VENERDÌ POMERIGGIO A3

Titolo nota 12/09/2008

$$(a^2+b^2)(c^2+d^2) = (ac-bd)^2 + (ad+bc)^2$$

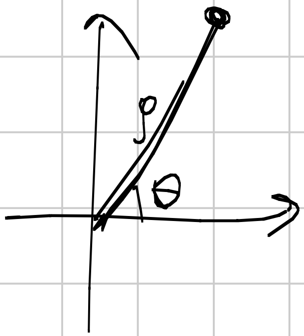
$a+ib$

$c+id$

$$|(a+ib)(c+id)|^2 = |ac-bd+i(bc+ad)|^2$$

$$\| \qquad = (ac-bd)^2+(bc+ad)^2$$

$$|a+ib|^2 \cdot |c+id|^2$$



$$\omega = \rho \cdot e^{i\theta}$$

$$|\alpha||\beta| = |\alpha\beta|$$

$$\left|\rho_1 \cdot e^{i\theta_1} \cdot \rho_2 e^{i\theta_2}\right| = \boxed{\rho_1 \cdot \rho_2} \, e^{i(\theta_1+\theta_2)}$$

MODULO

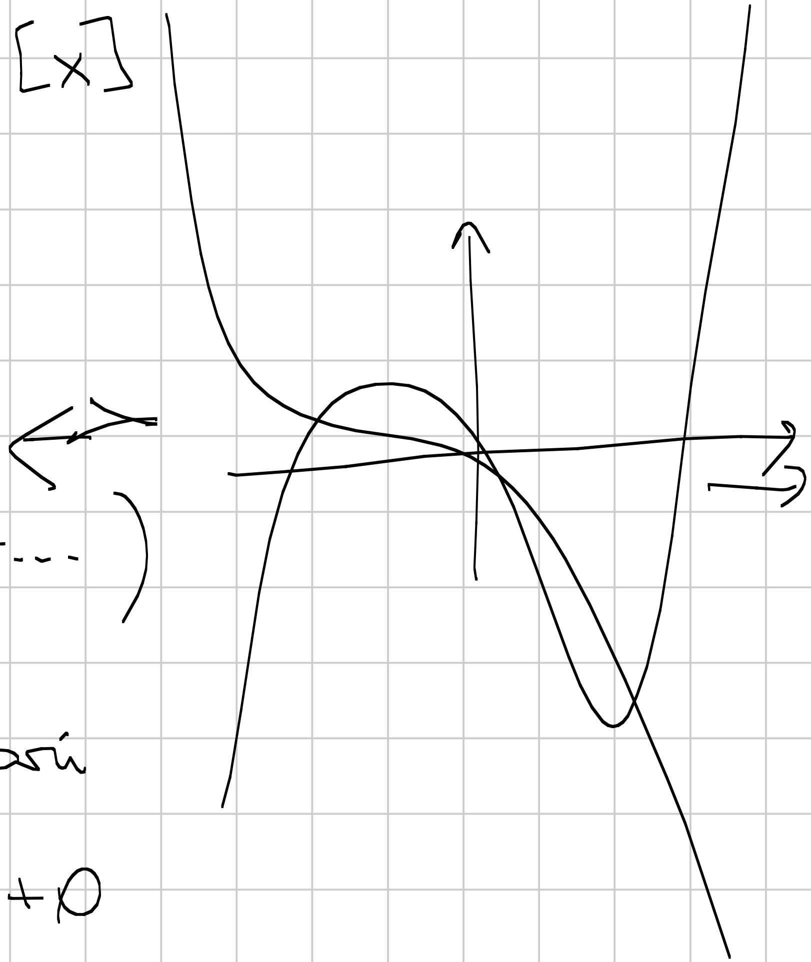
Esempio: sia $P(x)$ $\mathbb{R}[x]$

$P(x) \geq 0 \ \forall x$

$P(x) = p(x)^2 + q(x)^2$

$\sum^{n} a_i x^i = x^n \left(a_n + a_{n-1} \frac{1}{x} + \dots \right)$

$+$



sicuramente P ha grado pari

$P(x) \geq 0 \ \forall x \Rightarrow (x-\pi)^2 + 0$

$x^2 + bx + c$

$\left(x + \frac{b}{2}\right)^2 + \underbrace{c - \frac{b^2}{4}}_{+}$ $= \left(x+\frac{b}{2}\right)^2 + d^2$

