G3

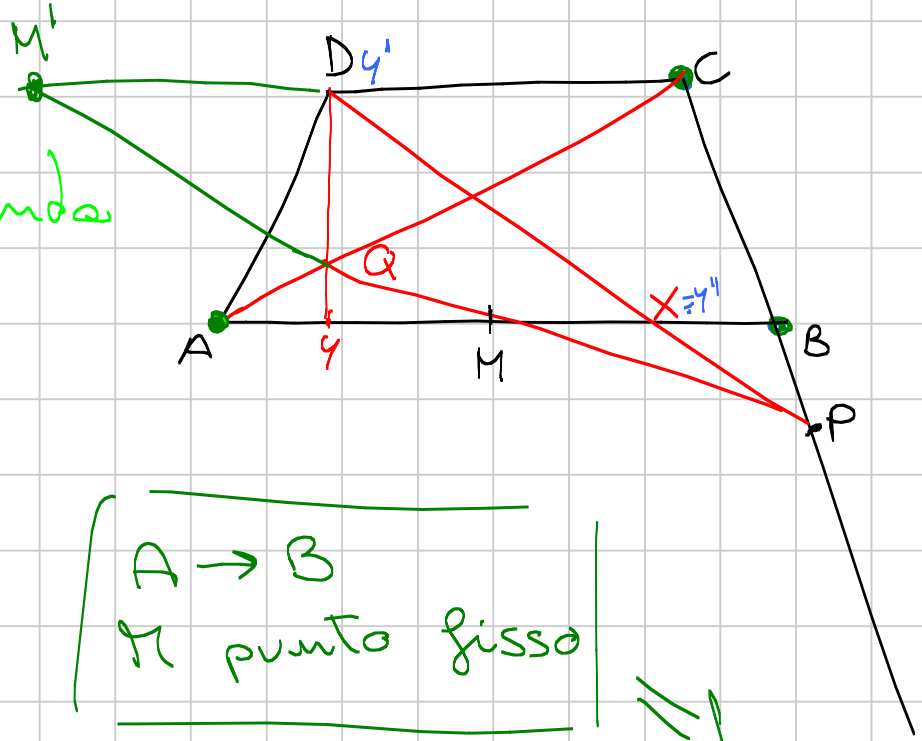
Titolo nota 06/09/2007

Th: $XM = MY$

Omotetia H_Q che manda
$A \to C$

Omotetia H_P manda
$C \to B$

Prima H_Q e poi H_P



$A \to B$
M punto fisso

$\Rightarrow$ simmetria centrale di centro M

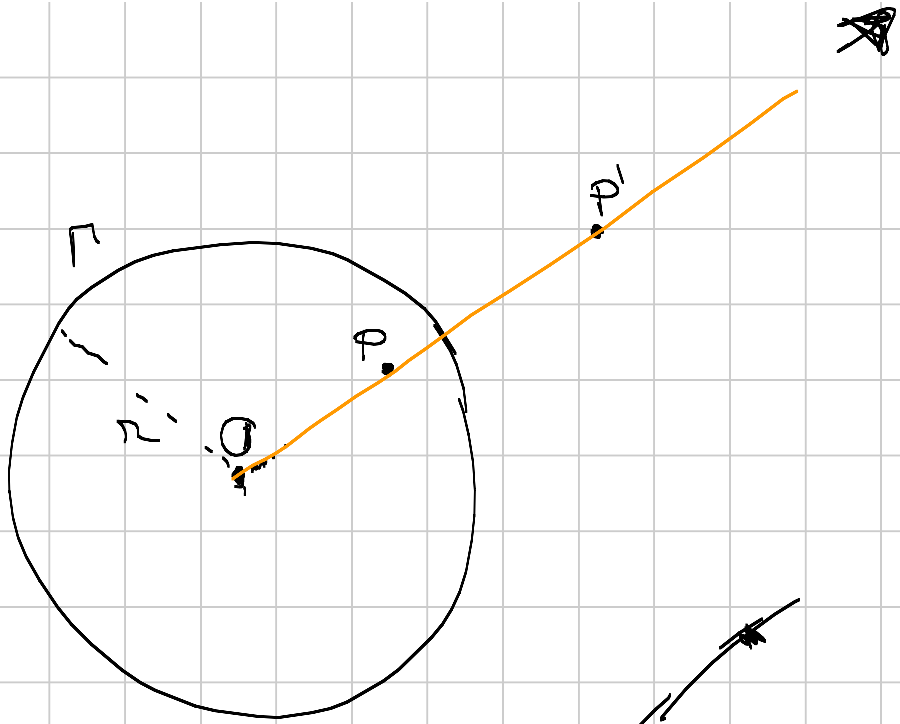
M è punto fisso della composizione

Vogliamo dimostrare che Y va in X

INVERSIONE

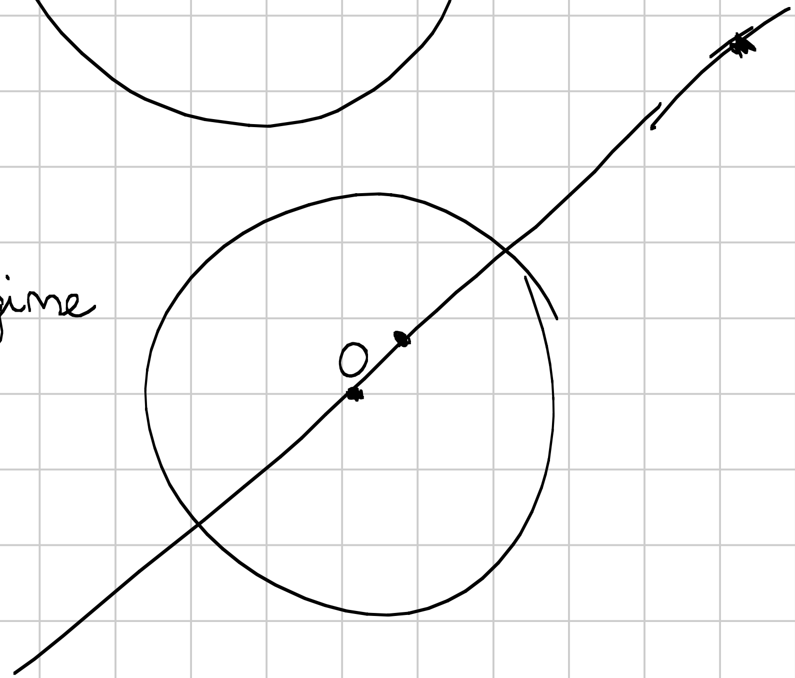
$P \to P'$

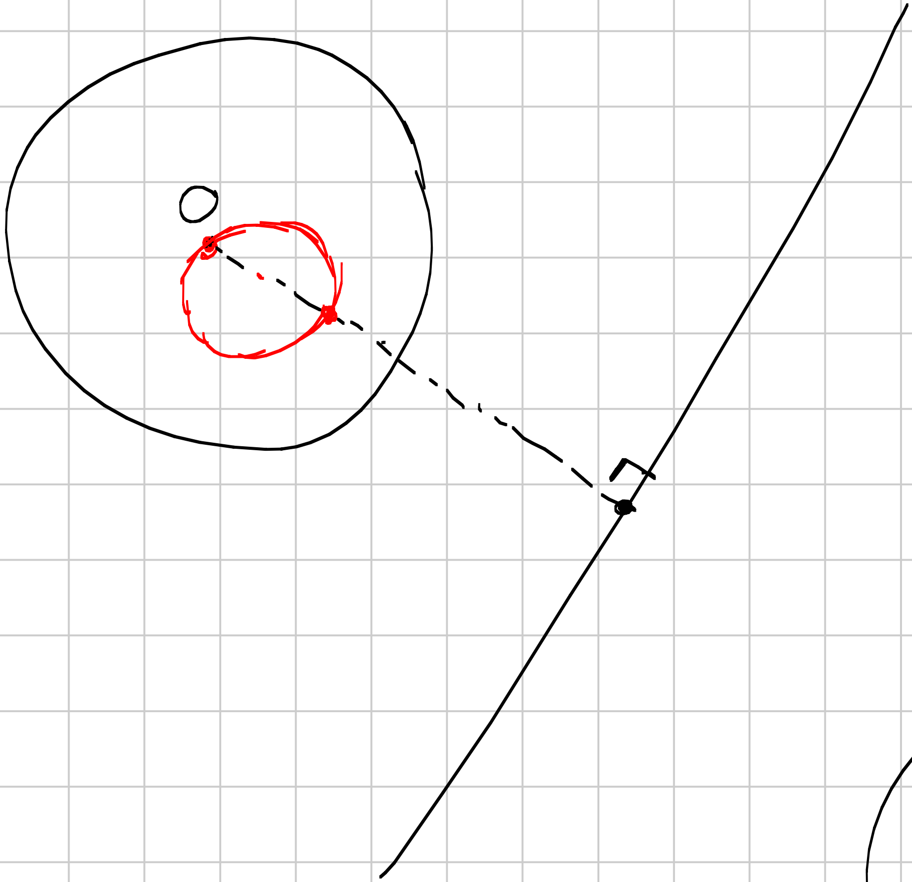
