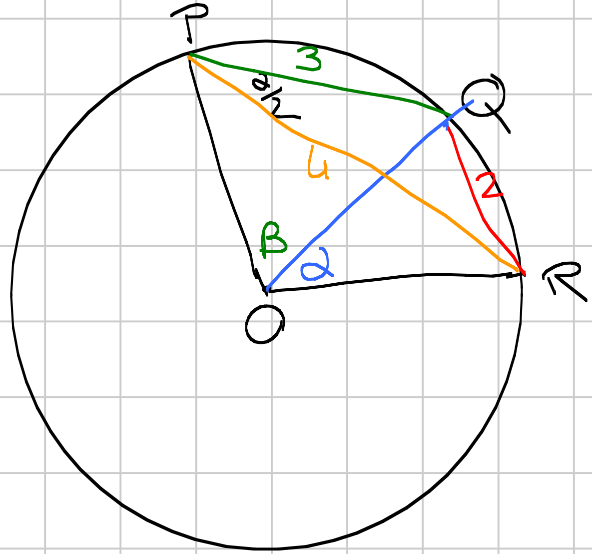
$QR^2 = PQ^2 + PR^2 - 2\cos\frac{\alpha}{2}\, PQ\cdot PR$

$\cos\frac{\alpha}{2} = \frac{PQ^2+PR^2-QR^2}{2\cdot PQ\cdot PR} = \frac{9+16-4}{2\cdot 3\cdot 4} = \frac{21}{3\cdot 8} = \frac{7}{8}$

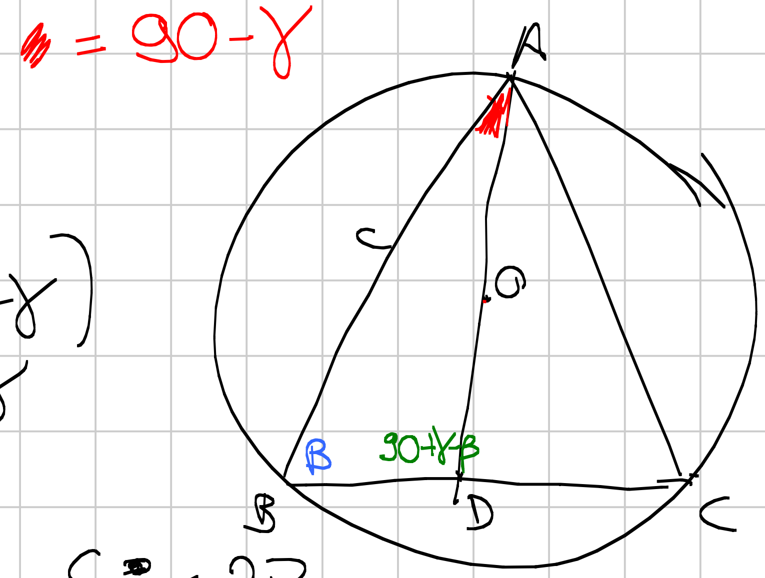
$\cos 2x = 2\cos^2 x - 1$

$\cos\alpha = 2\cos^2\frac{\alpha}{2} - 1 = 2\cdot\left(\frac{7}{8}\right)^2 - 1 = \frac{17}{32}$





⑧ $$\frac{1}{AD}+\frac{1}{BE}+\frac{1}{CF}=\frac{2}{R}$$



$\overset{\triangle}{ABD}$

$$\frac{AD}{\sin\beta}=\frac{c}{\sin(90+\gamma-\beta)}=\frac{c}{\cos(\beta-\gamma)}$$

