A 2

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Titolo nota

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DISUGUAGLIANZE

$$- - - - \leq - - - - \leq - - - -$$

$$2(ab+bc+ca) \leq 2(ab+bc+ca)+a^2+b^2+c^2 \leq$$

$$\leq (a+b+c)^2 \leq 3(a^2+b^2+c^2)$$

DISUGUAGLIANZE "BASE"

$x^2 \geq 0$ $\qquad a^2+b^2 \geq 2ab$ $\qquad (a-b)^2 \geq 0$

$$\frac{a}{b}+\frac{b}{a} \geq 2$$

$$ab+bc+ca \leq a^2+b^2+c^2$$

RIARRANGIAMENTO

$\sigma : \{1, \dots n\} \longrightarrow \{1 \dots n\}$ permutazione

$$\sum_{i=1}^{n} a_i b_{\sigma(i)}$$

$$\boxed{\begin{array}{l} a_1 \le a_2 \le \dots \le a_n \\ b_1 \le \dots\dots \le b_n \end{array}}$$

$\to$ <u>MAX</u>

1 — 1
2 — 2
3 — 3
⋮

<u>MIN</u>

1 — n
2 — n-1
⋮
n — 1

$$a_1 b_3 + a_2 b_2 + a_3 b_1 \le$$

$$\le a_1 b_2 + a_2 b_1 + a_3 b_3 \le a_1 b_1 + a_2 b_2 + a_3 b_3$$

RIARRANGIAMENTO FUNZIONA ANCHE SE

a_i .. b_i <u>NON</u> SONO POSITIVI

<u>DIM</u> : 2 elementi

$$a_1b_1 + a_2b_2 \overset{?}{\geq} a_2b_1 + a_1b_2$$

$$\underset{\substack{\vee\!\!\mid \\ 0}}{(a_2-a_1)}\,\underset{\substack{\vee\!\!\mid \\ 0}}{(b_2-b_1)} \overset{?}{\geq} 0$$

__DIM__ n elementi

$$\underbrace{a_1b_1+\dots+a_{k-1}b_{k-1}}_{k-1 \text{ sono a posto}} + a_k b_{k+l} + \boxed{\dots} + a_{k+l} b_k + \boxed{\dots}$$

(eventualmente nessuno)

$$\leq \boxed{} + a_k b_k + \boxed{} + a_{k+l} b_{k+l}$$

$$a_k b_k + a_{k+l} b_{k+l} \geq a_k b_{k+l} + a_{k+l} b_k$$

$$\Updownarrow$$

$$(a_{k+l} - a_k)(b_{k+l} - b_k) \geq 0$$

$$a_1 b_2 + a_2 b_3 + a_3 b_1 \leq a_1 b_1 + a_2 b_3 + a_3 b_2 \leq$$

$$\leq a_1 b_1 + a_2 b_2 + a_3 b_3$$

$$ab + bc + ca \leq a^2 + b^2 + c^2$$

$$\leq a \cdot a + b \cdot b + c \cdot c$$

<u>Ex</u> $a, b, c < 1$

$$\frac{\boxed{a}}{\boxed{1-b}} + \frac{\boxed{b}}{\boxed{1-c}} + \frac{\boxed{c}}{\boxed{1-a}} \leq \frac{\boxed{a}}{\boxed{1-a}} + \frac{\boxed{b}}{\boxed{1-b}} + \frac{\boxed{c}}{\boxed{1-c}}$$

$$a \cdot \frac{1}{1-b} + b \cdot \frac{1}{1-c} + c \cdot \frac{1}{1-a} \leq$$

$$\cancel{a \geq b \geq c}$$

$$\frac{b}{1-a} + \ldots\ldots$$

$$a \longleftrightarrow \frac{1}{1-a}$$

$$b \longleftrightarrow \frac{1}{1-b}$$

$$c \longleftrightarrow \frac{c}{1-c}$$

$$a \leq b \longleftrightarrow \frac{1}{1-a} \leq \frac{1}{1-b}$$

$$1-a \geqslant 1-b$$

$$\frac{1}{1-a} \not\leq \frac{1}{1-b}$$

$$\sum a_i b_{\sigma(i)} c_{\tau(i)} \leq \sum a_i b_i c_i$$

se $0 < a_1 \leq \dots \leq a_n$ $\quad 0 < b_1 \leq \dots b_n$ $\quad 0 < c_1 \leq \dots c_n$

" $\leq$ " (CONSIGLIO)

AM - GM

$a_1, \dots, a_n \geq 0$

$$\sqrt[n]{a_1 \cdot a_2 \cdot \dots \cdot a_n} \leq \frac{a_1 + a_2 + \dots + a_n}{n}$$

e vale l'= se e solo se sono tutti uguali

$$a_1 + a_2 + \dots + a_n = n \cdot AM$$

Quando $a_1 \cdot \dots \cdot a_n$ è maggiore?