$OP \cdot OP' = r^2$

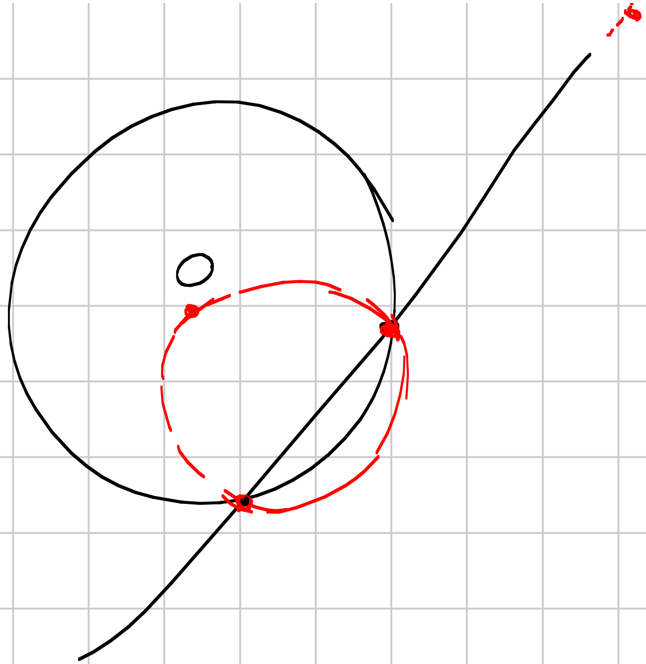


Pti fissi: circonferenza

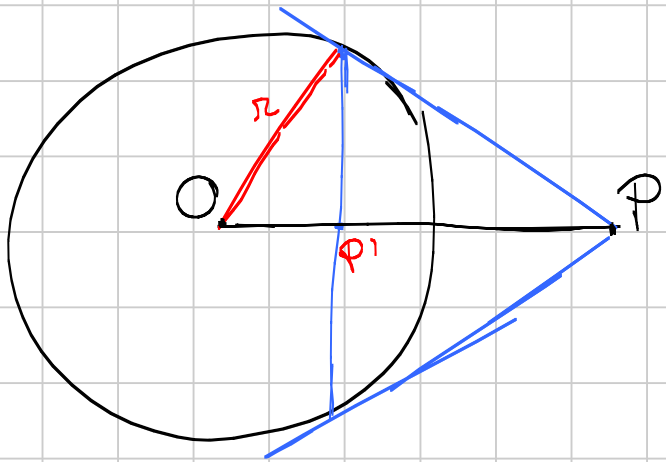
Retta x l'origine → retta x l'origine

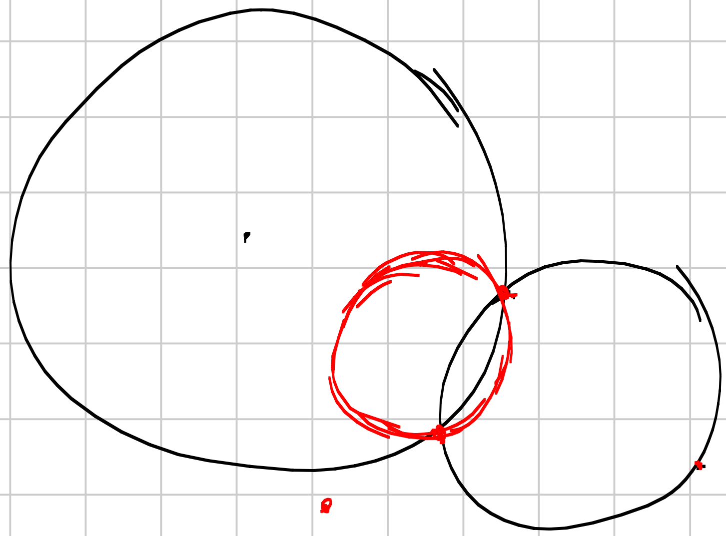


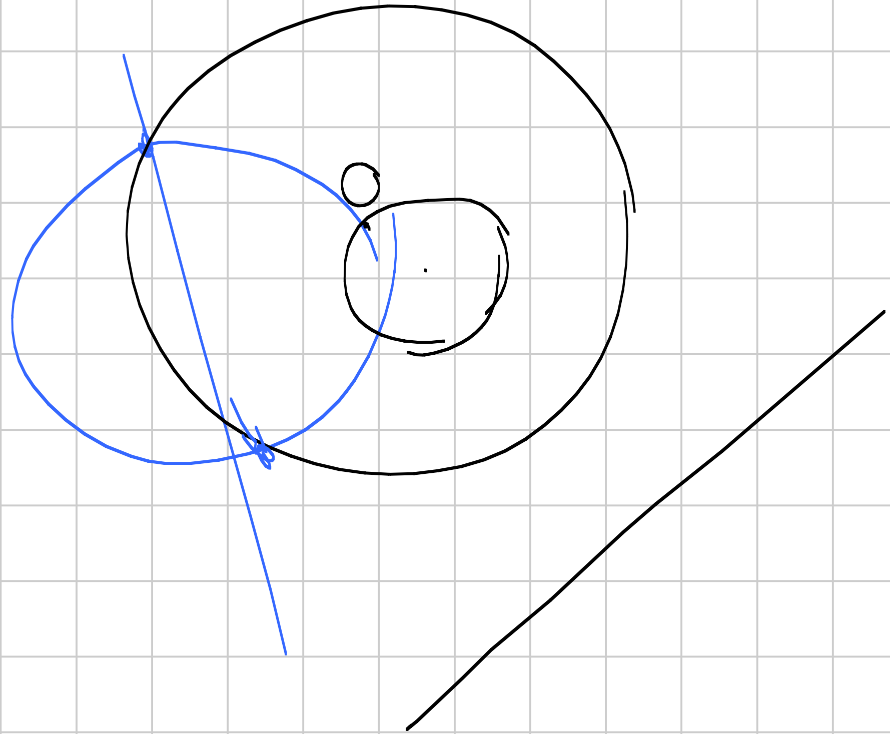


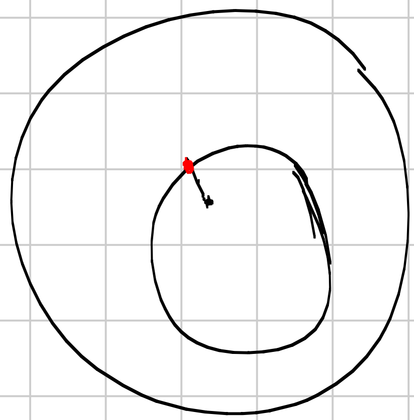


$$OP' \cdot OP = r^2$$

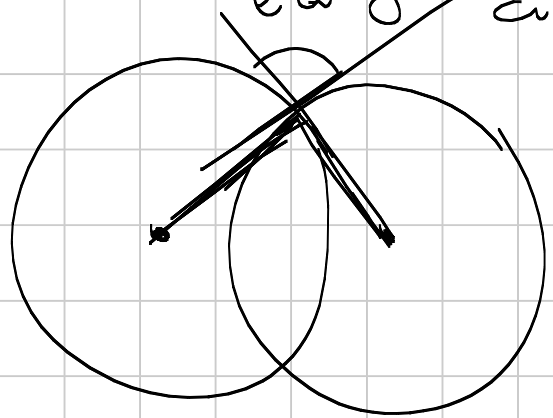






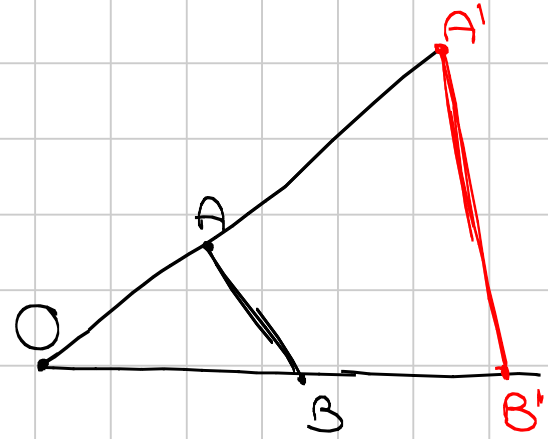


l'angolo tra 2 circonferenze



Cosa succede alle distanze?