$+$ positiva $= d^2$

$$P(x) = (\quad)^2(\quad)^2 \cancel{\prod (x-a_i)} \prod (x^2+c_i x+d_i)$$

$p(x)^2 \mid P(x)$ seconda parte

$p(x)^2 + q(x)^2$

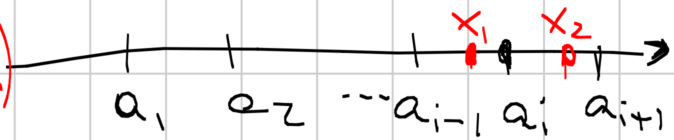
$(x_1 - a_{i-1})$

$(x_2 - a_{i-1})$

$P(x_1) = (x_1-a_1)\dots(x_1-a_m)$

stessa

$P(x_2) = (x_2-a_1)\dots(x_2-a_m)$



(x_1-a_i) è <0

x_2-a_i è >0

$\prod p_i(x)$

$P_1 \cdot P_2$ = somma di polinomi ciascuno al quadrato

$= q(x)^2 + r(x)^2$

$$x^n - y^n = (x-y)(x^{n-1} + \dots + y^{n-1})$$

$n =$ dispari $\quad x^n + y^n = (x+y)(x^{n-1} - \dots + y^{n-1})$

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$= 1 + 2 + 3 + 4 + \dots + n-2 + n-1 + n$$

$$= n+1 + n+1 + n+1 \dots$$

$$= (n+1) \cdot \frac{n}{2}$$

$$= \frac{n-1}{2}(n+1) + \frac{n+1}{2} = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

Dim:

1. Induzione

2. $(n+1)^3 - 1^3 = \sum_{k=1}^{n} (k+1)^3 - k^3 = \sum (3k^2 + 3k + 1)$

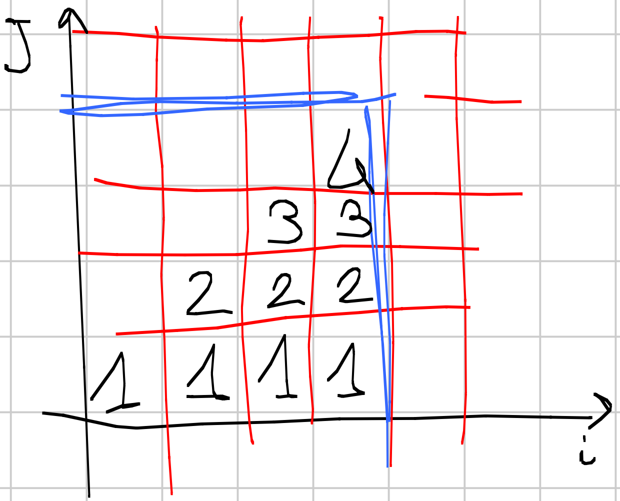
$$= \left[(n+1)^3 - \cancel{n^3}\right] + \left[\cancel{n^3} - (n-1)^3\right] - \ldots$$

$$= 3 \sum k^2 + 3 \sum k + \sum 1$$

$$= 3 \; I + 3 \; \frac{n(n+1)}{2} + n$$

$$\frac{(n+1)^3 - 1 - 3 \; \frac{n(n+1)}{2} - n}{3} = \sum_{k=1}^{n} k^2$$

3.



$1 +$

$1+2 +$

$1+2+3 +$

$1+2+3+4 +$

$1+1+1+1 +$

$2+2+2 +$

$3+3 +$

$4 +$

$\sum i^2$

$$\sum_{i=1}^{m}\left(\sum_{j=1}^{i} j\right) = \sum_{i=1}^{m}\frac{(i+1)i}{2} = \frac{1}{2}\sum i^2 + \frac{1}{2}\sum i$$

$$\sum_{j=1}^{m}\sum_{i=j}^{m} j = \sum_{j=1}^{m} j(m+1-j) =$$

$$= -\sum j^2 + (m+1)\sum j$$

$$\frac{1}{2}\sum i^2 + \frac{1}{2}\sum i = -\sum i^2 + (m+1)\sum i$$

$$\frac{3}{\cancel{2}}\sum i^2 = \frac{2m+1}{\cancel{2}}\sum i \Rightarrow \sum i^2 = \frac{2m+1}{3}\cdot\frac{m(m+1)}{2}$$

$$\sum i^3 = \left[\frac{n(n+1)}{2}\right]^2$$

$$\sum_{i=1}^{n} i^k = P(n)$$

$$\sum \binom{n}{i} = 2^n$$

$$\overbrace{1 \quad 2 \quad 3 \quad \dots}^{n} \;\Big|\; n+1 \dots 2n$$

<u>Esercizio</u>

$$\sum_{i=0}^{n} \binom{n}{i}^2 = \binom{2n}{n}$$

$\binom{2n}{n}$ = sottoinsiemi di n elementi di un ins di $2n$ elementi

$$\sum \binom{n}{i}^2 = \binom{n}{0}\binom{n}{n} + \binom{n}{1}\cdot\binom{n}{n-1} + \dots$$