(IDEA DI COME ANDRÀ A FINIRE

$$a_1 \cdot \dots \cdot a_n \leq AM \cdot AM \cdot \dots \cdot AM$$

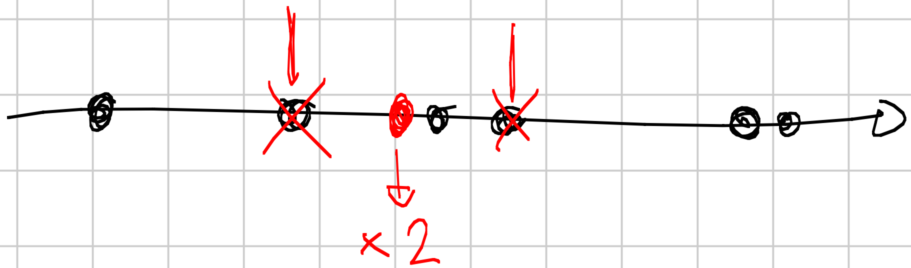
$$GM^n \leq AM^n$$

$$x, y \longrightarrow \frac{x+y}{2}, \frac{x+y}{2}$$

$$\cancel{a_1} \dots x \dots y \dots \cancel{a_n} \longrightarrow \cancel{a_1} \dots \frac{x+y}{2} \dots \frac{x+y}{2} \dots \cancel{a_n}$$

$$x \cdot y \leq \left(\frac{x+y}{2}\right)\left(\frac{x+y}{2}\right)$$

$$4xy \leq x^2 + y^2 + 2xy \qquad \text{VERA}$$



$$1 : 3 : 4$$

$$\wedge\!|$$

$$1 : \frac{7}{2} : \frac{7}{2}$$

$$\wedge\!|$$

$$\frac{9}{4} : \frac{9}{4} : \frac{7}{2}$$

$$\wedge\!|$$

$$\frac{9}{4} : \frac{25}{8} : \frac{25}{8}$$

$$\vdots$$

$\frac{8}{3} \quad \frac{8}{3} \quad \frac{8}{3}$

NON FUNZIONA

VERSIONE CHE FUNZIONA

$$a_1 + \ldots\ldots + a_n = n \cdot AM$$

CI SONO SICURAMENTE $x \leq AM \leq y$

$a_1 \gtrless AM$

$a_2 \geq AM$

$\vdots$

$$a_1 + \ldots + a_n \geq n \cdot AM$$

$a_1 \; \dots \; x \qquad y \; \dots \; a_n \qquad\qquad x \leq AM \leq y$

$AM \qquad x+y-AM$

$$xy \leq AM \cdot \underline{(x+y-AM)}$$

$$* \quad (AM-x)(y-AM) \geq 0$$

$$\downarrow$$

$$(x+y)\cdot AM \geq xy + AM^2$$

$$\downarrow$$

$$\boxed{(x+y-AM)AM \geq xy}$$

PRIMA DOPO

$$a_1 \dots x \dots y \dots a_n \leq a_1 \dots \underline{AM} \dots (x+y-AM) \dots a_n ?$$

$$\leq \quad \dots \quad \leq \quad \dots \quad \leq$$

TERMINA perché a ogni passo un a_i in più mi diventa uguale ad AM

= solo se "non c'è nulla da fare"

cioè solo se $a_1 = a_2 = \dots = a_n = AM$

$$\sqrt[3]{x^2y^2z^2} \cdot 3 \leq \frac{x^2+y^2+z^2}{3} \cdot 3$$

MIN $x+2y+3z$ QUANDO $x^3y^2z=1$

$$3 \cdot \frac{x+2y+3z}{3} \geq \sqrt[3]{x \cdot 2y \cdot 3z}$$

$$\frac{x}{3}+\frac{x}{3}+\frac{x}{3}+y+y+3z \geq 6\sqrt[6]{\frac{x^3y^2z\cdot 3}{3\cdot 3\cdot 3}}$$

$$x+2y+3z \geq \frac{6}{\sqrt[6]{9}}\sqrt{x^3y^2z} = 1$$

$$\frac{x}{3}=\frac{x}{3}=\frac{x}{3}=y=y=3z$$

$$s=\frac{x}{3}=y=3z$$

$$1=(3s)^3 s^2\cdot\frac{s}{3}=9s^6$$

$$s=\sqrt[6]{1/9}$$

$$x = 3 \cdot \sqrt[6]{1/9} \qquad y = \sqrt[6]{1/9} \qquad z = \frac{\sqrt[6]{1/9}}{3}$$

(BOOKLET - 4 SI FA NELLO STESSO MODO)

Media p-esima:

$a_1, \dots a_n > 0$

$$\sqrt[p]{\frac{a_1^p + a_2^p + \dots + a_n^p}{n}}$$

<u>TEOREMA</u> se fisso $a_1, \dots a_n$ e vario p,

questa cosa è crescente

$$\underset{a_i}{\text{MIN}} \leq \underbrace{\frac{n}{\frac{1}{a_1}+\dots+\frac{1}{a_n}}}_{\text{HM},\ p=-1} \leq \underbrace{\sqrt[n]{a_1\cdots a_n}}_{\text{GM},\ p=0} \leq \underbrace{\frac{a_1+\dots+a_n}{n}}_{\text{AM},\ p=1} \leq \underbrace{\sqrt{\frac{a_1^2+\dots a_n^2}{n}}}_{\text{QM},\ p=2} \leq \underbrace{\sqrt[3]{\frac{a_1^3+\dots+a_n^3}{n}}}_{p=3} \leq \underset{a_i}{\text{MAX}}$$

GM "prende il posto" di $p=0$

(seguono da convessità)

1) CASO DI UGUAGLIANZA

Sono equivalenti:

1) vale = in una qualsiasi

2) valgono tutti gli uguali

3) gli a_i sono tutti uguali tra loro

2) $$\frac{n}{\frac{1}{a_1}+\dots+\frac{1}{a_n}} \leq \frac{a_1+\dots+a_n}{n}$$ HM-AM

$$n^2 \leq (a_1+\dots+a_n)\left(\frac{1}{a_1}+\dots+\frac{1}{a_n}\right)$$

DIMOSTRAZIONE ALTERNATIVA: $\frac{a}{b}+\frac{b}{a} \geq 2$

$$9 \leq (a+b+c)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$$ (FACILE)

$$9 \geq \dots\dots$$ (DIFFICILE)

(SE VI ANNOIATE DURANTE LA LEZIONE:
COMINCIATE A FARE $\boxed{9}$ E $\boxed{10}$
(A2)

CAUCHY - SCHWARZ

$a_1, \dots, a_n, \quad b_1 \dots b_n$ ANCHE NEGATIVI

$$\sum a_i b_i \leq \sqrt{\left(\sum a_i^2\right)} \cdot \sqrt{\left(\sum b_i^2\right)}$$

$$\left|\begin{pmatrix} a_1 \\ \vdots \\ a_n \end{pmatrix} \cdot \begin{pmatrix} b_1 \\ \vdots \\ b_n \end{pmatrix}\right| \leq \left\|\begin{pmatrix} a_1 \\ \vdots \\ a_n \end{pmatrix}\right\| \cdot \left\|\begin{pmatrix} b_1 \\ \vdots \\ b_n \end{pmatrix}\right\|$$

NORME

↓

PRODOTTO SCALARE

$$\|a\|\cdot\|b\| \geq \|a\|\cdot\|b\|\cdot \cos \widehat{ab} = a\cdot b$$

$$\cos \widehat{ab} \leq 1$$

DIMOSTRAZIONE CS:

Inventarsi somme di quadrati

$$0 \leq \sum_i \left(a_i t - b_i\right)^2$$

$$0 \leq \sum \left(a_i^2 t^2 - 2 a_i b_i t + b_i^2 \right)$$

$$"B^2 \leq 4AC"$$

$$4\left(\sum a_i b_i\right)^2 \leq 4\left(\sum a_i^2\right) \cdot \left(\sum b_i^2\right)$$

CAUCHY-SCHWARZ

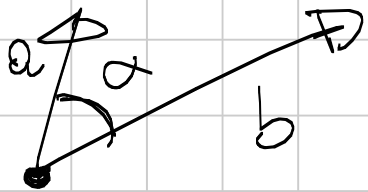
$(a_i \cdot t - b_i) = 0$ per ogni i

$a_i \cdot t = b_i$ per ogni i

$$\begin{pmatrix} a_1 \\ \vdots \\ a_n \end{pmatrix} = t \cdot \begin{pmatrix} b_1 \\ \vdots \\ b_n \end{pmatrix} \qquad t \geq 0$$

UGUAGLIANZA QUANDO C'È $t \geq 0$ tale che

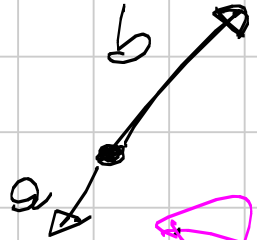
$$t \cdot \vec{a} = \vec{b}$$



$$a \cdot b \leq \|a\| \cdot \|b\|$$

$$\|a\| \cdot \|b\| \cdot \cos\alpha \leq \|a\| \cdot \|b\|$$

UGUALE SE $\cos\alpha = 1$



$\cos\alpha = -1$

$$(a_1 b_1 + a_2 b_2) = \sqrt{a_1^2 + a_2^2} \cdot \sqrt{b_1^2 + b_2^2}$$

$$-(a_1 b_1 + a_2 b_2) = \sqrt{a_1^2 + a_2^2} \cdot \sqrt{b_1^2 + b_2^2}$$

OMOGENEIZZAZIONE / DISOMOGENEIZZAZIONE

$$ab+bc+ca \leq a^2+b^2+c^2$$

$$ab+bc+ca \leq a^3+b^3+c^3 \quad (*)$$

MAI

SE è vera per tutte le terne di reali

Fisso A, B, C

DEVE VALERE PER $t \cdot A, \; t \cdot B, \; t \cdot C$ (QUALUNQUE t)