$$\sin(90-(\beta-\gamma))=\cos(\beta-\gamma)$$

$$\frac{c}{\sin\gamma}=2R$$

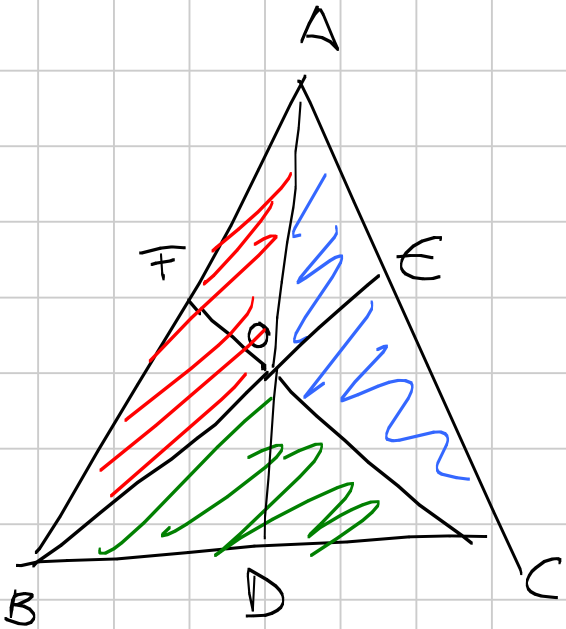
$$AD=\frac{\sin\beta\,\sin\gamma\cdot 2R}{\cos(\beta-\gamma)}$$

$$\sum\frac{1}{AD}=\sum\frac{\cos(\beta-\gamma)}{2R\sin\beta\sin\gamma}=\frac{1}{2R}\sum\frac{\cos\beta\cos\gamma+\sin\beta\sin\gamma}{\sin\beta\sin\gamma}=$$

$$=\frac{1}{2R}\sum 1+\frac{1}{\mathrm{tg}\beta\,\mathrm{tg}\gamma}=$$

$$=\frac{1}{2R}\left[3+\left(\frac{1}{\mathrm{tg}\beta\,\mathrm{tg}\gamma}+\frac{1}{\mathrm{tg}\beta\,\mathrm{tg}\alpha}+\frac{1}{\mathrm{tg}\alpha\,\mathrm{tg}\gamma}\right)\right]=$$

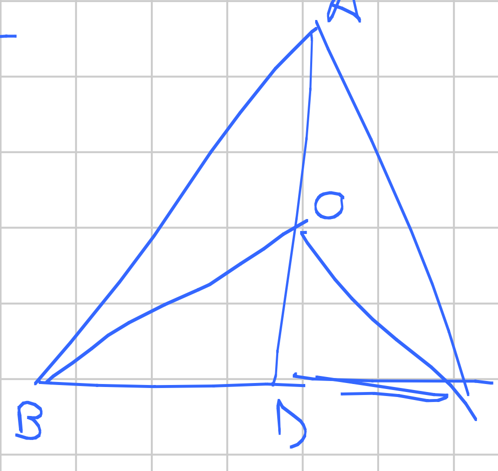
$$= \frac{1}{2R}\left[3 + \frac{\sum \operatorname{tg}\alpha}{\prod \operatorname{tg}\alpha}\right] = \frac{4}{2R} = \frac{2}{R}$$



$$\frac{AO}{AD} + \frac{BO}{BE} + \frac{CO}{CF} \overset{?}{=} 2$$

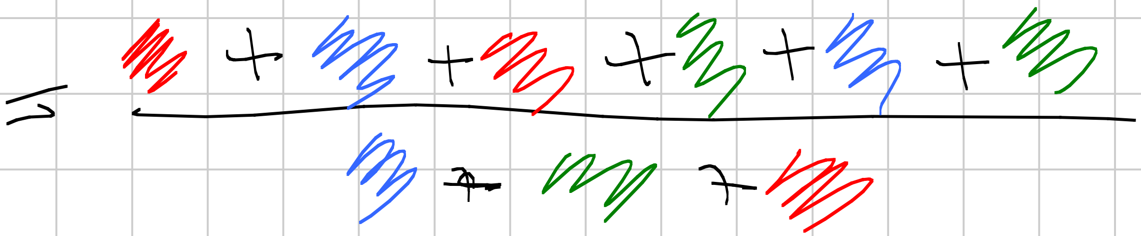
$$\frac{AO}{AD} = \frac{[ABOC]}{[ABC]}$$

