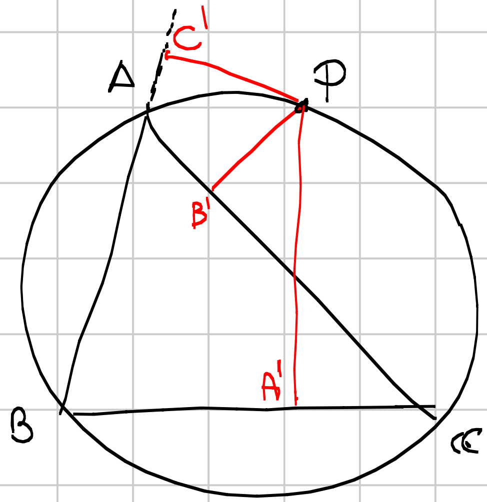
ESERCIZI

3, 6, 8

ESERCIZIO: Simson line

Dato un triangolo ABC e un punto P sulla circonferenza circoscritta, detta A' la proiezione di P su BC, B' quella su AC, C' quella su AB. Dimostrare che A', B', C' sono allineati.
(Chiedete tranquillamente hint se non vi viene).
Per chi le sa già: IMO 2007/2 (Hint: usare Simson)



IMO 2007/2

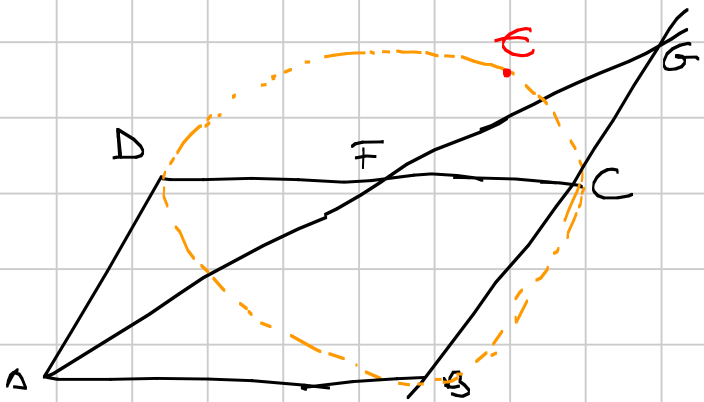
(Era scritto diverso ma fa niente, così è un pochino più chiaro)

ABCD è un parallelogramma.

Prendiamo una retta ℓ uscente da A che interseca il segmento CD in F e la retta BC in G.

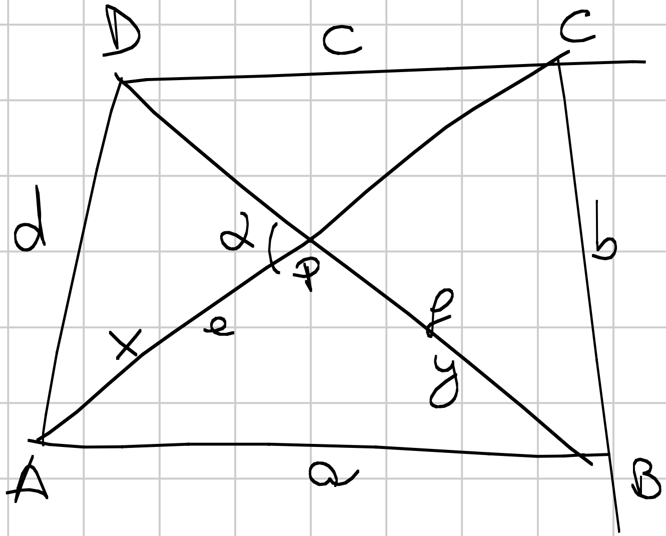
Consideriamo il circocentro E del triangolo CFG.