$$\binom{2n}{n} = \binom{n}{0}^2 + \binom{n}{1}^2 + \dots$$

SUCCESSIONI

$$a_k = 1 \quad \forall k$$

$$a_k = k^2$$

$$\begin{cases} a_0 = 5 \\ a_1 = 3 \\ a_{n+1} = a_n + 7a_{n-1} + n^5 + 3 \end{cases}$$

$$\begin{cases} a_1 = 1 \\ a_{n+1} = (n+1)\, a_n \end{cases} \Longrightarrow \quad a_n = n!$$

$$\sum i$$

$$a_1 = \alpha \qquad a_2 = \alpha + r \qquad a_3 = \alpha + 2r$$

$$a_{i+1} - a_i = r$$

$$a_1, \dots, a_n$$

$$\underbrace{\alpha \qquad\qquad \alpha + nr}$$

$$\sum_{i=1}^{n} a_i = (a_1 + a_n) \cdot \frac{n}{2}$$

$$(a_1 + a_n) \cdot \frac{n-1}{2} + \frac{a_1 + a_n}{2}$$

PROGRESSIONI GEOMETRICHE

$$a_1 = \alpha \qquad a_2 = \alpha r \qquad a_3 = \alpha r^2 \quad \dots \quad a_n = \alpha r^{n-1}$$

$$\sum_{i=1}^{n} a_i = \alpha + \alpha r + \alpha r^2 + \dots + \alpha r^{n-1}$$

$$r \sum_{i=1}^{n} a_i = \alpha r + \alpha r^2 + \dots + \alpha r^{n-1} + \alpha r^n$$

$$(r-1) \sum_{i=1}^{n} \alpha r^{i-1} = -\alpha + \alpha r^n$$

$$\sum_{i=1}^{n} \alpha r^{i-1} = \alpha \frac{r^n - 1}{r-1}$$

$r > 1 \qquad \alpha \sum r^{i-1} \quad n \to \infty$ diverge

$$r < 1 \qquad \alpha \sum r^{i-1} = \alpha \frac{-1}{r-1} = \alpha \frac{1}{1-r}$$

Esercizio

$$b_{n+1} = (n+1)\, b_n - n\, b_{n-1} \quad , \quad b_1 \text{ e } b_2 \text{ interi relativi.}$$

$\forall K$ intero positivo la succ è definitivamente costante mod K

$$b_1 , \dots b_n \dots$$

$$d_n = b_{n+1} - b_n \equiv 0 \pmod{K} \quad \forall\, n > \bar{n}$$

$$b_{n+1} - b_n = n\,(b_n - b_{n-1})$$

$$d_n = n \cdot d_{n-1}$$

$$d_n = n! \cdot d_0$$

$$d_K = K!\; d_0$$

divisibile x K

SUCC X RICORR LINEARI

$x_{n+1} = a x_n + b$, fissato x_0

$x_1 = a x_0 + b$

$x_2 = a(a x_0 + b) + b = a^2 x_0 + ab + b$

$\vdots \qquad a(ab+b) + b = a^2 b + ab + b$

$x_n = a^n x_0 + b(a^{n-1} + \dots + a + 1) =$

$= a^n x_0 + b \dfrac{a^n - 1}{a - 1}$ SE $a \neq 1$

SE $a = 1$:

$x_1 = x_0 + b$

$x_2 = x_0 + b + b = x_0 + 2b$

$\vdots$

$x_n = x_0 + nb$

SUCC X RIC DA 2 TERMINI PRECEDENTI

$$\begin{cases} x_0 \\ x_1 \quad \text{fissati} \\ \boxed{x_{n+2} = \alpha x_{n+1} + \beta x_n} \end{cases}$$

$$\begin{cases} F_0 = 0 \\ F_1 = 1 \\ F_{n+2} = F_{n+1} + F_n \end{cases}$$

b_n e a_n sono succ. che soddisfano

$S_n = a_n + b_n$

$$\underline{a_{n+2}} + b_{n+2} = \underline{\alpha (a_{n+1}} + b_{n+1}) + \underline{\beta (a_n} + b_n)$$