$$t^2 \underbrace{(AB+BC+CA)}_{\text{NUMERO}} \leq t^3 \underbrace{(A^3+B^3+C^3)}_{\text{NUMERO}} \quad (*)$$

PER t GRANDE, $t^3 \geq \square\, t^2$

PER t PICCOLO $t^2 \geq \square\, t^3$

QUINDI (*) NON VALE PER TUTTI GLI

$a, b, c \geq 0$

$$(*) \quad \frac{1}{a^3+b^3+1} + \frac{1}{b^3+c^3+1} + \frac{1}{c^3+a^3+1} \leq 1$$

$$\boxed{abc = 1}$$

$$\Uparrow \qquad\qquad \Downarrow$$

$$(**)\quad \frac{1}{a^3+b^3+abc}+\frac{1}{b^3+c^3+abc}+\frac{1}{c^3+a^3+abc}\leq\frac{1}{abc}$$

TH $(*)$ È VERA PER TUTTI GLI A, B, C

CON $ABC=1$ (e $A, B, C > 0$)

SE E SOLO SE

$(**)$ È VERA PER TUTTI GLI $A, B, C > 0$

$$(*)\quad \frac{1}{a^3+b^3+1}+\frac{1}{b^3+c^3+1}+\frac{1}{c^3+a^3+1}\leq 1$$

$$\Downarrow \Uparrow \qquad \boxed{abc=1}$$

$$(**)\quad \frac{1}{a^3+b^3+abc}+\frac{1}{b^3+c^3+abc}+\frac{1}{c^3+a^3+abc}\leq\frac{1}{abc}$$

$\underline{\text{TR}}$: $\forall\, a, b, c$ vale $(**)$ $\underrightarrow{\text{PRENDO}}$ (a, b, c)

e considero $(*)$ (ta, tb, tc)

con t tale che $ta\cdot tb\cdot tc=1$, $t=\sqrt[3]{\frac{1}{abc}}$

$$\frac{1}{t^3a^3+t^3b^3+\cancel{1}_{\,t^3abc}}+\ \dots\ \geq \cancel{1}\ \frac{1}{t^3abc}\qquad \boxed{ta\cdot tb\cdot tc=1}$$

$$\cancel{\frac{1}{t^3}}\left(\frac{1}{a^3+b^3+abc}+\dots\right)\geq\cancel{\frac{1}{t^3}}\cdot\frac{1}{abc}$$

E QUESTA ERA LA TESI

$$a^3+b^3+c^3+a^2+b^2+c^2 \overset{?}{\geq} 5$$

HOM $\Big\downarrow$ su $abc=1$

$$a^3+b^3+c^3+(a^2+b^2+c^2)\sqrt[3]{abc} \overset{?}{\geq} 5abc$$

DATA UNA DISUGUAGLIANZA + VINCOLO, POSSO <u>SEMPRE</u> OMOGENEIZZARLA (GRATIS)

DATA UNA DISUGUAGLIANZA <u>OMOGENEA</u>

$$ab+bc+ca \leq a^2+b^2+c^2$$

$$t^2(ab+bc+ca) \leq t^2(a^2+b^2+c^2)$$

POSSO IMPORRE UN VINCOLO "UTILE"

$$\frac{1}{c}+\frac{1}{a}+\frac{1}{b} \leq a^2+b^2+c^2 \quad \text{SU} \quad abc=1$$

OPPURE

$$ab+bc+ca \leq 1 \quad \text{SU} \quad a^2+b^2+c^2=1$$

$$\boxed{9,\ 10}$$

NON POSSO IMPORRE COME VINCOLO

$$\frac{b}{a} + \frac{c}{b} + \frac{a}{c} = 10$$

TIIM SE HAI $f(a,b,c) \overset{?}{\geq} 0$ OMOGENEA

$$f(ta,tb,tc) = t^{\square} \cdot f(a,b,c)$$

PUOI DIMOSTRARLA PER UN SOLO VALORE DI t

CIOÈ: PUOI SCEGLIERE $abc = 1$

FISSATI a, b, c, $\quad ta\, tb\, tc = 1$

SCELGO T TALE CHE

FISSATI A, B, C

NON POSSO SCEGLIERE t IN MODO CHE $\frac{a}{b}+\frac{b}{c}+\frac{c}{a}=10 \Rightarrow \frac{\cancel{t}a}{\cancel{t}b}+\frac{\cancel{t}b}{\cancel{t}c}+\frac{\cancel{t}c}{\cancel{t}a}=10$