$\overline{A'B'}$ $\overline{AB}$



$$OB \cdot OB' = r^2 = OA \cdot OA' = r^2$$

$$\frac{OB}{OA} = \frac{OA'}{OB'}$$

$OAB \sim OB'A'$

$$AB = K\, A'B'$$

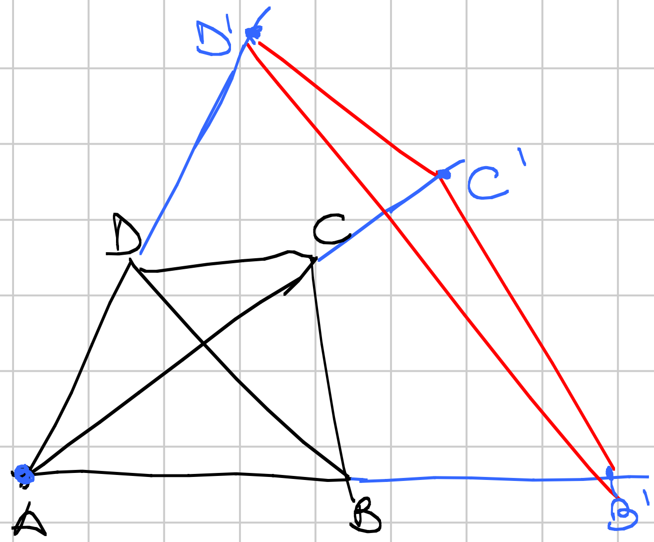
$$\frac{A'B'}{AB} = \frac{OB'}{OA} = \frac{r^2}{OA \cdot OB}$$

ES: TEOREMA DI TOLOMEO

$$\overline{AD} \cdot \overline{BC} + \overline{AB} \cdot \overline{CD} \geq \overline{AC} \cdot \overline{BD}$$

con uguaglianza sse ABCD è ciclico

Inversione con centro A e raggio r



$$B'C' = \frac{BC \cdot r^2}{AB \cdot AC}$$

$$B'D' = \frac{BD \cdot r^2}{AB \cdot AD}$$

$$C'D' = \frac{CD \cdot r^2}{AC \cdot AD}$$

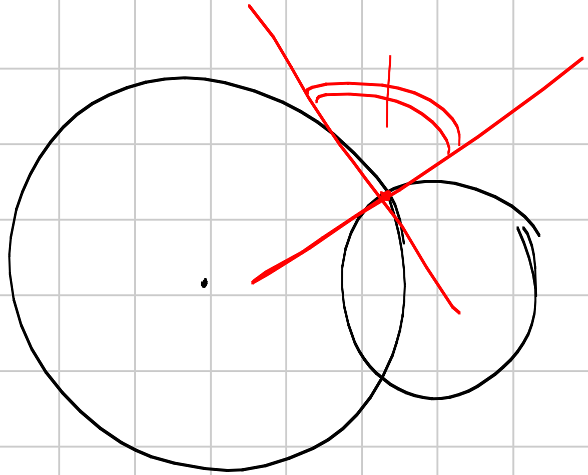
$$B'D' \leq B'C' + C'D'$$

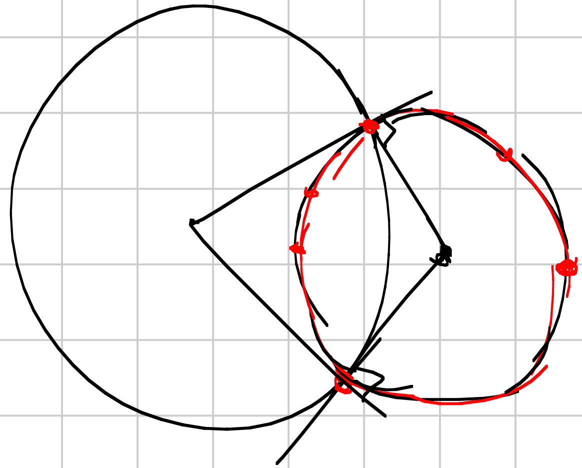
$$\frac{BD \cdot \cancel{r^2}}{AB \cdot AD} \leq \frac{BC \cdot \cancel{r^2}}{AB \cdot AC} + \frac{CD \cdot \cancel{r^2}}{AC \cdot AD}$$

$$AC \cdot BD \leq BC \cdot AD + CD \cdot AB$$

uguaglianza sse i punti B'C' D' sono allineati

$\Longleftrightarrow$ ABCD è ciclico



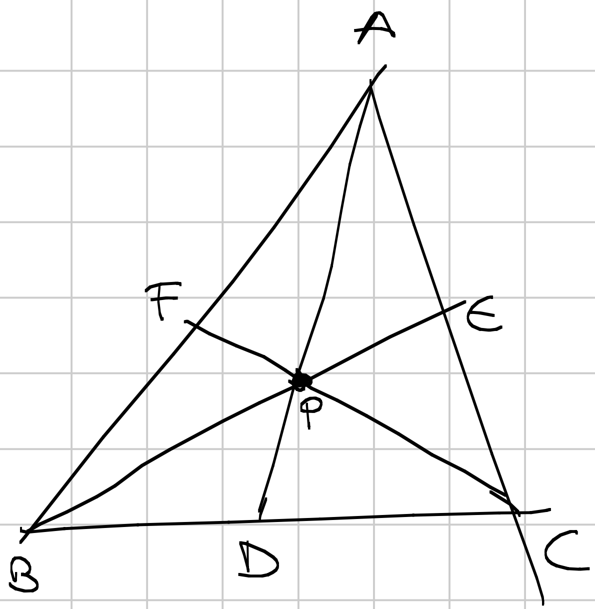


- CEVA

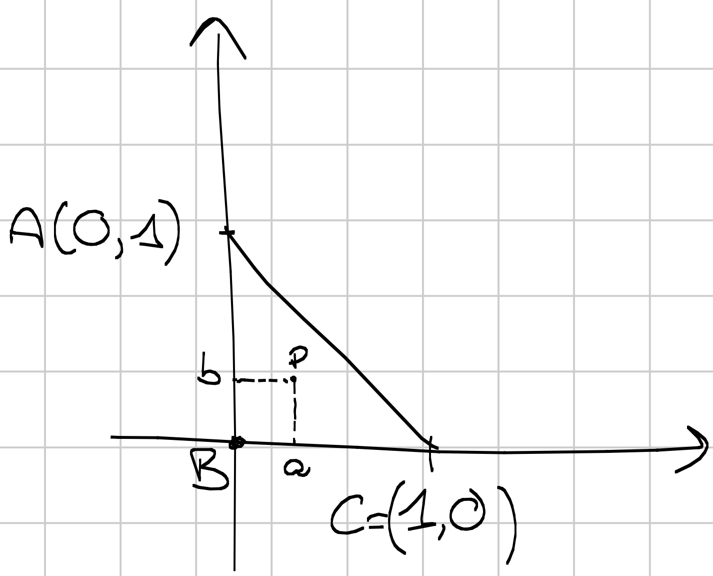
$$\frac{AF}{FB} \cdot \frac{BD}{DC} \cdot \frac{CE}{EA} = 1$$

$$\Updownarrow$$

AD, BE, CF concorrono



① Dim:



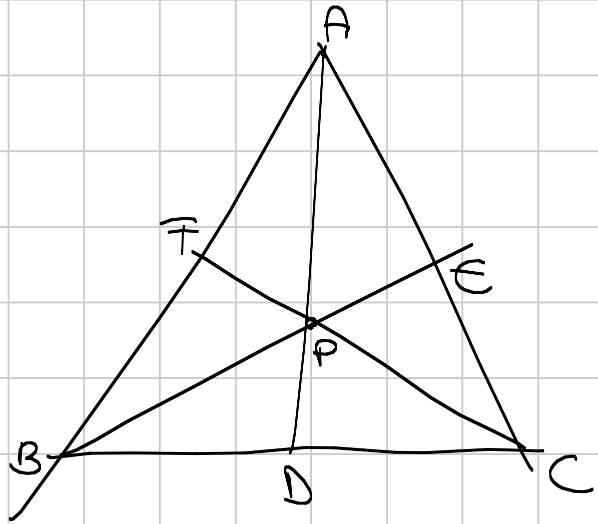
② Dim: AREE

$$\frac{BD}{DC} = \frac{[BPD]}{[PDC]} = \frac{[ABD]}{[ADC]}$$

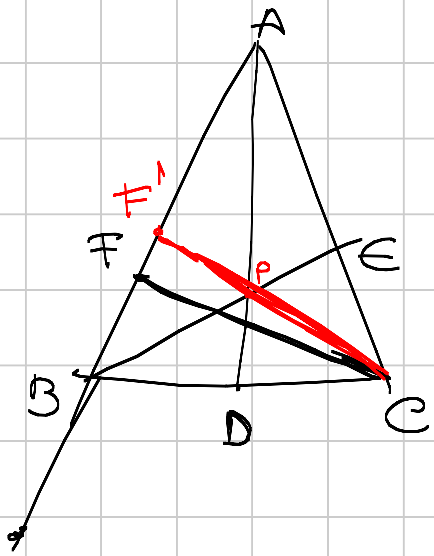
$$\frac{BD}{DC} = \frac{[APB]}{[ACP]}$$

$$\frac{CE}{EA} = \frac{[BCP]}{[ABP]} \qquad \frac{AF}{FB} = \frac{[APC]}{[BPC]}$$