λa_n

$a_n = x^n$

$$x^{\cancel{n+2}} = \alpha x^{n+1} + \beta \cancel{x^n}$$

$$x^2 - \alpha x - \beta = 0$$

chiamiamo r_1 e r_2 le radici

$$x_n = a r_1^n + b r_2^n$$

$$\begin{cases} x_0 = a + b = x_0 \\ x_1 = a r_1 + b r_2 = x_1 \end{cases} \qquad b = \frac{x_1 - r_1 x_0}{r_2 - r_1}$$

$x^2 - x - 1$

FIBONACCI: $x_n = \frac{1}{\sqrt{5}} \left\{ \left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n \right\}$

$$x^2 - \alpha x - \beta = (x - r)^2$$

$$\alpha = 2r \quad \beta = -r^2$$

$x_n = r^n$

$x_n = n r^n$

$x_n = a r^n$

$$x_{n+2} = 2r x_{n+1} - r^2 x_n$$

$$(n+2)\cancel{r^{n+2}} = \cancel{2}r(n+1)\cancel{r^{n+1}} - \cancel{r^2}\, n\, \cancel{r^n}$$

x_0

x_1

$$x_n = a r^n + b n r^n = (a+nb) r^n$$

$$x_0 = a = 1$$

$$x_1 = (a+b) r$$

STUDIO QUALITATIVO

$$\begin{cases} a_{n+1} = \dfrac{a_n^2+3}{4} \\ a_0 = 2 \end{cases}$$

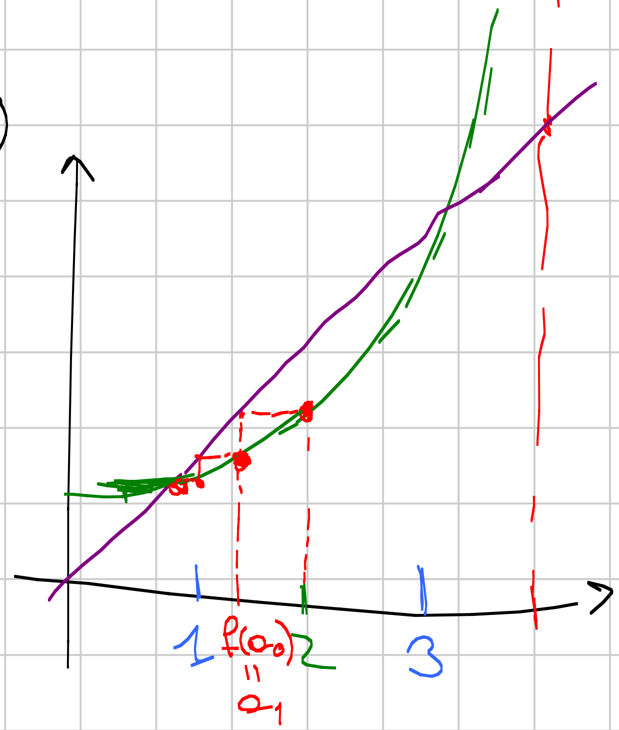
$$a_1 = f(a_0)$$

$$a_2 = f(a_1)$$

$$f(x) = \frac{x^2+3}{4}$$

$$f(x) = x \qquad x^2+3 = 4x$$

$$x = 1 \qquad x = 3$$



Es 9 libretto

$$x_{n+2} = x_{n+1} + x_n \qquad x_0 = 0 \qquad x_1 = 1$$

Dimostrare che $4091 \mid x_{4091} - 1$

x_1, x_2 le radici di $x^2 - x - 1$

$x^2 - x - 1 \mod 4091$

$$x_n = c\, \cancel{x_1}^n + d\, \cancel{x_2}^n$$

$$x_{4091} = c \cdot \cancel{x_1}^{4091} + d\, \cancel{x_2}^{4091} \equiv c \cdot \cancel{x_1} + d\, \cancel{x_2} = x_1$$

$$\equiv 1 \quad (4091)$$

$$c^2 \equiv 5 \quad (4091) \qquad r_1 = \frac{1+\sqrt{5}}{2} \overset{1,64}{} \, , \quad \frac{1-\sqrt{5}}{2} \overset{64}{} = r_2$$

$64 = 4096$

$(1+64)\, 2^{-1}$

$\sqrt{-1} = i \qquad \sqrt{5} = k$

$$\boxed{\begin{array}{ll} x_0 = 1 & x_{n+1} = 4x_n - 4x_{n-1} \\ x_1 = 3 & \end{array}}$$

$$x_n = a \cdot 2^n$$

$$\begin{cases} x_0 = a \cdot 1 = 1 & \Rightarrow a = 1 \\ x_1 = a \cdot 2 & \Rightarrow a = \frac{3}{2} \end{cases}$$

assurdo

EQUAZIONI FUNZIONALI

$f(x+y) = f(x) + f(y)$ (eq. di Cauchy)