$$\sum \frac{x_i}{\sqrt{1-x_i}} \overset{?}{\geq} \sqrt{\frac{n}{n-1}} \qquad \text{con } \sum x_i = 1$$

a_i, b_i crescenti

$$\geq \frac{\sum a_i b_i}{n} \geq \frac{\sum a_i}{n} \cdot \frac{\sum b_i}{n}$$

CS CHEB

[PER CASA: DIMOSTRA CHEBYSHEV:

PRENDI TUTTI I POSSIBILI RIARRANGIAMENTI [TUTTE LE PERMUTAZIONI] $\le a_1 b_1 + \dots + a_n b_n$

E SOMMI

$$\sum \frac{x_i}{\sqrt{1-x_i}} \overset{?}{\geq} \sqrt{\frac{n}{n-1}} \qquad \text{con } \sum x_i = 1$$

$a_i = x_i$
$\Updownarrow$
$b_i = \frac{1}{\sqrt{1-x_i}}$

$$\frac{LHS}{\cancel{n}} \geq \frac{\overset{\overset{1}{\shortparallel}}{\sum x_i}}{\cancel{n}} \cdot \frac{\sum \frac{1}{\sqrt{1-x_i}}}{n} =$$

$$= \frac{\sum \frac{1}{\sqrt{1-x_i}}}{n} \geq$$

$$c_i = \frac{1}{\sqrt{1-x_i}} \qquad 1-x_i = c_i^{-2}$$

$$\frac{\sum c_i}{n} \geq \left(\frac{\sum 1-x_i}{n} \right)^{-\frac{1}{2}} =$$

AM

$$= \left(\frac{n-1}{n} \right)^{-1/2} = \text{RHS}$$

$$\sum \frac{x_i}{\sqrt{1-x_i}} \overset{?}{\geq} \sqrt{\frac{n}{n-1}} \qquad \text{con } \sum x_i = 1$$

$$\underline{\sum (1-x_i)^{-1/2}} = \underline{\sum (1-x)^{1/2}} \geq$$

$$p=1,\ p=-\frac{1}{2} \quad \frac{n-1}{n} = \frac{\sum (1-x_i)}{n} \geq \left(\frac{\sum (1-x_i)^{-\frac{1}{2}}}{n}\right)^{-2} = \frac{n^2}{\left(\sum (1-x_i)^{-\frac{1}{2}}\right)^2}$$

$$\left[\sum (1-x_i)^{-\frac{1}{2}}\right]^2 \geq \frac{n^2 \cdot n}{n-1}$$

$$\frac{n-1}{n} = \frac{\sum (1-x_i)}{n} \geq \left(\frac{\sum (1-x_i)^{1/2}}{n} \right)^2$$

come viene in mente la sostituzione?

$$\sqrt{1-x_i} = \sin \alpha_i \qquad \cos^2(\alpha_i) \stackrel{\cdot}{=} x_i$$

$$\sum \cos^2(\alpha_i) = 1$$

$$\frac{x_i}{\sqrt{1-x_i}} = \frac{\cos^2 \alpha_i}{\sin \alpha_i} = \frac{1-\sin^2 \alpha_i}{\sin \alpha_i} = \frac{1}{\sin \alpha_i} - \sin \alpha_i =$$

$$= \frac{1}{\sqrt{1-x_i}} - \sqrt{1-x_i}$$

$$\sum \frac{1}{\sin\alpha_i} - \sin\alpha_i \geq 1$$

SAPENDO CHE

$$\sum \cos^2\alpha_i = 1$$

$$\Updownarrow$$

$$\sum \sin^2\alpha_i = n-1$$

$\boxed{10}$ NESBITT

$$\frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b} \geq 3/2$$

$a+b = x$, $b+c = y$, $c+a = z$

[IDEA: NUMERATORI DIFFICILI
DENOMINATORI FACILI È BENE]

[A CASA: APPLICATE A (9)]

$$\begin{cases} a+b = x \\ b+c = y \\ c+a = z \end{cases}$$

$$y+z = b+2c+a$$

$$y+z-x = 2c \qquad c = \frac{y+z-x}{2}$$

$$a = \frac{z+x-y}{2}$$

$$b = \frac{y+x-z}{2}$$

$$\frac{x+y-z}{z}+\frac{y+z-x}{x}+\frac{z+x-y}{y}\overset{?}{\geq}3$$

$$\frac{x}{z}+\frac{y}{z}-1+\frac{y}{x}+\frac{z}{x}-1+\frac{z}{y}+\frac{x}{y}-1\overset{?}{\geq}3$$

s somma $\frac{x}{z}+\frac{z}{x}\geq 2$ $\quad \frac{y}{z}+\frac{z}{y}\geq 2$ $\quad \frac{y}{x}+\frac{x}{y}\geq 2$