$$\frac{BD}{DC} \cdot \frac{CE}{EA} \cdot \frac{AF}{FB} = \frac{\cancel{[APB]}}{\cancel{[ACP]}} \cdot \frac{\cancel{[BCP]}}{\cancel{[APB]}} \cdot \frac{\cancel{[APC]}}{\cancel{[BPC]}} = 1$$



$$\frac{AF}{FB} \cdot \frac{BD}{DC} \cdot \frac{CE}{EA} = 1$$



$P = BE \cap AD$

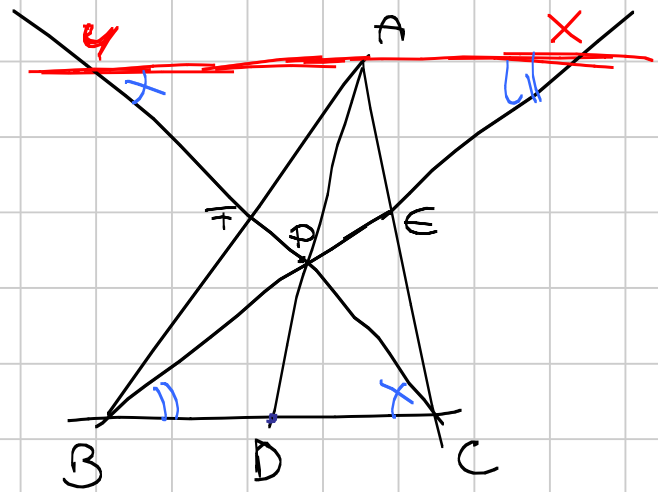
$$\frac{AF'}{F'B} \cdot \frac{BD}{DC} \cdot \frac{CE}{EA} = 1$$

$$1 + \frac{AF}{FB} = \frac{AF'}{F'B} + 1 \Rightarrow \frac{AB}{FB} = \frac{AB}{F'B} \Rightarrow F \equiv F'$$

3) $\overset{\triangle}{BCF} \sim \overset{\triangle}{AFY}$

$$\frac{BF}{FA} = \frac{BC}{AY}$$

$\overset{\triangle}{BCE} \sim \overset{\triangle}{AEX}$ $\quad \frac{CE}{EA} = \frac{BC}{AX}$

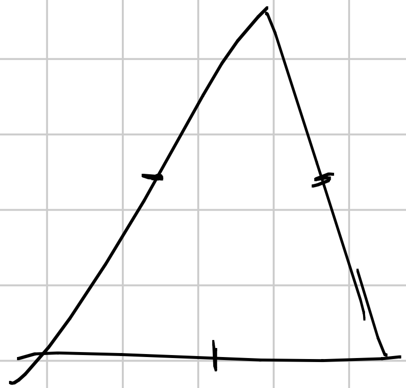


$PXY \sim BCP$

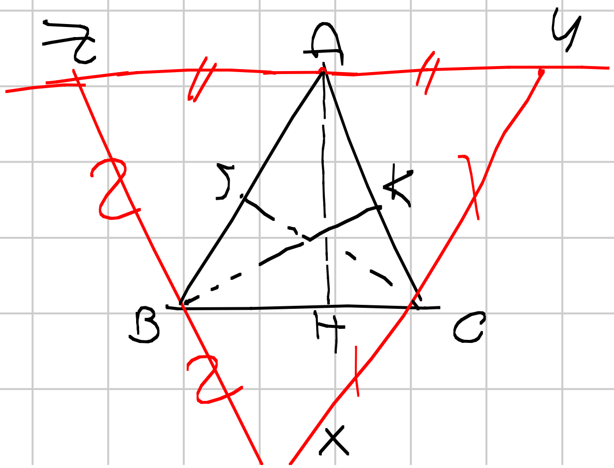
$$\frac{BD}{DC} = \frac{AX}{AY}$$

$$\frac{AF}{FB} \cdot \frac{BD}{DC} \cdot \frac{CE}{EA} = \cancel{\frac{AY}{BC}} \cdot \cancel{\frac{AX}{AY}} \cdot \cancel{\frac{BC}{AX}} = 1$$

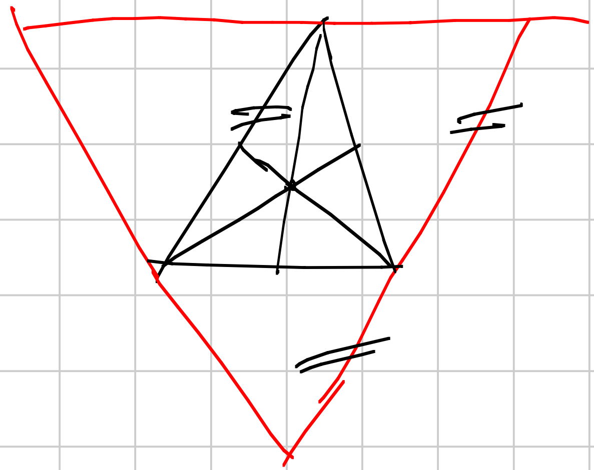
ES Esiste il baricentro



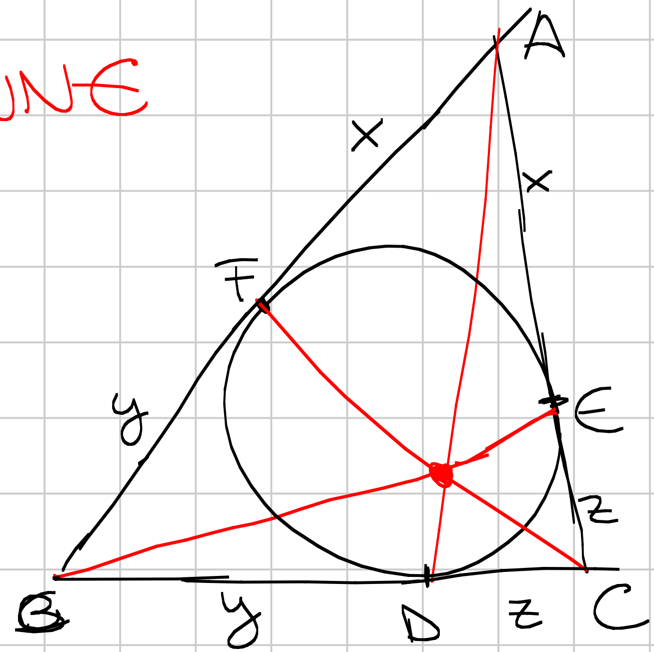
ES: le altezze concorrono



AH è l'asse di YZ



ES: PUNTO DI ~~NAGEL~~ GERGONNE



$AF = AE = x$

$BF = BD = y$

$CD = CE = z$

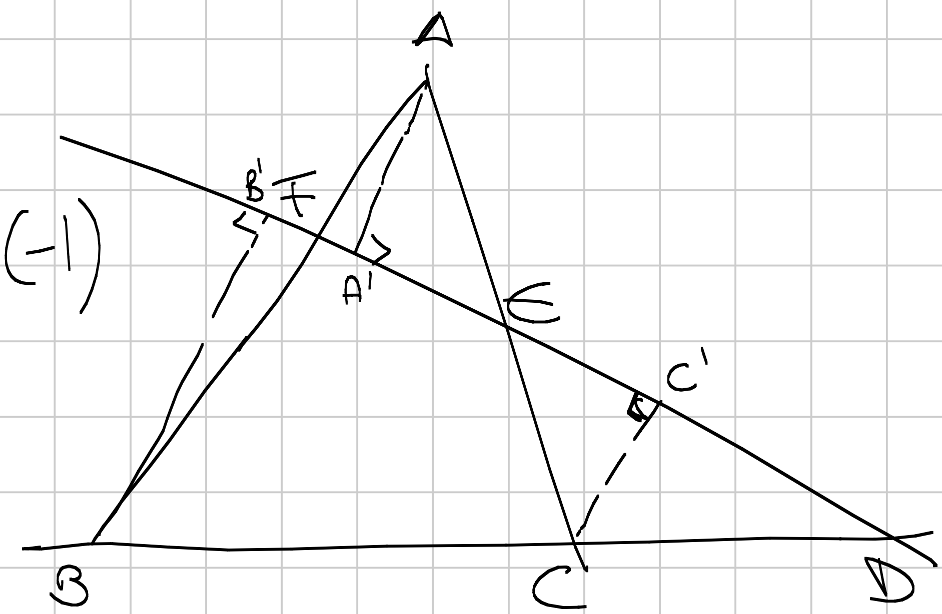
$$\frac{BD}{DC}\cdot\frac{CE}{EA}\cdot\frac{AF}{FB} \overset{?}{=} 1$$

$$\frac{\cancel{y}}{\cancel{z}}\cdot\frac{\cancel{z}}{\cancel{x}}\cdot\frac{\cancel{x}}{\cancel{y}} = 1$$