$f(x - f(y)) = 1 - x - y$

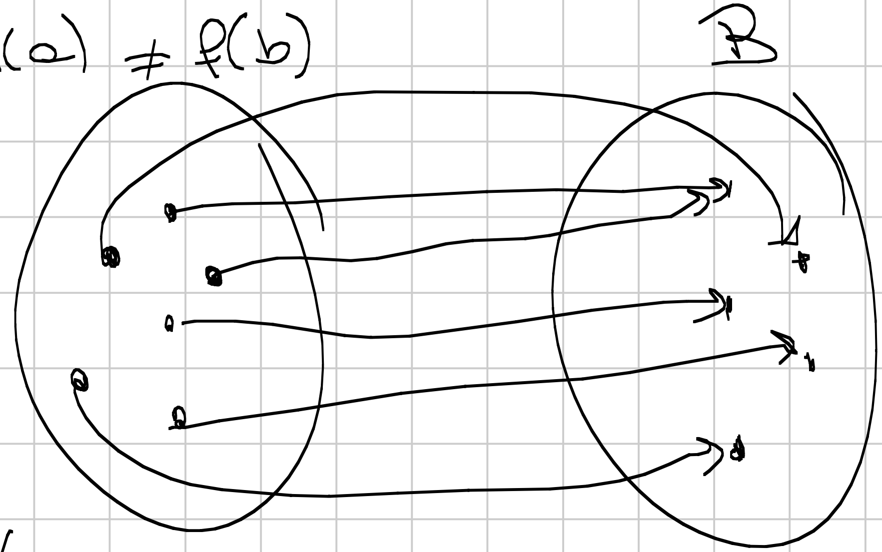
$f(x) = ax$ $\quad a(x+y) = ax + ay$

$\cancel{f(1)} = \cancel{f(1)} + f(0)$

$0 = f(0)$

DEF: f INIETTIVA

$a \neq b \Rightarrow f(a) \neq f(b)$

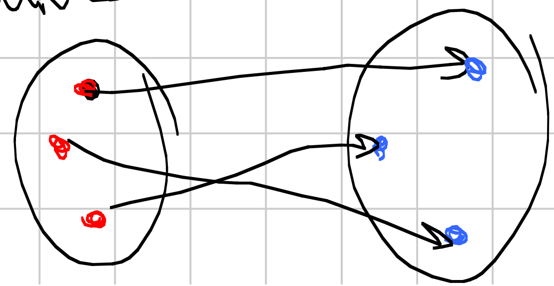


f SURIETTIVA

$\forall b \in B \quad \exists a \mid f(a) = b$

f BIUNNOCA o INVERTIBILE

iniettiva e suriettiva



$f(x) = x$ $\qquad f(x) = x^2$ $\qquad \dfrac{\text{su } \mathbb{R}}{\text{su } \mathbb{R}^+}$

