Titolo nota 03/09/2007

$$11 \mid 2a + 3b \Rightarrow 11 \mid \underline{a^2 - 5b^2}$$

$$2a + 3b \equiv 0 \quad (11)$$

$$2a \equiv -3b \quad (11)$$

$$\underline{4a^2 \equiv 9b^2} \quad (11) \Rightarrow 4a^2 \equiv -2b^2$$

$$4a^2 \equiv -2b^2 \quad (11)$$

$$2a^2 \equiv -b^2 \quad (11)$$

$$4a^2 \equiv 9b^2$$

$$-2a^2 \equiv -10b^2 \Rightarrow a^2 \equiv 5b^2$$

$$4a^2 - 9b^2 \equiv 0 \quad (11)$$

$$12a^2 - 27b^2 \equiv 0 \quad (11)$$

$$a^2 - 5b^2 \equiv 0 \quad (11)$$

$\forall n \in \mathbb{Z}^+$

$$d_n = (100+n^2, 100+(n+1)^2) =$$

$$= (100+n^2, n^2+2n+1-n^2) =$$

$$= (100+n^2, 2n+1) =$$

$$= (100+n^2-2n^2-n, 2n+1) =$$

$$= (100-n-n^2, \underline{2n+1})$$

$$\begin{array}{l} d_n \mid 100+n^2 \\ d_n \mid 100-n-n^2 \end{array} \Rightarrow \begin{array}{l} d_n \mid 200-n \\ d_n \mid 2n+1 \end{array} \Rightarrow d_n \mid 401$$

$=$

$\hookrightarrow \quad (4m^2 + 400,\ 2m+1) =$

$= (2m+1,\ 401)$

$$\begin{array}{l|l} 4m^2 \qquad +400 & 2m+1 \\ \hline -4m^2 - 2m & 2m-1 \\ \quad - 2m + 400 & \\ \quad\ \ 2m + 1 & \\ \hline \qquad\ \ 401 & \end{array}$$

$$y^2 = x^5 - 4$$

| | | | |
|---|---|---|---|
| 11 | 0 | → | 0 |
| 10 | 1 | → | 1 |
| 9 | 2 | → | 4 |
| 8 | 3 | → | 9 |
| 7 | 4 | → | 5 |
| 6 | 5 | → | 3 |

$= \pm 1 - 4$

$-3 \equiv 8$

$-5 \equiv 6$

$-4 \equiv 7$

$-3 \qquad -3$

$x^{10} \equiv 1 \quad (11)$

$\sqrt{x^{10}} \equiv \pm 1$

$x^{10} \equiv 0$

$(p-1, 5) = 5 \qquad 5 \mid p-1$

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$x^{10} \equiv 1 \pmod{11}$

$x^5 \equiv \pm 1 \pmod{11}$

8

$x^2 \equiv 1 \pmod{8}$

$x \equiv \pm 1 \pmod{8}$

$x^{10} \equiv 1 \ (11)$ $\qquad$ $11 \mid x^{10}-1$

$11 \mid \underline{(x^5+1)}\,\underline{(x^5-1)}$

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$\boxed{p^4 \overset{?}{\equiv} 1}\ (5)$ $\quad 2 \rightarrow$ $5 \nmid p$

$$p^4 - q^4 \equiv 0 \quad (5)$$

$p^2 \equiv 1 \qquad q^2 \equiv 1$ $\qquad$ $p \equiv \pm 1 \quad 4$

$p^4 \equiv 1 \quad 4$

$p^2 - q^4 \equiv 1 - 1 \equiv 0$

$4 \cdot 5 \cdot 3$ $\boxed{60}$

$p = 11$ $q = 13$

$p^4 - q^4$

2, 3, 5, 7

$(p - q)($

37

$37 = 3\cancel{7}^4 - 11^4$

7

$p = 11$ $q = 13$

$$(p-q)(p+q)(p^2+q^2)$$

$$\boxed{\begin{matrix} 2 & \cdot & 24 & \cdot & 290 \\ 2 & & 3 \cdot 8 & \cdot & 29 \cdot 2 \cdot 5 \end{matrix}}$$

$d \qquad (17, 11)$

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$$\boxed{d} = MCD(p^4 - q^4)$$

$$13^4 - 11^4 = 2^5 \cdot 3 \cdot 5 \cdot \cancel{28}$$

$\equiv \equiv 1$

$29 \mid p^4 - q^4$ ?

$$29 \mid p^4 - \cancel{2q^4} + \cancel{2q^4}$$

$3 \mid$

$p^4 \equiv 1 \quad (5) \qquad p^4 - q^4 \equiv 1 - 1 \equiv 0$

$\equiv 1 \quad (16)$

$d = 2^4 \cdot 3 \cdot 5$

$17^4 - 11^4 = (17-11)(17+11)(17^2+11^2)$

1 2

$$\begin{array}{r} 289 \\ 121 \\ \hline 410 \end{array}$$

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$$\Longrightarrow \quad f(2n) = 2f(n)+1$$

$$\longrightarrow \quad f(2n+1) = 2f(n)$$

$$\longrightarrow \quad \begin{cases} \boxed{n}\,0 \xmapsto{f} \boxed{f(n)}\,1 \\ \boxed{n}\,1 \xmapsto{f} \boxed{f(n)}\,0 \end{cases}$$

0
1
2
3
4
5

5

| n | f(n) |
|---|---|
| ~~0~~ | ~~0~~ |
| 1 | 0 |
| 10 | 01 |
| 11 | 00 |
| 101 | 010 |
| 110 | 001 |

no2

$$\underbrace{10101010101}$$

1010101 _ _ 0