USO "MODERNO" DI C-S

$$\sum a_i b_i \leq \dots\dots$$

$$\left(\sum a_i^2\right) \geq \frac{\left(\sum a_i b_i\right)^2}{\left(\sum b_i^2\right)}$$

$$a_1 = \sqrt{\frac{a}{b+c}} \qquad b_1 = \sqrt{b+c}\,\sqrt{a}$$

$$\sqrt{\frac{b}{c+a}} \qquad b_2 = \sqrt{c+a}\,\sqrt{b}$$

$$\sqrt{\frac{c}{a+b}} \qquad b_3 = \sqrt{a+b}\,\sqrt{c}$$

$$(a+b+c)^2 \leq LHS \cdot \big(a(b+c)+b(c+a)+c(a+b)\big)$$

$$LHS \geq \frac{(a+b+c)^2}{2(ab+bc+ca)} \geq \frac{a^2+b^2+c^2}{2(ab+bc+ca)} + \frac{2(ab+bc+ca)}{2(ab+bc+ca)}$$

$$\frac{1}{2} \geq \quad = 1$$

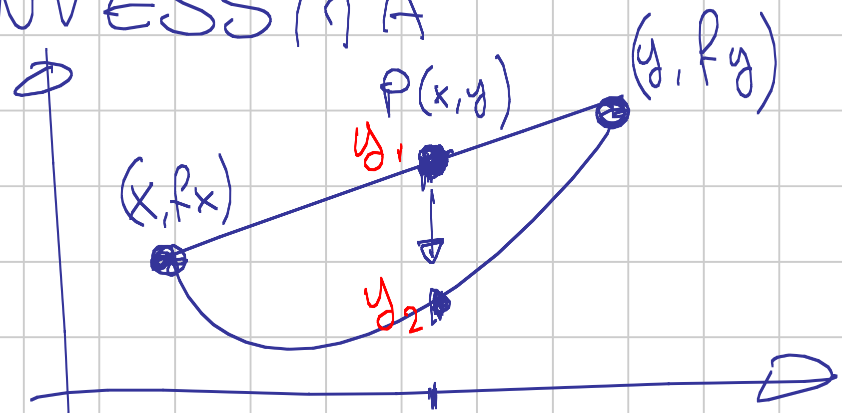
$$a^2+b^2+c^2 \geq ab+bc+ca$$

$$a_i = \sqrt{a+b}$$

$$b_i = \frac{1}{\sqrt{a+b}}$$

[PER CASA PROVATE]

CONVESSITÀ



$$\lambda \cdot \begin{pmatrix} x \\ f_x \end{pmatrix} + (1-\lambda) \cdot \begin{pmatrix} y \\ f_y \end{pmatrix}$$

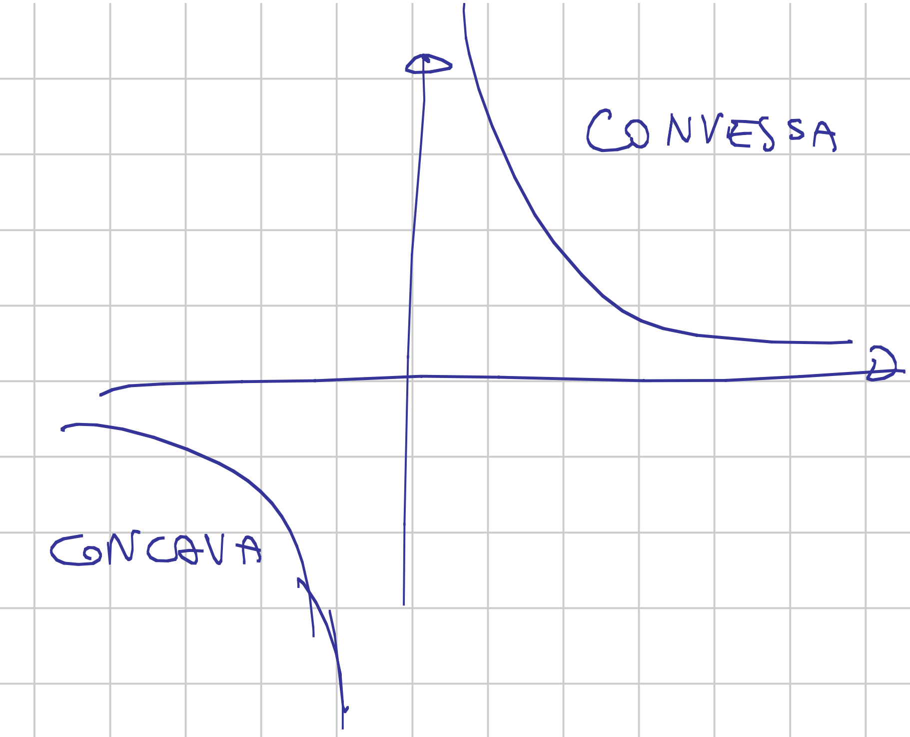
$$f(\overbrace{\lambda x + (1-\lambda) y}^{y_2}) \leq \overbrace{\lambda f(x) + (1-\lambda) f(y)}^{y_1}$$