$f: A \longrightarrow B$

$f(A) = \text{Im } A$ $\qquad f(A) = B \iff$ suriettiva

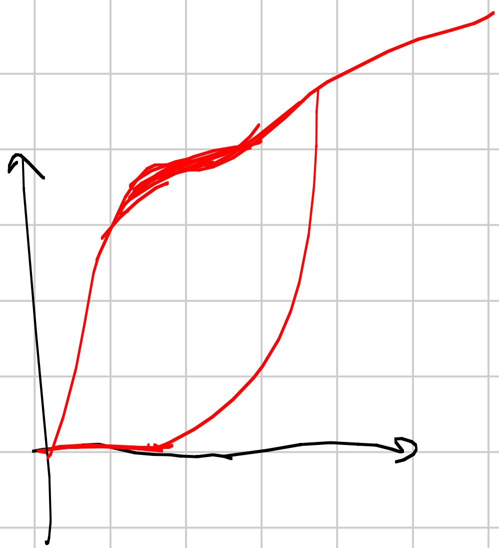
$f:$

$a \leq b \Rightarrow f(a) \leq f(b)$

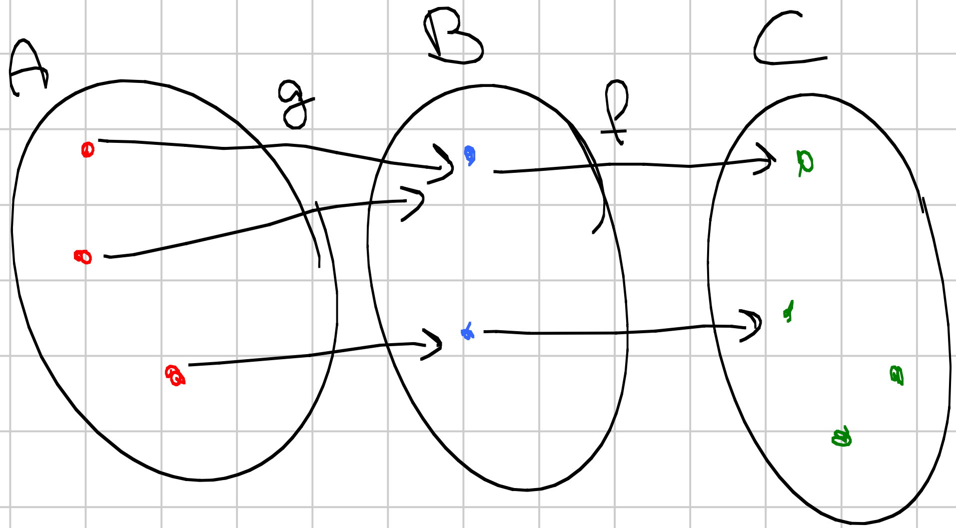
$a < b$

$a \neq b \quad f(a) = f(b)$

$a < b \quad f(a) < f(b)$



$$(f \circ g)(x) = f(g(x))$$



$g: A \to B$

$f: B \to C$

f, g iniettive

$$(f \circ g)(a) = f \circ g\,(b)$$

$$g(a) = g(b)$$

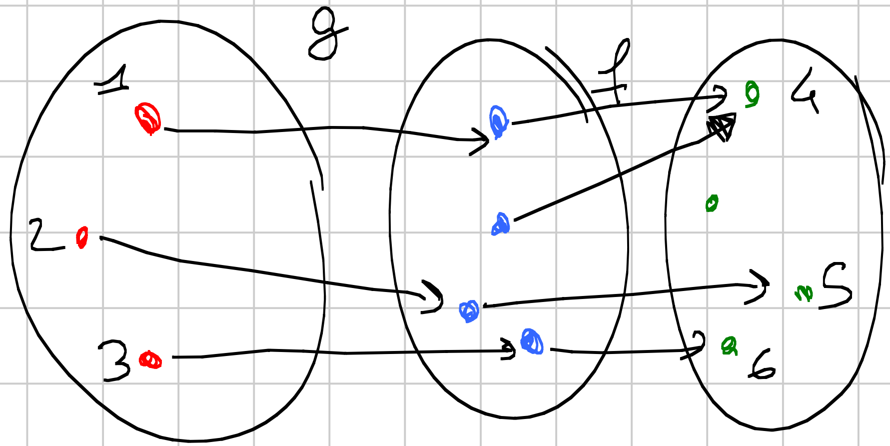
$f($

$f(qlc) = f(altro)$

$\Downarrow$

$qlc = altro$

$f \circ g$ iniettiva



$f \circ g$ iniettiva $\Longrightarrow$ g iniettiva

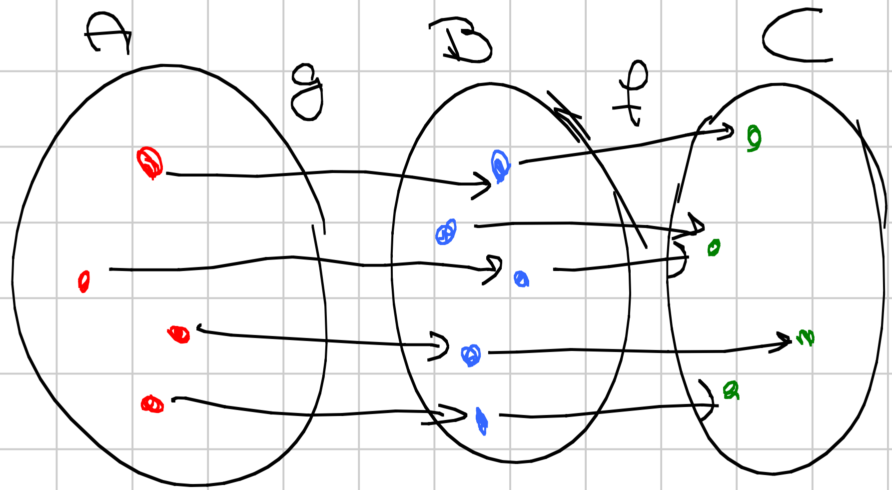
$$g(a) = g(b)$$

$$f(g(a)) = f(g(b))$$

$f_1 \circ \dots \circ f_k \Longrightarrow f_k$ iniettiva

$f \circ g$ suriettiva

f è suriettiva



$c \in C \mid \nexists\, b \mid f(b) = c$

$f \circ g$ $\quad\quad f(g(d)) = c$

$f(f(x)) = x + 3$ = biunivoca

f è suriettiva e iniettiva

$f(3f(x) + x) = 2x + 5$

f è surgettiva

$g(x) = 3x +$

$f(g(f(x))) = 2x + 5$

f è iniettiva

1 SOSTITUZIONI

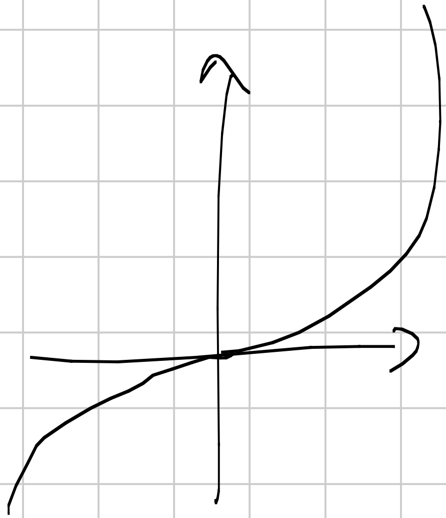
$$f(x+y) = f(x) + f(y)$$