Dimostrare che se E appartiene alla circonferenza circoscritta a BCD allora ℓ è bisettrice di $D\hat{A}B$



ES

$$S = \frac{1}{2} e f \operatorname{sen}\alpha$$



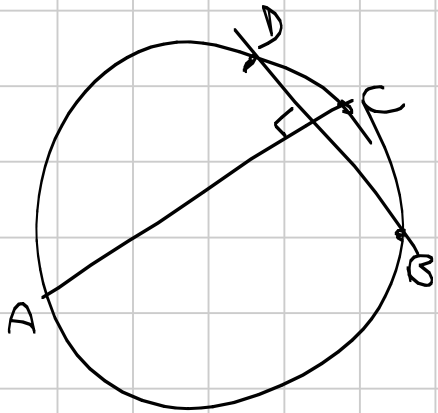
$$\frac{1}{2} AP \cdot PB \cdot \operatorname{sen}\alpha +$$
$$+ \frac{1}{2} PB \cdot PC \operatorname{sen}\alpha +$$
$$\frac{1}{2} PC \cdot PD \operatorname{sen}\alpha +$$
$$+ \frac{1}{2} PD \cdot PA \operatorname{sen}\alpha =$$

$AP = x \qquad PB = y$

$$= \frac{1}{2} \operatorname{sen}\alpha \left(AP + PC\right)\left(BP + PD\right) = \frac{1}{2} e f \operatorname{sen}\alpha$$