TEOREMA DI MENELAO

D, E, F sono allineati

$\Longleftrightarrow \dfrac{AF}{FB} \cdot \dfrac{BD}{DC} \cdot \dfrac{CE}{EA} = 1 \quad (-1)$



$AA'F \sim BB'F$

$$\frac{AF}{FB} = \frac{AA'}{BB'}$$

$$\frac{CE}{EA} = \frac{CC'}{AA'}$$

$$\frac{BD}{CD} = \frac{BB'}{CC'}$$

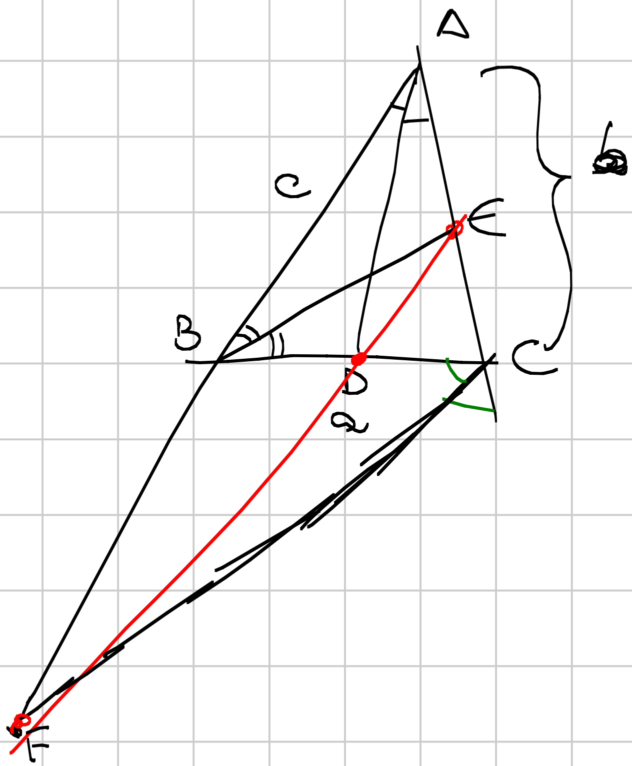
ESERCIZIO:

$$\frac{AF}{FB}\cdot\frac{BD}{DC}\cdot\frac{CE}{EA} \overset{?}{=} 1$$

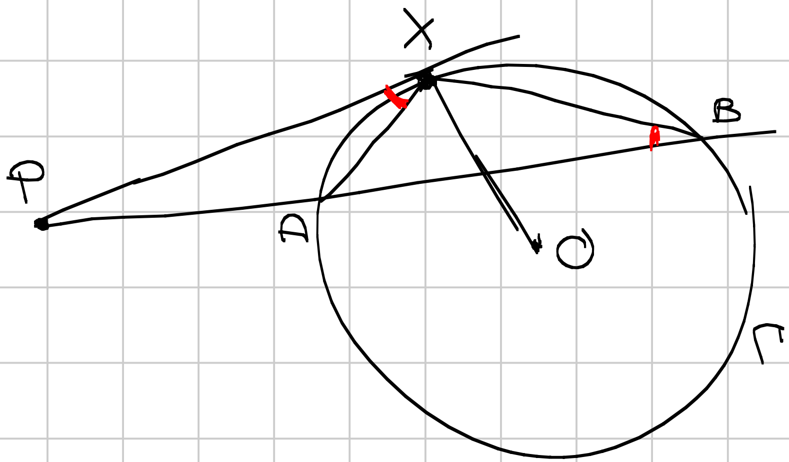
$$\frac{BD}{DC} = \frac{c}{b}$$

$$\frac{CE}{EA} = \frac{a}{c}$$

$$\frac{AF}{FB} = \frac{b}{a}$$



$\mathrm{Pow}_\Gamma(P)$

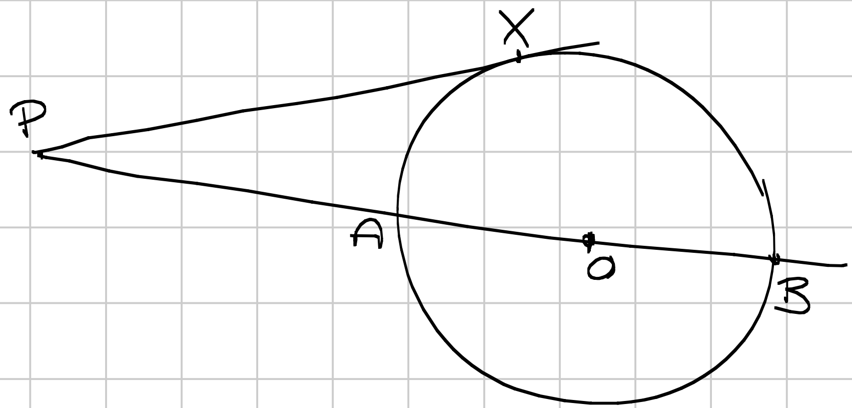


$P\hat{X}A = X\hat{B}A$
(insistono sull'arco $\overset{\frown}{AX}$)

$P\overset{\triangle}{X}A \sim P\overset{\triangle}{X}B$

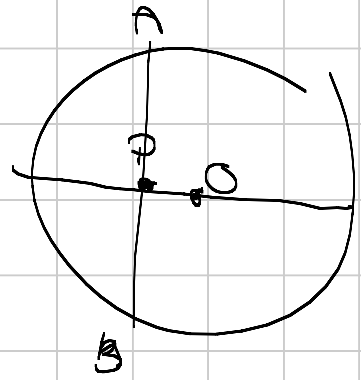
$$\frac{PX}{PA} = \frac{PB}{PX} \implies PX^2 = PA \cdot PB = \mathrm{pow}_\Gamma(P)$$

$d = \overline{PO}$



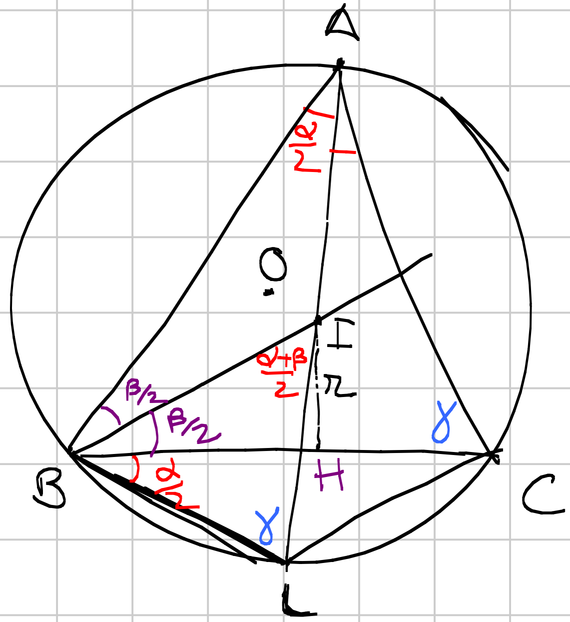
$PX^2 = PA \cdot PB =$
$= (d-R)(d+R) = d^2 - R^2$

$\mathrm{Pow}_\Gamma(P) = d^2 - R^2$



Esercizio:

calcolare $\mathrm{Pow}(I)$



$$\mathrm{Pow}\, I = OI^2 - R^2 = AI \cdot IL \stackrel{?}{=} -2rR$$

Lemma: $BL = LI$

BLI è isoscele di base BI

th dei seni al triangolo $AIB \Rightarrow \dfrac{c}{\operatorname{sen}\frac{\alpha+\beta}{2}} = \dfrac{AI}{\operatorname{sen}\frac{\beta}{2}}$

$$AI \cdot IL = AI \cdot BL$$

th dei seni $A\hat{B}L$ $\quad \dfrac{c}{\operatorname{sen}\cancel{\gamma}} = \dfrac{BL}{\operatorname{sen}\frac{\alpha}{2}}$

$$AI \cdot BL = \frac{c \operatorname{sen}\frac{\beta}{2}}{\operatorname{sen}\frac{\alpha+\beta}{2}} \cdot \frac{c \operatorname{sen}\frac{\alpha}{2}}{\operatorname{sen}\gamma} = 2R \, c \, \frac{\operatorname{sen}\frac{\beta}{2} \cdot \operatorname{sen}\frac{\alpha}{2}}{\operatorname{sen}\frac{\alpha+\beta}{2}}$$

$$\frac{c}{\operatorname{sen}\frac{\alpha+\beta}{2}} \cdot \operatorname{sen}\frac{\alpha}{2} \overset{?}{=} \frac{r}{\operatorname{sen}\beta/2} \qquad c \operatorname{sen}\frac{\beta}{2} \cdot \operatorname{sen}\frac{\alpha}{2} \overset{?}{=} r \operatorname{sen}\frac{\alpha+\beta}{2}$$

th seni $\overset{\triangle}{ABI}$: $\frac{c}{\operatorname{sen}\frac{\alpha+\beta}{2}} = \frac{BI}{\operatorname{sen}\frac{\alpha}{2}}$