DEFINIZIONE DI CONVESSITÀ

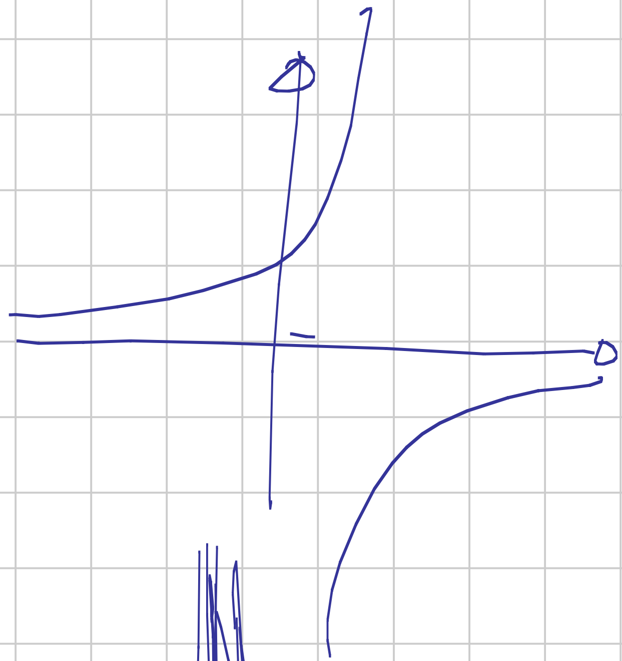
$f''(x) \geq 0 \iff f$ CONVESSA ($\in C^2$)

$f(x) = \frac{x}{1-x}$ è convessa o no?

