INDUZIONE

Titolo nota 02/09/2007

$$\left.\begin{array}{ll} P_0 & \text{PASSO BASE} \\ \forall n \quad P_n \Rightarrow P_{n+1} & \text{PASSO INDUTTIVO} \end{array}\right\} \Rightarrow \forall n \; P_n$$

$$\left.\begin{array}{l} P_k \\ \forall n \geq k \quad P_n \Rightarrow P_{n+1} \end{array}\right\} \Rightarrow \forall n \geq k \; P_n$$

$$\left.\begin{array}{l} P_0 \\ \forall n \quad P_0 \wedge P_1 \wedge \dots \wedge P_n \Rightarrow P_{n+1} \end{array}\right\} \Rightarrow \forall n \; P_n$$

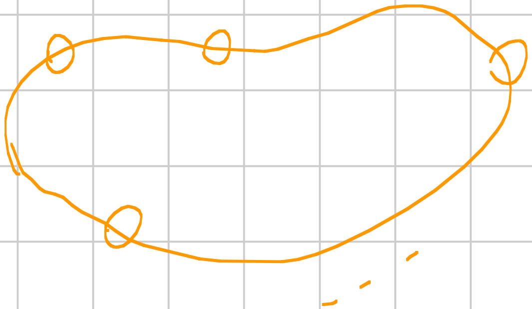
(INDUZIONE ESTESA)

$$1 + \ldots + n = \frac{n(n+1)}{2}$$

$$1 = \frac{1 \cdot 2}{2}$$

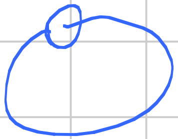
$$1 + \ldots + n + 1 = \frac{n(n+1)}{2} + n + 1 =$$

$$= \frac{n(n+1) + 2(n+1)}{2} = \frac{(n+1)(n+2)}{2}$$

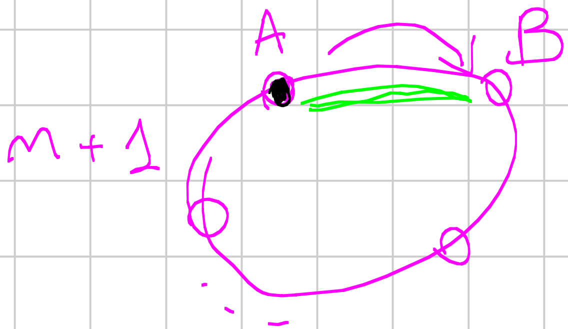


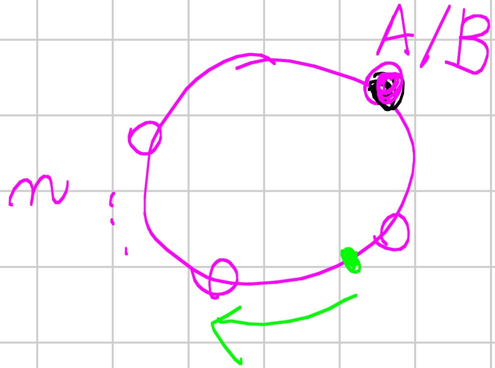
n macchine

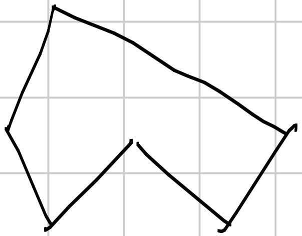
abbastanza carburante (in tutto) per 1 giro

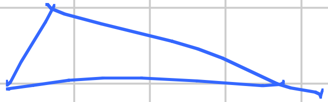


PASSO BASE









OK

n + 1 lati

$\leq n$ lati

$\leq n$ lati

(PICK)

$$A = I + \frac{B}{2} - 1$$

pti sul bordo

pti interni

(Bernoulli) $(1+x)^n \geq 1+nx$

$x > -1$

$n=0 \quad 1 \geq 1$

$$(1+x)^{n+1} \geq (1+x)(1+nx) =$$
$$= 1 + nx + nx^2 + x =$$
$$= 1 + (n+1)x + \underbrace{nx^2}_{\geq 0} \geq 1+(n+1)x$$

GIOCHI FINITO

1 mossa

- A vince
- A perde

$\leq n+1$ mosse

A muove $\rightarrow$ gioco con $\leq n$ mosse

- B vince
- B perde

(Minimo intero) $S \subseteq \mathbb{N}$ $\quad S \neq \emptyset$ $\quad S$ ha un minimo

$\# S = 1$ ok

$\# S = n+1$

$x < \min \rightarrow x$

$x > \min \Rightarrow \min$

n $\quad x$

min

per assurdo $T \subseteq \mathbb{N}$ $\quad T \neq \emptyset$ $\quad T$ non ha minimo