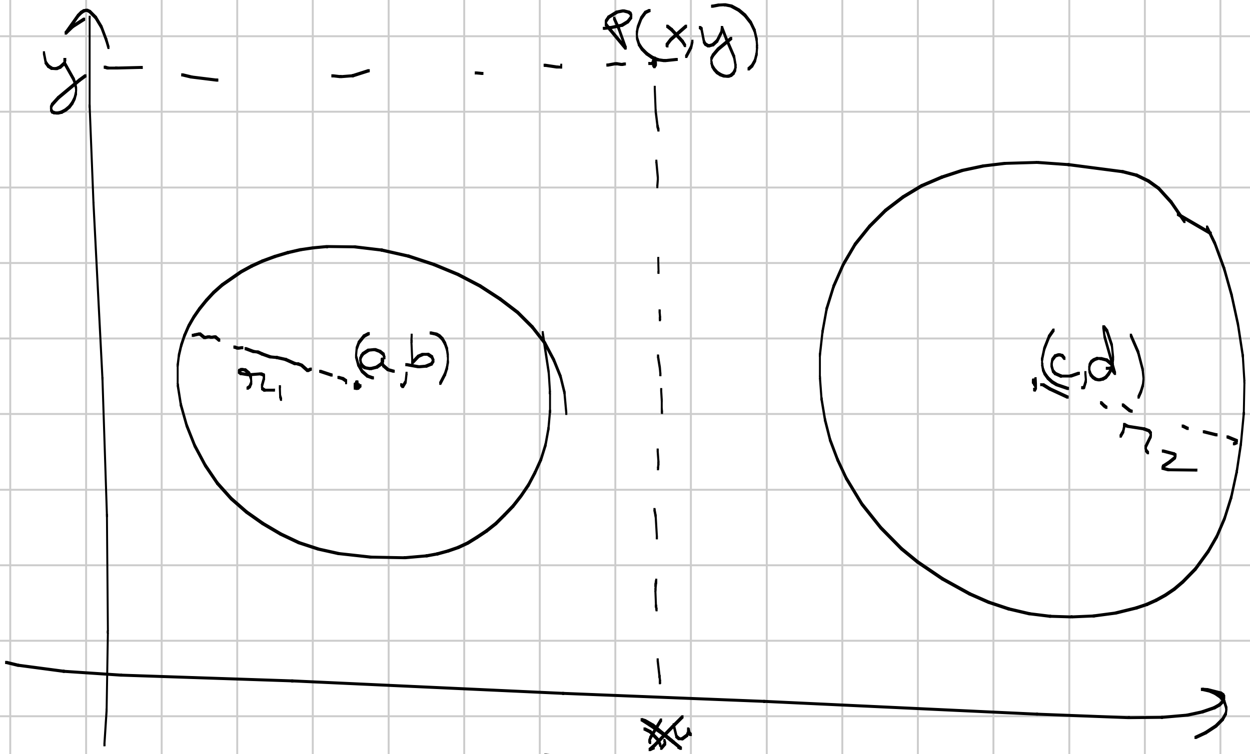
$$BI \overset{?}{=} \frac{r}{\operatorname{sen}\beta/2} \quad \text{VERA}$$

$$OI^2 - R^2 = -2rR$$

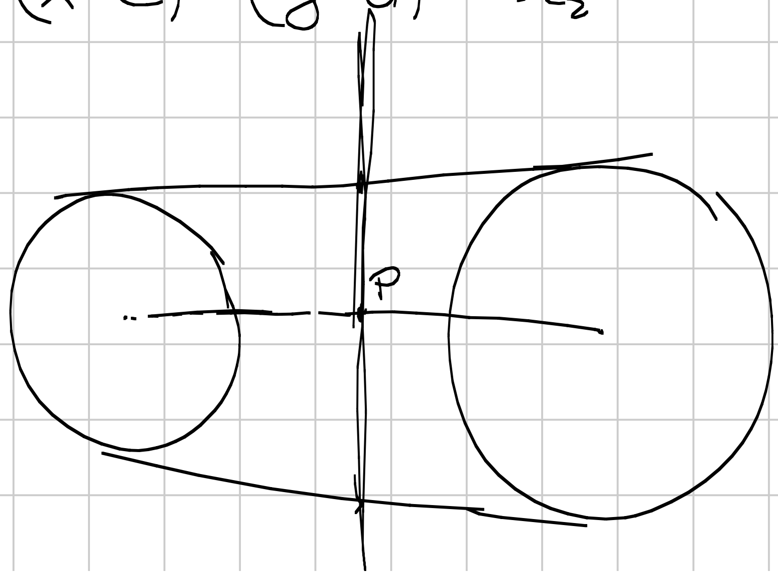
$$OI^2 = R^2 - 2rR > 0$$

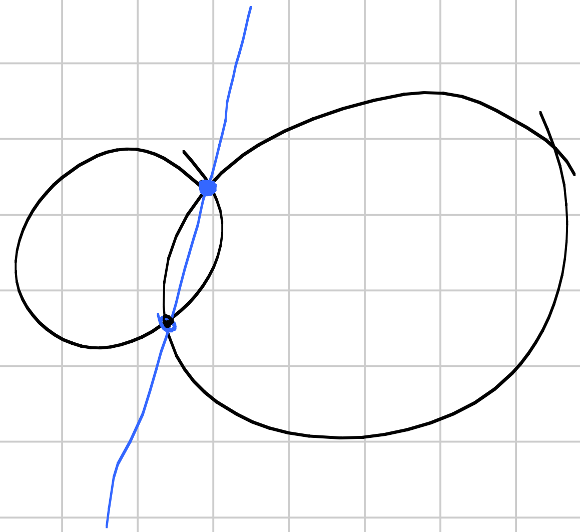
$$R - 2r > 0 \implies r < \frac{R}{2}$$

$$(x-a)^2 + (y-b)^2 - r_1^2 = 0$$



$$(x-a)^2 + (y-b)^2 - r_1^2 = \mathrm{pow}(P) = (x-c)^2 + (y-d)^2 - r_2^2$$





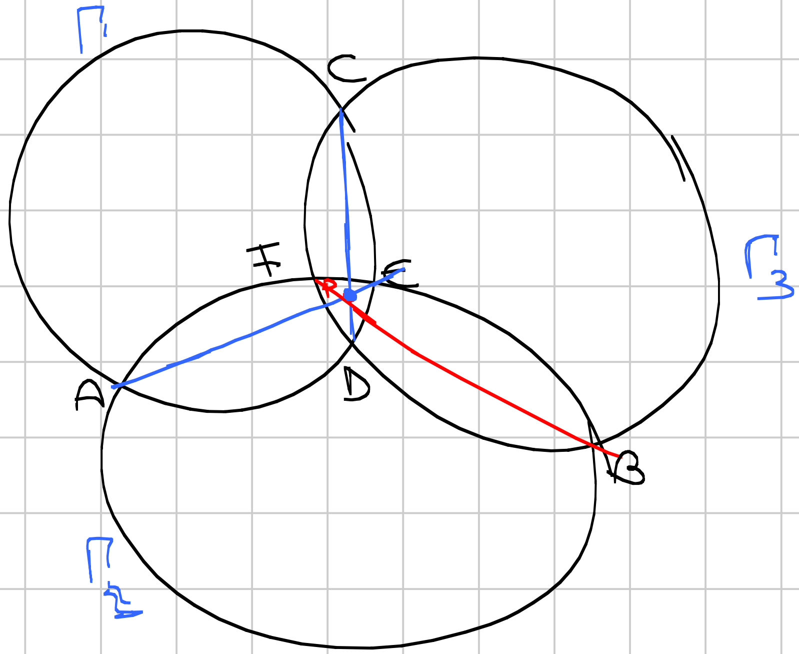
ES

AE, BF, CD concorrono.

