GEOMETRIA PROIETTIVA - ESERCIZI

Titolo nota 06/09/2007

IRAN TST 2007 - 2° giorno

Sia ABC un triangolo, ω la cfr. inscritta, sia $PQ \parallel BC$ e tg a ω con $P \in AB$, $Q \in AC$. Siano D, E, F i punti di tg di ω sui lati e sia $T = EF \cap BC$. Sia infine M pt. medio di PQ. Allora TM tange ω.

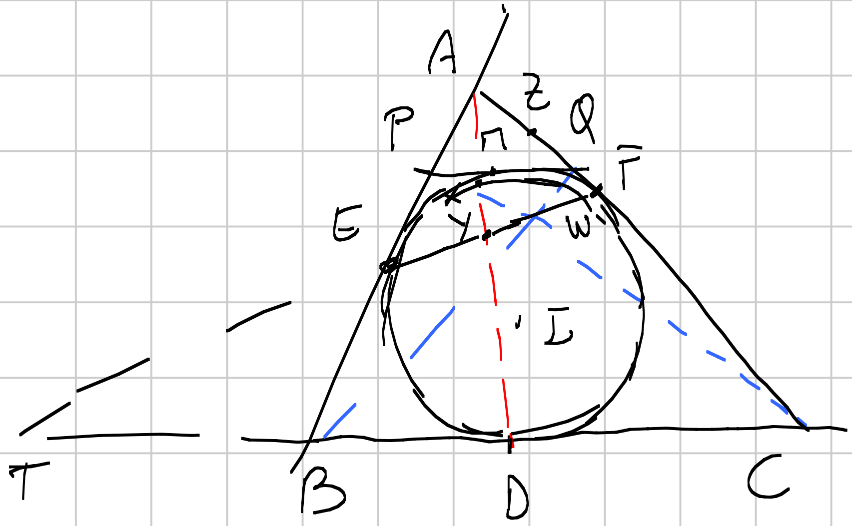
— o — o —

1 • Notare che $AD = pd_\omega(T)$ ⇒ riscrivo la tesi come $AD \cap TM \in \omega$

2 • Notare che $(B, C, D, T) = -1$ e usarlo tramite similitudini

3 • Provare a "mandare all' ∞" la parallela a BC per A

2) $(P, Q, M, U_\infty) = -1$ $\qquad PQ \cap \omega = J$



$X = AD \cap \omega \qquad Y = AD \cap EF$

$\boxed{PC, BQ, EF \text{ concorrono}}$

$TX \text{ tg } \omega$

$(A, Y, X, D) = -1$

$TX \cap AC = Z$

CA, CY, CX, CD

BA, BY, BX, BD

$\text{pol}(Y) = AT$

$\text{pol}(B) = DE$

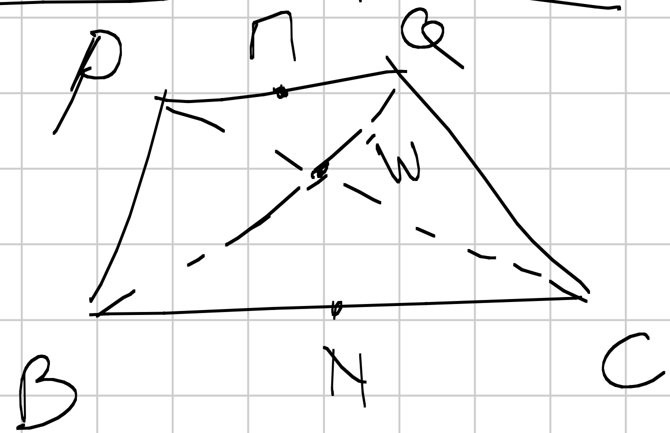
$\Pi' = TX \cap PQ$

$T \Pi' Q C$ circoscritto

$X J F D$

$\boxed{TQ, \Pi' C, XF, SD \text{ concorrono}}$

Oss estemporanea :