$$f(0) = 2f(0) \Rightarrow f(0) = 0$$

$$y = -x$$

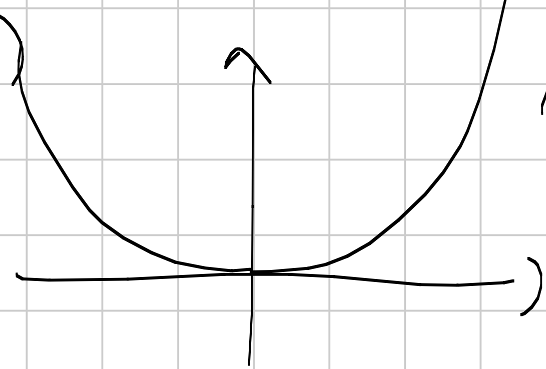
$$f(0) = f(x) + f(-x)$$

$$f(x) = -f(-x)$$ DISPARI



$$f(x) = f(-x)$$

PARI



$$f(f(x)) = -f(x) + 3$$

$$\boxed{f(x) = y}$$

$$f(y) = -y + 3 \qquad \forall y \text{ NO}$$

$$\forall y \in \operatorname{Im} f$$

$$f(x) = \frac{3}{2}$$

$f: \mathbb{R} \to \mathbb{R}$

Esempio 1: $f(x - f(y)) = 1 - x - y \quad \forall x, y \in \mathbb{R}$

$$y = 0$$

$$f(\underbrace{x - f(0)}_{z}) = 1 - x$$

$$f(\underbrace{x - \dots}_{z}) = (\quad)x + \dots$$

$$f(z) = (\quad)z + \dots$$

$$z = x - f(0)$$

$$f(z) = 1 - (z + f(0)) =$$

$f(z) = -z + c$

VERIFICA!!

$f(x+y-c) = 1-x-y$

$\cancel{-x}\cancel{-y}+c+c = 1\cancel{-x}\cancel{-y}$

$2c = 1$

$c = \frac{1}{2}$

$f(z) = -z + \frac{1}{2}$

Esempio: $f: \mathbb{R} \to \mathbb{R}$

$$\boxed{x\left[\frac{1}{x}f(-x) + f\left(\frac{1}{x}\right) = x\right]} \quad \forall x \in \mathbb{R}$$

$y = -\frac{1}{x}$

$x \longrightarrow -\frac{1}{x}$

$$-y\, f\left(+\frac{1}{y}\right) + f(-y) = -\frac{1}{y} \qquad \forall y \in \mathbb{R}$$