$Pow_{\Gamma_1} = Pow_{\Gamma_2}$

$Pow_{\Gamma_1} = Pow_{\Gamma_3}$

$Pow_{\Gamma_1}(P) = Pow_{\Gamma_2}(P) = Pow_{\Gamma_3}(P)$

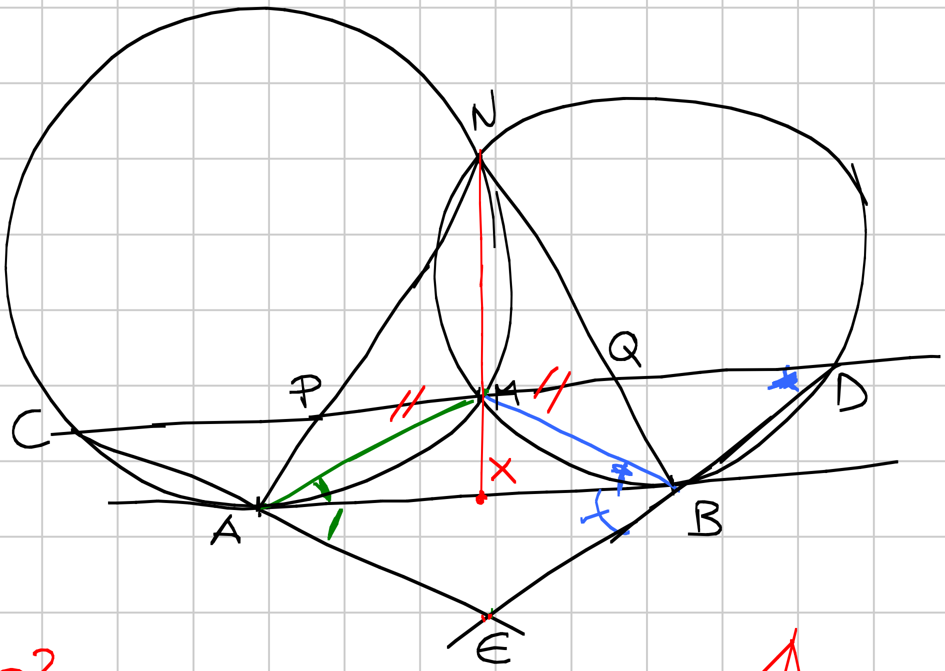


IMO 2000

$CD \parallel AB$

Th: $PE \overset{?}{=} EQ$

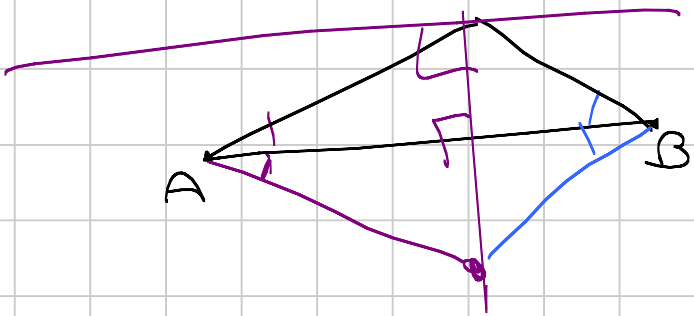
Lemma: $PM \overset{?}{=} MQ$

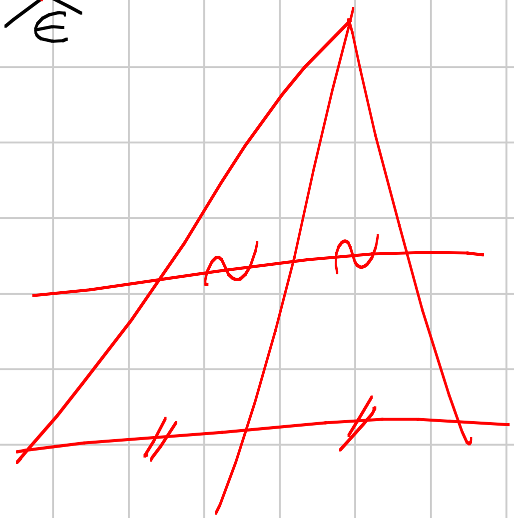


$pow_{\Gamma_1}(X) = pow_{\Gamma_2}(X) = XA^2 = XB^2$

$ME \perp CD$

$M\hat{B}A = A\hat{B}E$





$$\mathrm{pow}_{\Gamma_1}(E) = EA \cdot CE = \frac{EB \cdot EC}{ED} \cdot EC$$

$$\mathrm{pow}_{\Gamma_2}(E) = EB \cdot ED$$

$$\frac{EA}{EC} = \frac{EB}{ED} \Rightarrow EA \cdot ED$$

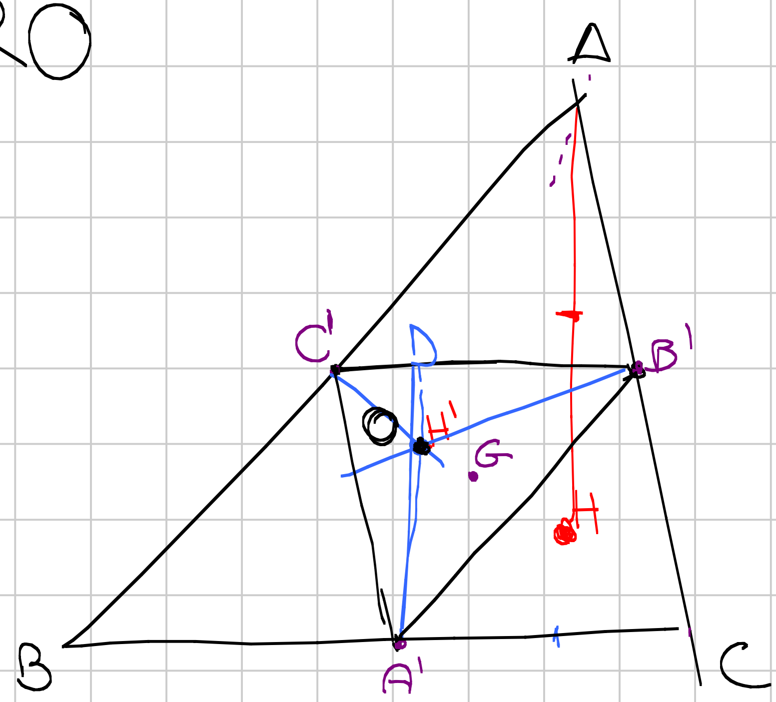
RETTA DI EULERO

G, O, H sono allineati

OGH

$$OG = \frac{1}{2} OH$$

Omotetia di centro G e rapporto $-\frac{1}{2}$

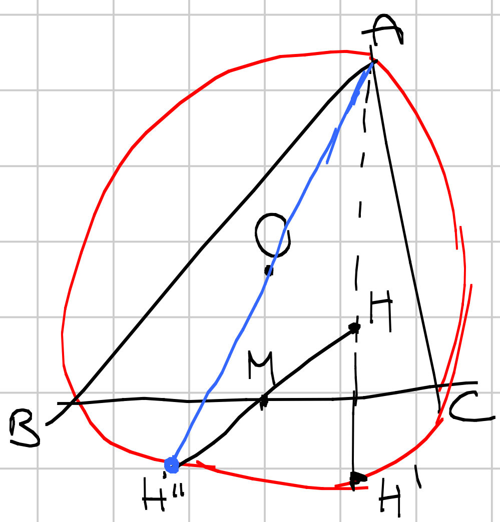


A'D = l'asse di BC

H' = O

LEMMA (usefue!!!):

il simmetrico dell'ortocentro rispetto a un lato o a un punto medio sta sulla circonferenza circoscritta.



$$G = \frac{A+B+C}{3} \qquad H = 3G = A+B+C$$

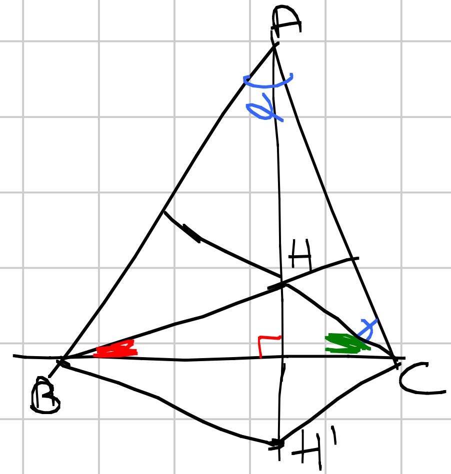
$$M = \frac{B+C}{2} \qquad H'' = 2M - H = B+C-(A+B+C) = -A$$

$$BH'C \cong BHC$$

$$B\hat{A}C + B\hat{H'}C \overset{?}{=} 180$$

$$B\hat{H'}C = BHC$$

$$B\hat{A}C + B\widehat{H'}C = \alpha + BHH' + CHH' =$$
$$= \alpha + \gamma + \beta = 180$$