$$2S = e f \operatorname{sen}\alpha \leq e f \leq ac + bd$$

$\operatorname{sen}\alpha = 1$



6 TS: $BE \stackrel{?}{=} EC$

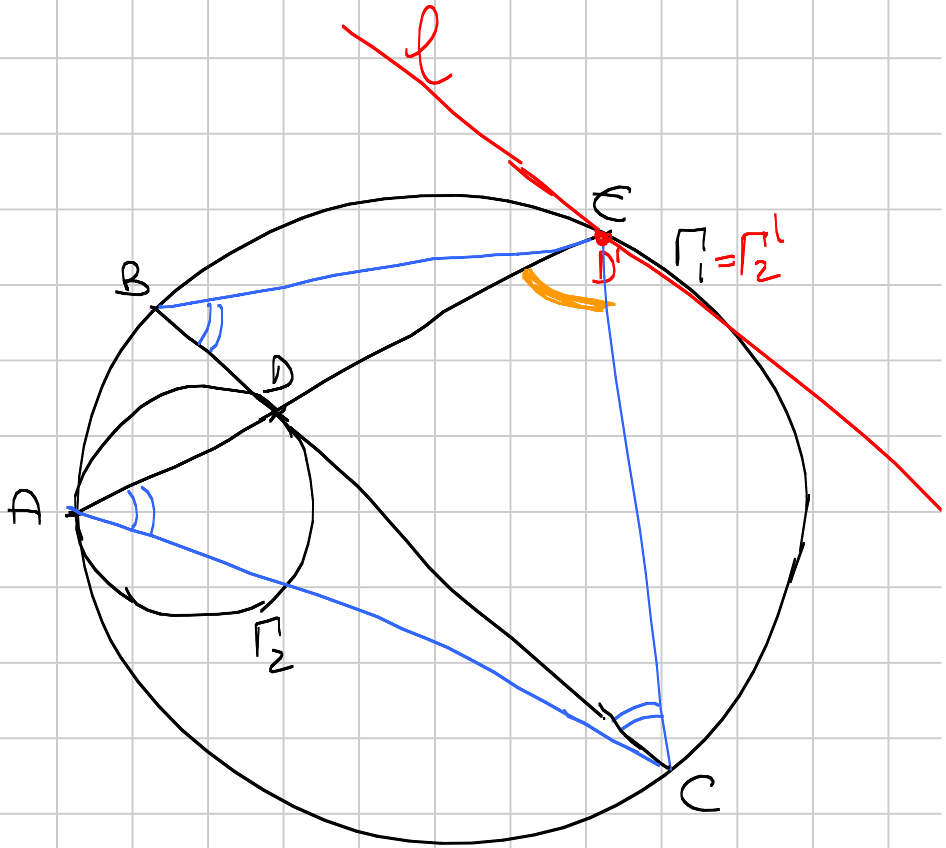
$BC \parallel \ell$

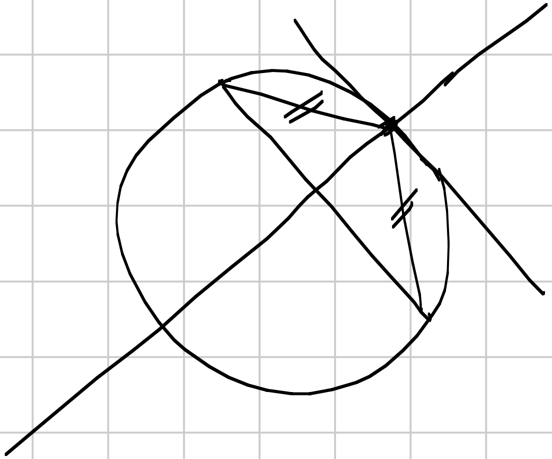
$\operatorname{pow}_{\Gamma_2}(E) = ED \cdot EA$

$\triangle ACE \sim \triangle CDE$

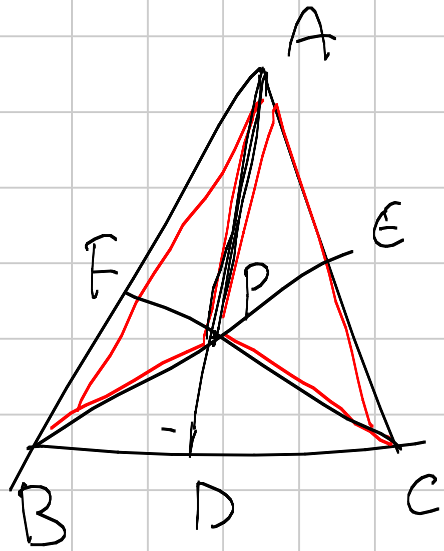
$$\frac{ED}{EC} = \frac{EC}{EA}$$

$\operatorname{pow}_{\Gamma_2}(E) = EC^2$





8



$$\frac{AF}{FB} = \frac{[APF]}{[FPB]} = \frac{[ACF]}{[FCB]} = \frac{[APC]}{[BPC]}$$

$$\frac{AE}{EC} = \frac{[APB]}{[BPC]}$$

$$\frac{AF}{FB} + \frac{AE}{EC} = \frac{[ABPC]}{[BPC]}$$

$$\frac{AP}{PD} = \frac{[ABPC]}{[BPC]} = \frac{AF}{FB} + \frac{AE}{EC}$$

SIMSON LINE

AB'PC'

BA'PC'

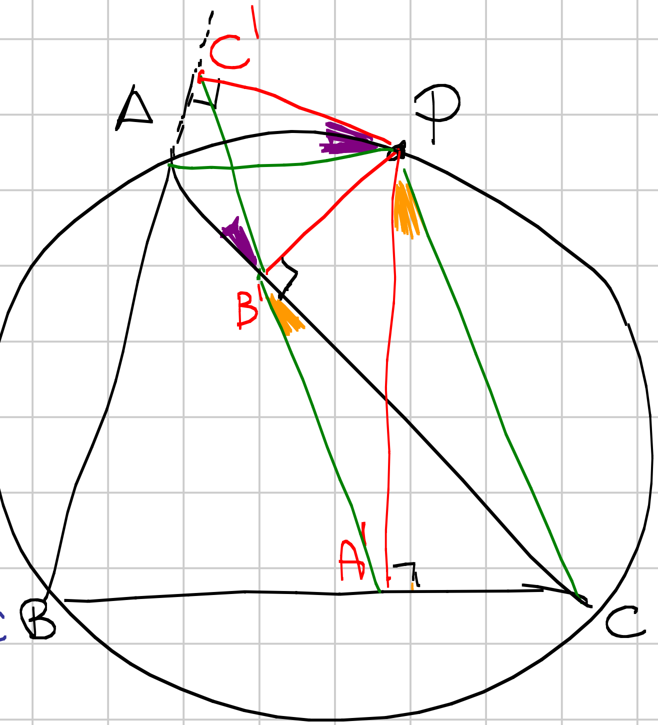
PB'A'C

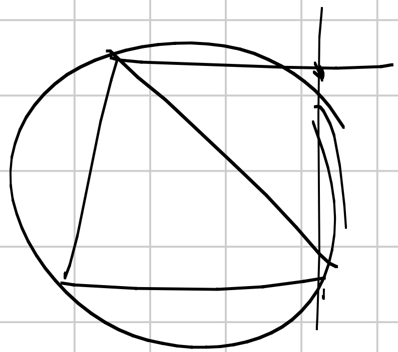
$PAC' = \underline{90 -}$

$PAC' = PCB = \underline{90 -}$

$PAC' = 180 - P\hat{A}B = PCB$ (ciclicità di ABCP)