$$\frac{AO}{AD} = \frac{[AOB]}{[ABD]}$$



$$\frac{AO}{AD} + \frac{BO}{BE} + \frac{CO}{CF} =$$

$$= \frac{[ABOC] + [BCOA] + [CAOB]}{[ABC]} =$$



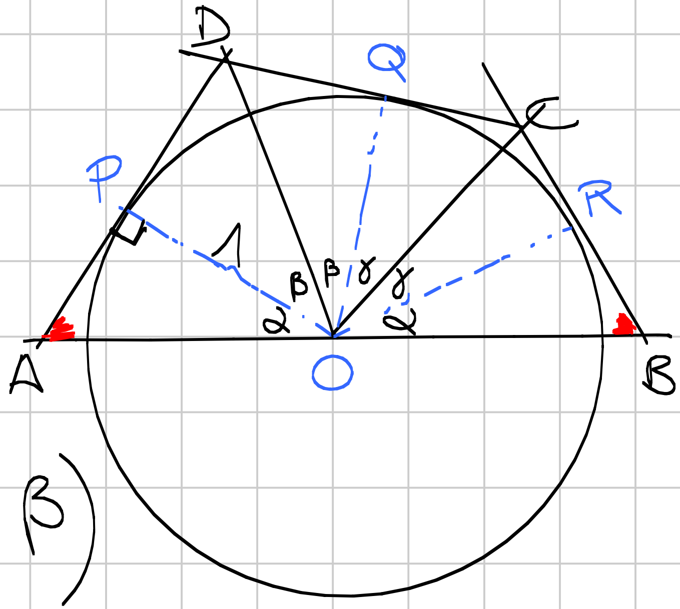
$$= 2$$

$Th: AB^2 = 4 BC \cdot AD$

$\hat{APO} \cong \hat{BOR}$

$AB = 2 \cdot AO = \frac{2}{\cos\alpha}$

$\alpha + \beta + \gamma = \frac{\pi}{2}$



$BC = BR + RC =$

$= tg\alpha + tg\gamma$

$AD = AP + PD = tg\alpha + tg\beta$

$\cancel{\frac{4}{\cos^2\alpha}} = \cancel{4}(tg\alpha + tg\gamma)(tg\alpha + tg\beta)$

$$\frac{1}{\cos^2\alpha} = \boxed{\frac{\sin\alpha + \cos\alpha \frac{\sin\gamma}{\cos\gamma}}{\cancel{\cos\alpha}}} \cdot \frac{\sin\alpha + \cos\alpha \frac{\sin\beta}{\cos\beta}}{\cancel{\cos\alpha}}$$

$\alpha + \beta + \gamma = \frac{\pi}{2}$

$\alpha = \frac{\pi}{2} - \beta - \gamma$

$\sin\alpha = \cos\beta + \gamma$

$$\cos\alpha = \sin\beta+\gamma$$

$$\sin\alpha + \cos\alpha \,\frac{\sin\gamma}{\cos\gamma} = \cos\beta+\gamma + \sin\beta+\gamma \cdot \frac{\sin\gamma}{\cos\gamma} =$$

$$= \cos\beta\cos\gamma - \sin\beta\sin\gamma + (\sin\beta\cos\gamma + \sin\gamma\cos\beta)\frac{\sin\gamma}{\cos\gamma}$$

$$= \cos\beta\cos\gamma - \cancel{\sin\beta\sin\gamma} + \cancel{\sin\beta\sin\gamma} + \frac{\sin^2\gamma\cos\beta}{\cos\gamma} =$$

$$= \cos\beta\left(\frac{\cos^2\gamma + \sin^2\gamma}{\cos\gamma}\right) = \frac{\cos\beta}{\cos\gamma}$$

$$\frac{1}{\cancel{\cos^2\alpha}} \stackrel{?}{=} \boxed{\frac{\sin\alpha + \cos\alpha\,\frac{\sin\gamma}{\cos\gamma}}{\cancel{\cos\alpha}}} \cdot \frac{\sin\alpha + \cos\alpha\,\frac{\sin\beta}{\cos\beta}}{\cancel{\cos\alpha}}$$

$\cos\beta/\cos\gamma$ $\qquad$ $\cos\gamma/\cos\beta$

$$1 = \frac{\cancel{\cos\beta}}{\cancel{\cos\gamma}} \cdot \frac{\cancel{\cos\gamma}}{\cancel{\cos\beta}}$$