$0 \notin T$

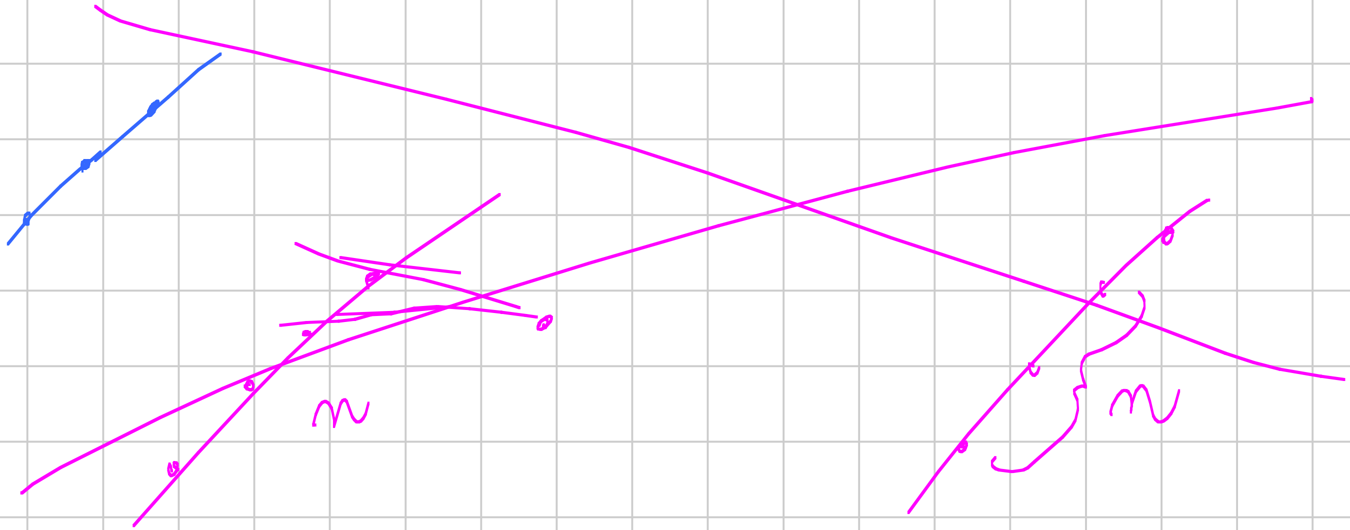
$n \notin T$

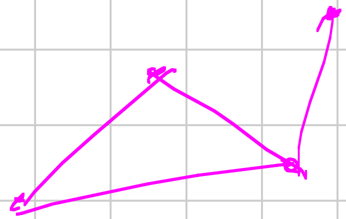
$0, 1, \ldots, n \notin T \Rightarrow n+1 \notin T$

$\forall n \quad n \notin T \Rightarrow T = \emptyset$

(Sylvester) n punti nel piano

$\forall$ coppia la retta che li unisce ne contiene almeno un terzo $\Longrightarrow$ tutti i pti sono allineati.





(Discesa infinita) Una successione decrescente (debolmente) di naturali è costante da un certo punto in poi.

$$(a^2+b^2) = 3(x^2+y^2)$$

$a = 3a_1$

$b = 3b_1$

$$(3a_1)^2 + (3b_1)^2 = 3(x^2+y^2)$$

$$3(a_1^2+b_1^2) = (x^2+y^2)$$

(a, b, x, y) $\Downarrow$ $|a|+|b|+|x|+|y|$ minimo, >0

$\left(x, y, \frac{a}{3}, \frac{b}{3}\right)$ è sol.

PIGEONHOLE

n cassetti, $n+1$ oggetti $\Longrightarrow$ $\exists$ un cassetto con almeno 2 oggetti.

n cassetti, $nk+1$ oggetti $\Longrightarrow$ $\exists$ un cassetto con almeno $k+1$ oggetti.

n interi $a_1 \dots a_n$

$\exists$ un sottinsieme tc $n \mid$ somma

$S_1 = a_1$

$S_2 = a_1 + a_2$

$\vdots$

$S_n = a_1 + \dots + a_n$

$\exists\, i$ tc $n \mid S_i$

$\exists\, i, j$ tc $n \mid S_i - S_j$

(Dirichlet) α irrazionale $\exists$ infinite frazioni $\frac{p}{q}$ ridotte ai minimi termini t.c.

$$\boxed{\left|\alpha - \frac{p}{q}\right| < \frac{1}{q^2}}$$

$\forall\, n$

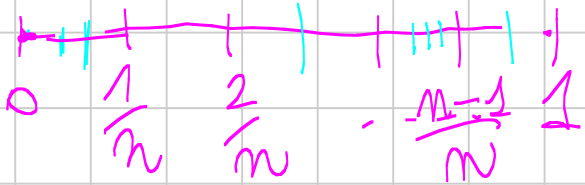
$\alpha,\ 2\alpha \ \ldots\ n\alpha$

$\{0\cdot\alpha\}, \{\alpha\} \ \ldots\ \{n\alpha\},$

$h, k : \left|\{h\alpha\} - \{k\alpha\}\right| < \frac{1}{n}$

$a = [h\alpha]$

$b = [k\alpha]$



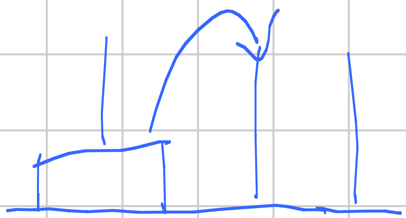
$$\left|k\alpha - b - h\alpha + a\right| < \frac{1}{n} \Rightarrow \left|(k-h)\alpha + a - b\right| \leq \frac{1}{n}$$

$$\left| \alpha - \frac{b-a}{k-h} \right| < \frac{1}{n|k-h|} \leq \frac{1}{(k-h)^2}$$

$$n \geq k-h$$

(Hanoi)

Per risolvere una torre di Hanoi con n dischi sono necessarie (e sufficienti) $2^n - 1$ mosse.



ok (1 disco)

per $n+1$ dischi $\quad 2^n + 2^n - 1 = 2^{n+1} - 1$

$1 \dots 2n$

$a_1 < a_2 < \dots a_i < a_n$

$b_1 > b_2 > \dots b_i \dots > b_n$

$$|a_1 - b_1| + |a_2 - b_2| + \dots + |a_n - b_n| = n^2$$

$\forall i = 1 \dots n$ $\quad a_i \in [1 \dots n] \Rightarrow b_i \geq n+1$

$a_i \geq n+1 \Rightarrow b_i \leq n$

$i + n - i + 1 = n + 1$ (piccioni)

n (tane)

$n+1 + n+2 + \dots + 2n - (1 + 2 + \dots + n) =$

$$= n + \ldots + n = n^2$$