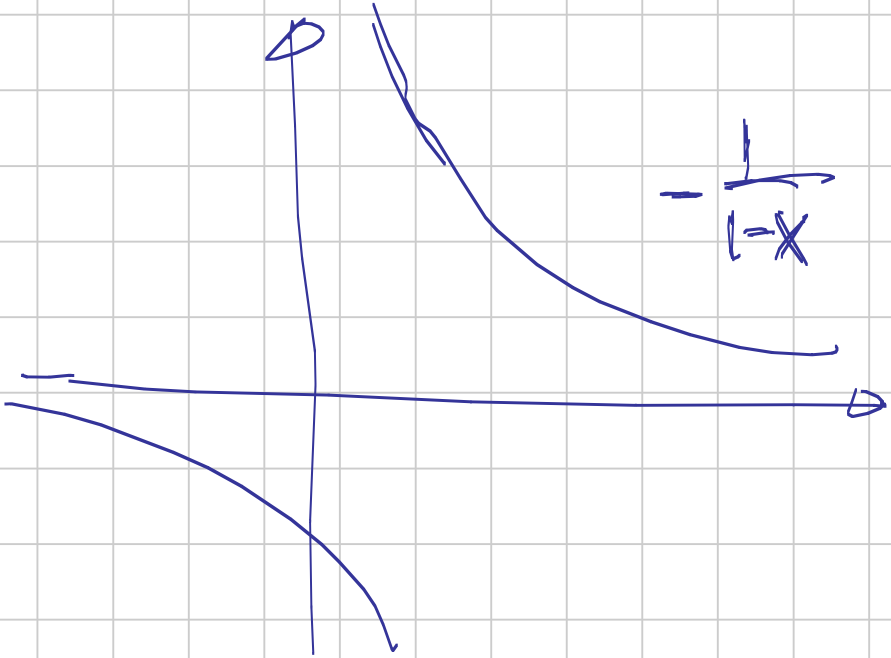
$$\frac{x}{1-x} = 1 - \frac{1}{1-x}$$



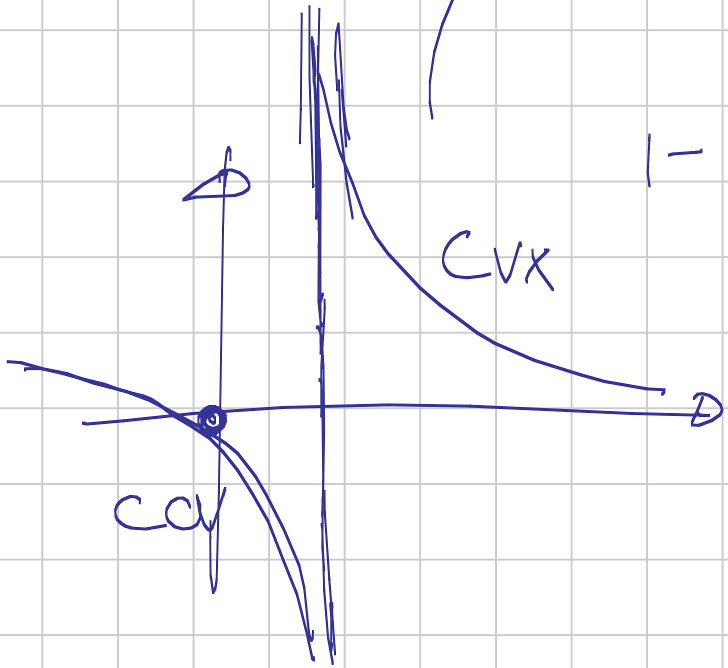
$1/x$



$\frac{1}{1-x}$



$-\frac{1}{1-x}$

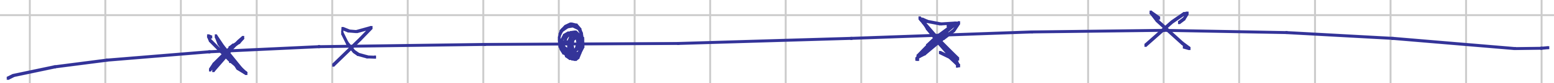


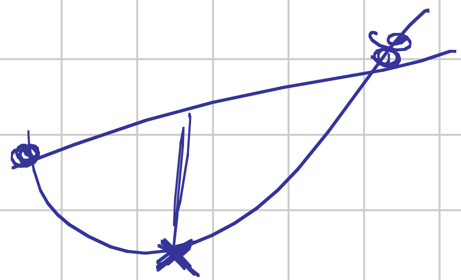
$1-\frac{1}{1-x}$

$\dfrac{x}{1-x}$ è convessa per $x>1$ e concava per $x<1$

JENSEN: f CONVESSA, ALLORA $x_1, \dots x_n$ NEL DOMINIO DELLA F.

$$f\left(\frac{x_1+x_2+\dots+x_n}{n}\right) \leq \frac{\sum f(x_i)}{n}$$





10)

$$\frac{a}{b+c}+\frac{b}{c+a}+\frac{c}{a+b} \overset{?}{\geq} \frac{3}{2}$$

IDEA: DISOMOGENEIZZIAMO $\quad a+b+c=1$

$$\frac{\boxed{a}}{1-a}+\frac{\boxed{b}}{1-b}+\frac{\boxed{c}}{1-c} \overset{?}{\geq} \frac{3}{2}$$

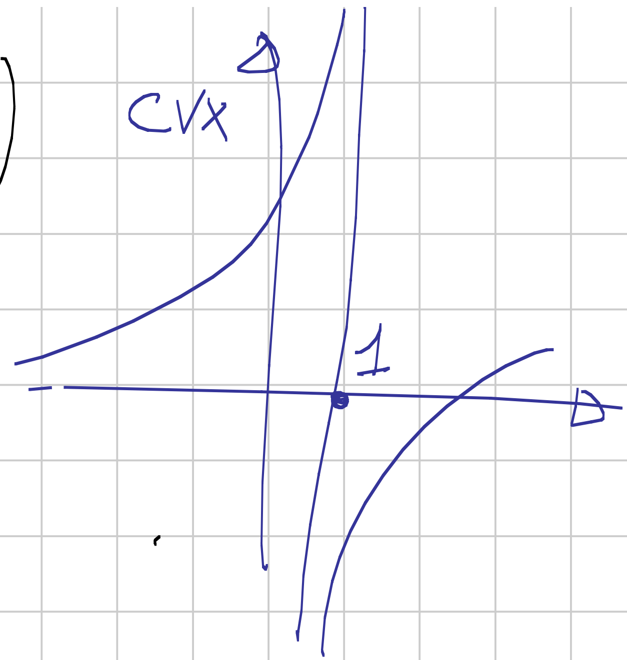
+ APPLICO JENSEN A $\quad f(x)=\dfrac{x}{1-x}$

ALTRA IDEA

JENSEN SU $\quad f(x)=\dfrac{1}{1-x}$

$$a\cdot f(a)+b f(b)+c f(c) \geq f(\underbrace{a^2+b^2+c^2})$$

$$\frac{1}{1-a^2+b^2+c^2}$$



$a+b+c$ NOT ∞

$a^2+b^2+c^2 \geq \dots ?$

QM AM

4

9A 9B

10A 10B

(10C)