M, N pt medi

MNW allineati

$$(N, M, W, A) = -1$$

$$T(N, M, W\ A) = -1$$

intersecata con AD

$$(D, X_2, Y, A) = -1$$

$$(D, X, Y, A) = -1$$

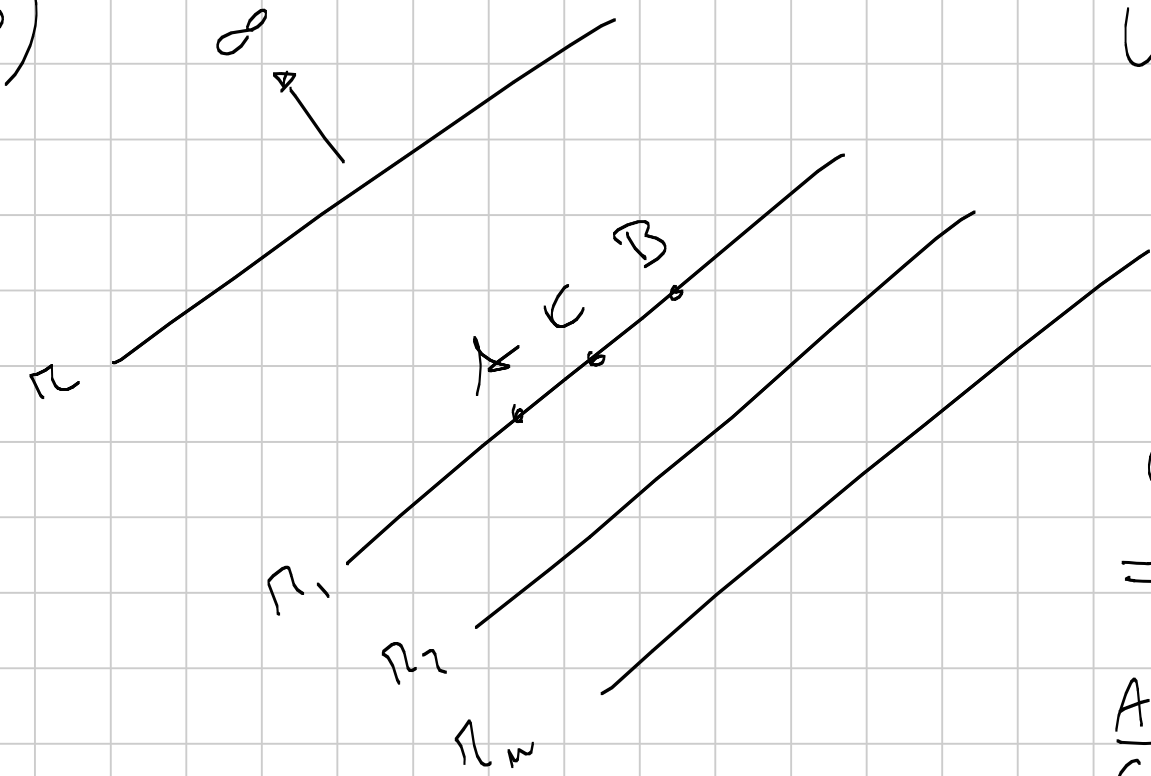
$X = TD \cap W$

$\underline{\cup}$

$TX \cap yw$

$X_1 = X$

3)



$U = r_i \cap r \in \{z=0\}$

se $n \to \infty$

U si muove all'∞

$(A, B, C, U) =$

$= (T(A), T(B), T(C), T(U))$

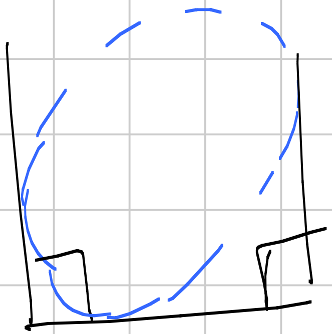
$$\frac{AC}{CB} = \frac{T(A)T(C)}{T(C)T(B)}$$