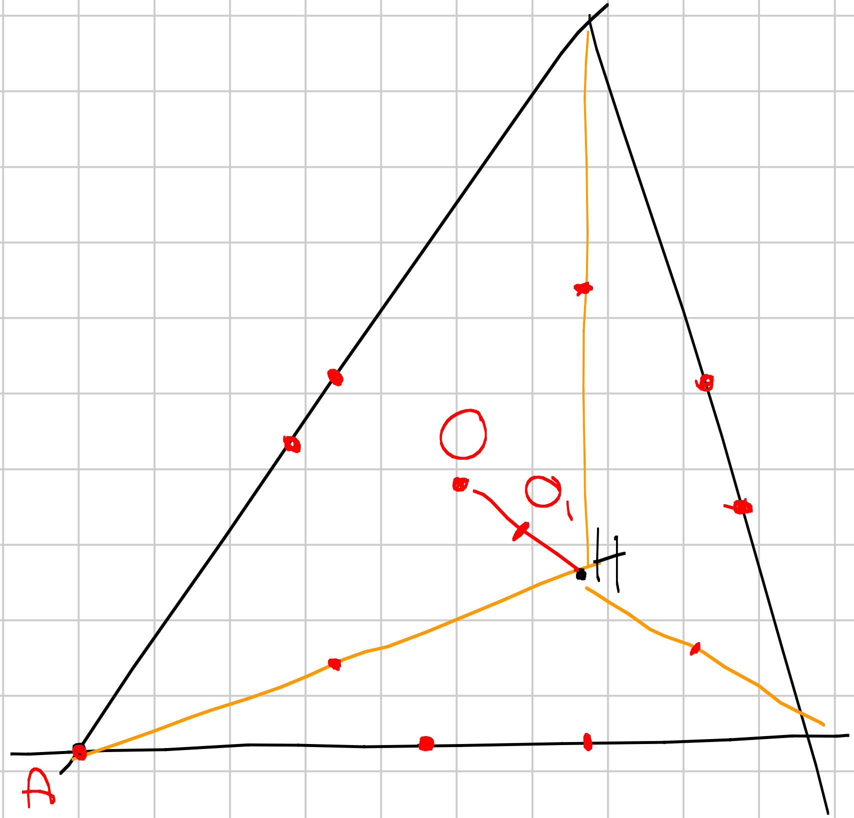
$$= 90 - \gamma$$

$$BHH' = \gamma$$

$$= 90 - \beta$$

$$CHH' = \beta$$

H_H $\frac{1}{2}$

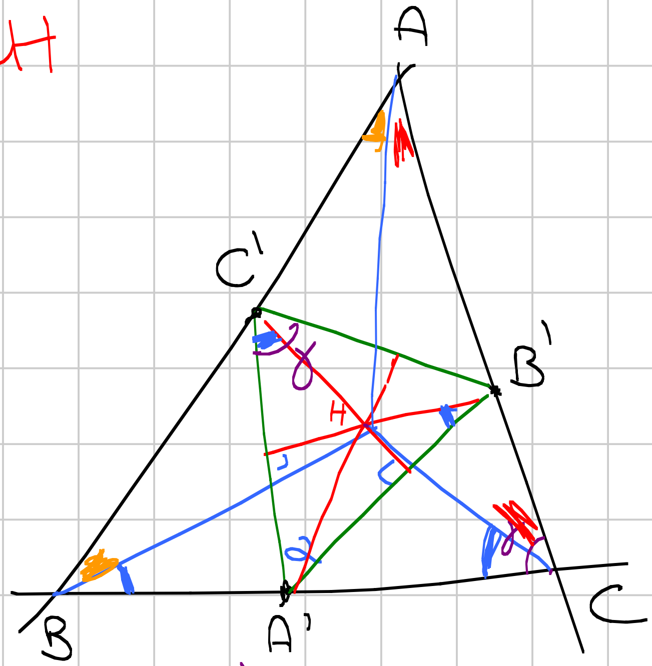


H'' H'

$ABC \sim A'B'C'$

$AH = BH = CH$

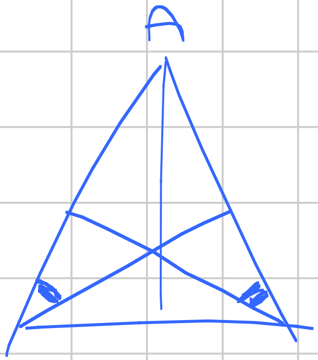
Dimostrare che l'ortocentro di $A'B'C'$ è il circocentro di ABC

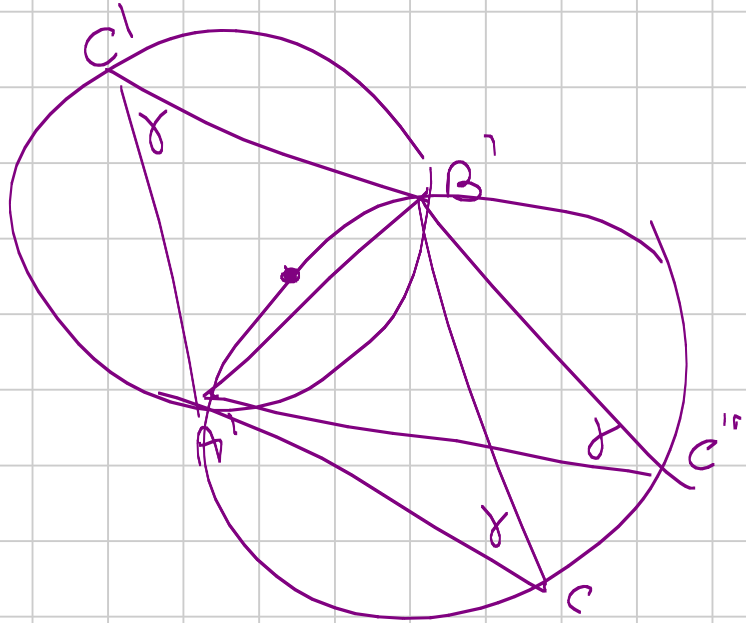


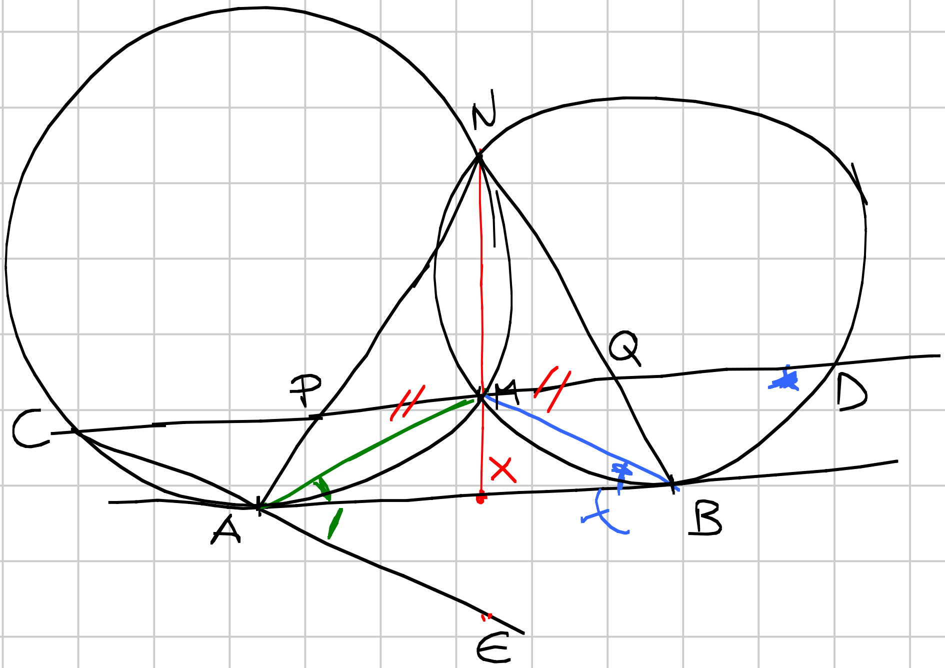
$A'CB'H$ sono conciclici

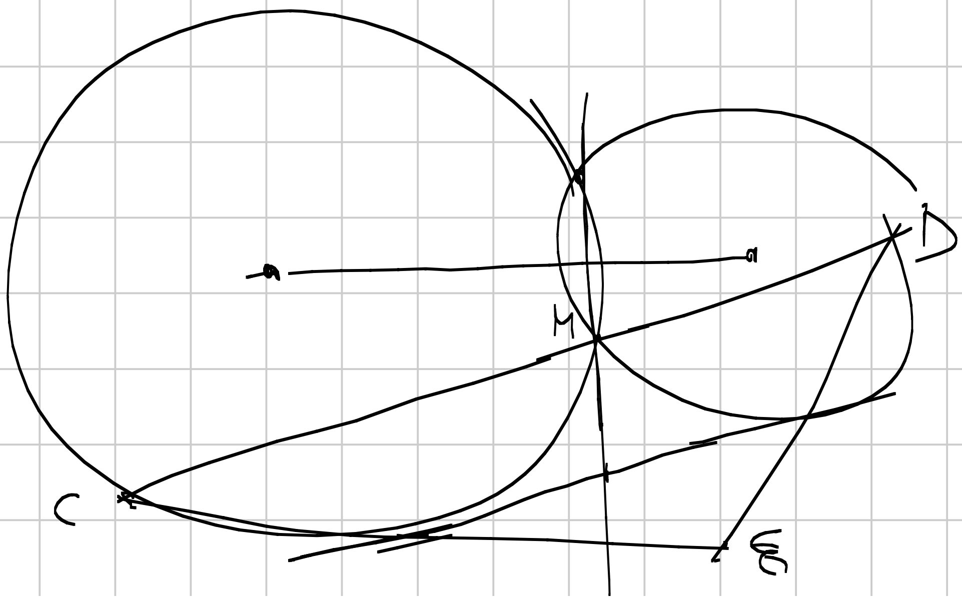
$A'\hat{C}H = A'\hat{B}'H = A'\hat{C}'H = H\hat{B}A'$

$A'HC'B$ ciclico









$ME \perp CD$