$=$

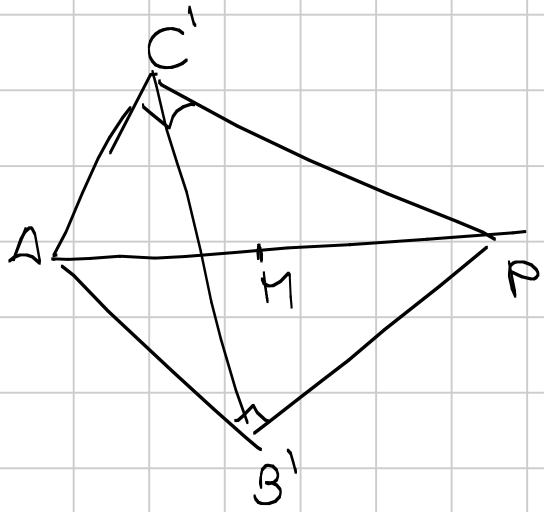


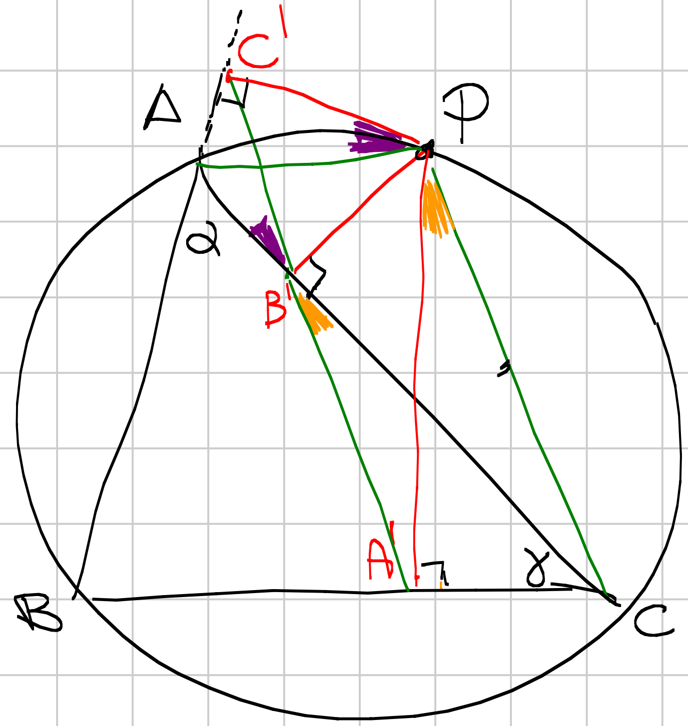


Ts: $AB \cdot PC + BC \cdot AP = BP \cdot AC$

$A'C' = A'B' + B'C'$

$$\frac{B'C'}{\operatorname{sen}\alpha} = 2\,\frac{AP}{2}$$





$$B'C' = AP \operatorname{sen}\alpha = \frac{BC}{2R} \cdot AP$$

$$A'B' = \frac{PC \cdot AB}{2R}$$

$$\frac{A'B'}{\operatorname{sen}\gamma} = PC$$

$$A'B' = PC \cdot \operatorname{sen}\gamma = \frac{AB}{2R} \cdot PC$$

$A'C' = \underline{AC \cdot PB}$

$2R$

$$\frac{AC \cdot PB}{\cancel{2R}} \leq \frac{PC \cdot AB}{\cancel{2R}} + \frac{BC \cdot AP}{\cancel{2R}}$$