T t.c. $r \longrightarrow \{z=0\}$

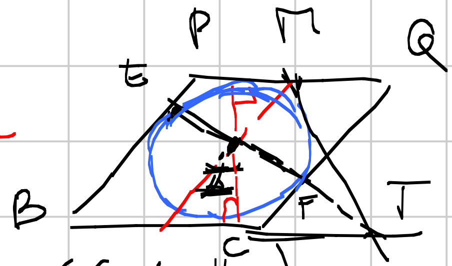
ℓ retta per A // BC

$\ell \longrightarrow \infty$ $\Rightarrow$ si conservano i rapporti su PQ, BC

$\Rightarrow$ M rimane pt medio di PQ



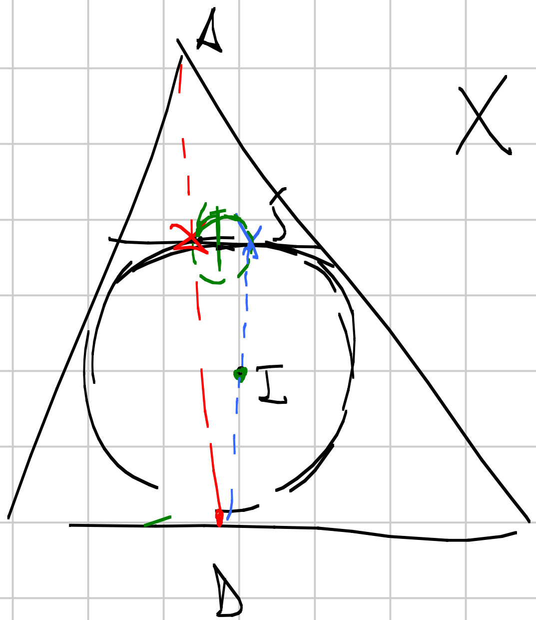
Oppure

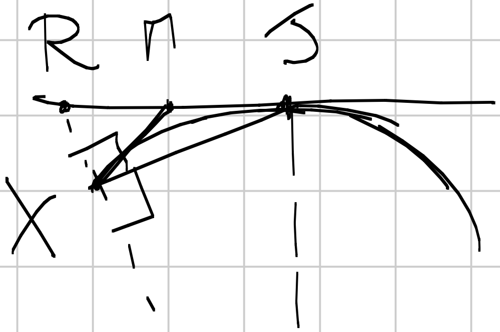


(Galeotti)

è //gramma $\Rightarrow$ è un rombo perché è circoscritto

1)





$X = AD \cap \omega$ vede DJ sotto $\frac{\pi}{2}$

$\widehat{DXJ} = \frac{\pi}{2}$ $\widehat{RXJ} = \frac{\pi}{2}$

$AD \cap PQ = R$ $X, J =$ cfr verde $\cap\, \omega$

$MX \perp IX \Rightarrow MX$ tg

TX tg $\Rightarrow T, X, M$ allineati

$\Rightarrow TM$ tangente

IMO Shortlist 2005 - G6

$\hat{ABC}$, M pt med di BC, γ cfr inscr.

$AM \cap \gamma = \{K, L\}$ X, Y su γ t.c.

$KX \parallel LY \parallel BC$. Siano $P = AX \cap BC$

$Q = AY \cap BC$. Allora $BP = CQ$.

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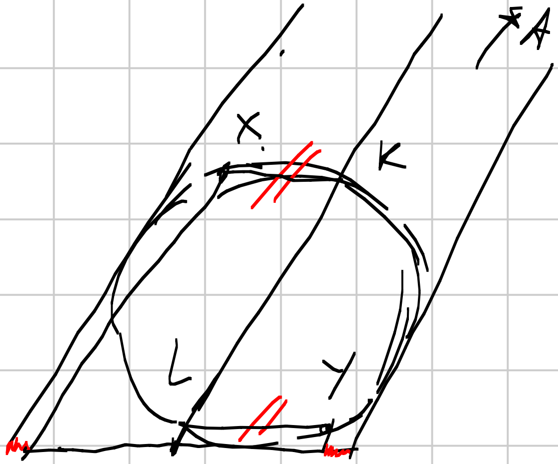
1. Mandare all'∞ una retta per $A \parallel BC$.
2. Birapporti
3. Trasf omot di centro A

1)



3) Γ circ per Π e P tg AC, AB in T, S

Γ = img di γ sotto inv omot di centro A

Π = img di K   P = img di X

$BΠ \cdot BP = BS^2$

$CP \cdot CΠ = TC^2$

$BS = AS - AB$

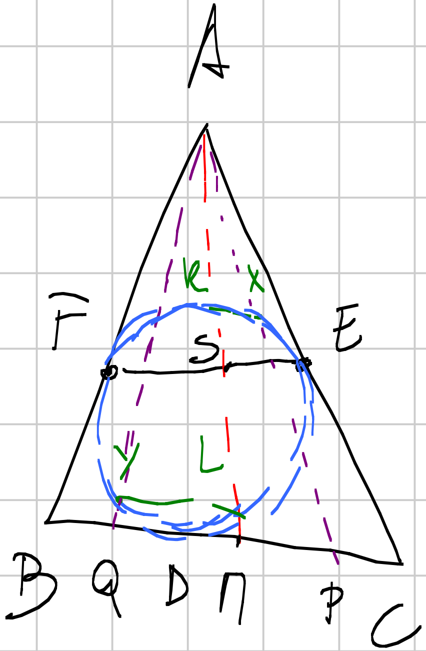
$TC = AT - AC$

$$\sqrt{CP} - \sqrt{BP} = \frac{b-c}{\sqrt{\frac{a}{2}}}$$

$$\sqrt{CQ} - \sqrt{BQ} = \frac{b-c}{\sqrt{\frac{a}{2}}}$$

EF, KL,

2)



$CP = BQ$

$$\boxed{DS \ni I}$$??

$S = EF \cap AM$

$(K, L, S, A) = -1$

I H piede dell'altezza da A

$$\frac{HD}{DM} = \frac{AS}{SM} \qquad \frac{HD}{DM} = \frac{b+c-a}{a}$$

$$\boxed{\frac{FB}{FA} \cdot MC + \frac{EC}{EA} DB = \frac{SM}{SA} BC}$$

Transversal theorem (?)

$\Downarrow$ $\dfrac{SM}{SA} = \dfrac{a}{b+c-a}$ $\Rightarrow$ D, S, I allineati

II $D : \left[0, \cos^2\frac{C}{2}, \cos^2\frac{B}{2}\right]$

$E : \left[\cos^2\frac{C}{2}, 0, \cos^2\frac{A}{2}\right]$ $\quad I : [1, 1, 1]$

$F : \left[\cos^2\frac{B}{2}, \cos^2\frac{A}{2}, 0\right]$ $\quad A : [1, 0, 0]$

$M : [0, c, b]$

$AM : \{ by - cz = 0 \}$

$EF : \left\{ -\cos^2\frac{A}{2}\, x + \cancel{\cos^2\frac{A}{2}}\cos^2\frac{B}{2}\, y + \cos^2\frac{C}{2}\cancel{\cos^2\frac{A}{2}}\, z = 0 \right\}$

$DI : \left\{ \left(\cos^2\frac{C}{2} - \cos^2\frac{B}{2}\right) x + \cos^2\frac{B}{2}\, y - \cos^2\frac{C}{2}\, z = 0 \right\}$

$DS \perp BC$ harmonic.

$$\boxed{\frac{AR}{AL} = \frac{SK}{SL}}$$

SEDI

$$\boxed{\frac{SK}{SL} = \frac{KX}{YL}}$$

$$\frac{AM}{AK} = \frac{MP}{KX} \qquad \frac{AL}{AM} = \frac{YL}{MQ}$$

$$\frac{SL}{SK} = \frac{AL}{AK} = \frac{MP}{MQ} \cdot \frac{YL}{KX} =$$

$$\| $$

$$\frac{YL}{KX}$$

$$\Rightarrow \frac{MP}{MQ} = 1$$

$A' =$ simm. di A risp. a DI

$$(S, A', X, Y) = -1$$

$$(AS, AA', AX, AY) = -1$$

$(M, U_\infty, P, Q) = -1 \Rightarrow M$ p.to med. di P, Q

Iran 2002 - round 3

ABC, γ inscr., A', B', C' tg di γ con i lati

P ult. int. di AA' con γ, M ult. int. di BP con γ

N ult. int. di CP con γ

Dim che AA', BN, CM concorrono.

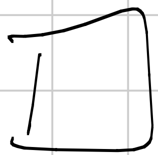
— ∘ — ∘ —

1. Desargues sui triangoli ABC $A'NM$,
2. Sapendo tanta teoria
3. Birapporti

2) ABC triangolo con Γ cfr circo.

$\exists\, T$ proiett. t.c. $T(\Gamma) = \Gamma$

e $T(ABC)$ è tri. equilatero

$ABC \xrightarrow{\text{affinità}}$ equil. $\xrightarrow{\text{“pro”}}$ 

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con. isogonale

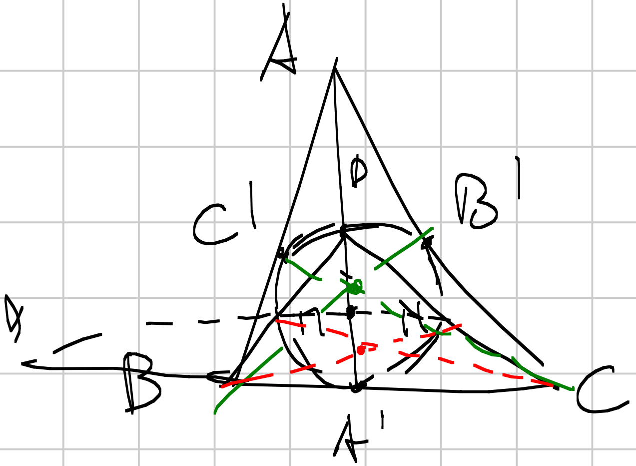
del con. isotomico

- Fisse i vertici
- Mande l'ellisse di Steiner nella cfr circoscritta.

1) Collineari sse $(\Leftarrow)$ $MN \cap BC$, $NA' \cap BA$, $MA' \cap AC$

$(\Rightarrow)$ $CB \cap MN$, $NA' \cap BP$, $MA' \cap CP$

$A'P \cap MN = X = pol\left(L\left(BP \cap A'N,\ CP \cap A'M\right)\right)$



$pol(X) \cap BC = pol(X) \cap pol(P)$

X, A', P sono all. sse $pol(X)$, $pol(A')$, $pol(P)$ concorrono.

$D = MN \cap BC$ $\qquad$ $K = BC \cap B'C'$

$(D, M, X, N) = (DP, PM, PX, PN) = (D, B, A', C)$

$AA' \cap C'B' = Z$ $\qquad$ $(C', B', Z, K) = -1$