$x = y$

$$\boxed{-x\, f\left(\frac{1}{x}\right) + f(-x) = -\frac{1}{x}}$$

$$f(-x) + f(-x) = x^2 - \frac{1}{x}$$

$$f(-x) = \frac{x^2 - \frac{1}{x}}{2} = \frac{x^3 - 1}{2x}$$

$-x \to x$

$$f(x) = \frac{-x^3 - 1}{-2x} = \frac{x^3 + 1}{2x}$$

ES 10

$f: \mathbb{R} \to \mathbb{R}$

$$f(xf(x) + f(y)) = f(x)^2 + y \qquad 0$$

$x = 0$ $\qquad f(f(y)) = f(0)^2 + y \qquad 1$

f è INIETTIVA e SURIETTIVA

$\exists!\, \alpha \qquad f(\alpha) = 0$

$x = \alpha$

$$f(f(y)) = y \qquad 2$$

$f(0) = 0$

$x = f(z)$

$$f(f(z) \cdot z + f(y)) = z^2 + y \qquad 3$$

Confrontando 0 e 3 :

$$x^2 = f(x)^2$$

$$f(x) = \pm x$$

$f(1) = 1$

$f(2) = -2$

ATTENZIONE!!!

$$f(x f(x) + f(y)) = f(x)^2 + y$$

0

$$f(0) = 0 = \begin{cases} -0 \\ +0 \end{cases}$$

$x = a \neq 0 \quad f(a) = a$

$y = b \neq 0 \quad f(b) = -b$

$$f(x) = \begin{cases} x \\ \cancel{-x} \end{cases}$$

$$f(a^2 - b) = a^2 + b$$

$$\cancel{a^2} - b = \cancel{a^2} + b \qquad b = 0$$

$$f(a^2 - b) = -a^2 + \cancel{b} = a^2 + \cancel{b}$$

Esempio: $f(0)+g(0)=0$

$$\underbrace{f(\underbrace{x+g(y+1)})}+y = xf(y)+\underbrace{f(\underbrace{x+g(y)})}$$

$$x\,g(y+1) = x+g(y)$$

$$x\,(g(y+1)-1) = g(y)$$

Esempio $f: \mathbb{R}^+ \to \mathbb{R}^+$

1. $f(x+yf(x)) = f(x)\cdot f(y)$
2. $f(x)=1$ ha un num finito di soluzioni.

$$f(x)\cdot f(y) = \underbrace{f(x+yf(x)) = f(y+xf(y))}$$

SIMMETRIZZARE VARIABILI

$$x + y f(x) = y + x f(y)$$

$$c = \frac{(f(x)-1)}{x} = \frac{(f(y)-1)}{y} \qquad \forall\, x,y \in \mathbb{R}$$

$$\frac{f(x)-1}{x} \qquad \begin{matrix} x_0 \\ y_0 \end{matrix} \qquad \frac{f(x_0)-1}{x_0} \neq \frac{f(y_0)-1}{y_0}$$

$$f(x) = cx + 1$$

$$f(x + y f(x)) = f(x) \cdot f(y)$$

$$f(x + y(cx+1)) = (cx+1) \cdot (cy+1)$$

$$c[x + y(cx+1)] + \cancel{1} = c^2xy + cx + cy + \cancel{1}$$

$$cx + c^2xy + cy =$$

Dimostrazione iniettività:

$$f(x + y f(x)) = f(x) \cdot f(y)$$

$f(x_0) = 1$

$$f(x_0 + y) = f(y)$$

$x = x_0$, $y = x_0$

$$f(2x_0) = f(x_0) = 1$$

$$f(3x_0) = f$$

$x = x_0$, $y = 2\cancel{x}_0$

$$f(x_0 + 2x_0 \cdot 1) = f(x_0) \cdot f(2x_0) = 1$$

X INDUZ

$$f(kx_0) = 1 \qquad \forall k \in \mathbb{N}$$

$$f((k+1)x_0) = 1$$

se nell'originale mettiamo

$x = x_0$, $y = kx_0$ otteniamo

$$f(x_0 + kx_0 \cdot 1) = f(x_0) \cdot f(kx_0) = 1$$

$$\Rightarrow \nexists \ x_0 \mid f(x_0) = 1$$

$$f(\underbrace{x + y f(x)}_{x + \ldots}) = f(x) \cdot f(y)$$

$a < b$ e $f(a) = f(b)$

$x = a$ $\quad a + y f(a) = b \Rightarrow y = \dfrac{b-a}{f(a)}$

$$f(b) = f(a) \cdot \boxed{f\left(\frac{b-a}{f(a)}\right)} = 1$$

Dimostra

Lemma 1: $f(x) \neq 1 \quad \forall x$

Dim

Lemma 2: f è iniettiva

Simmm variabili

Verifico

RICONDURSI A CAUCHY

$f: \mathbb{Q} \to \mathbb{Q}$

$$f(x+y) = f(x) + f(y)$$

$$\Rightarrow \quad f(x) = ax \quad \forall x$$

$x = y = 0 \qquad f(0) = 0$

$y = -x \qquad 0 = f(x) + f(-x) \Rightarrow f$ è dispari

$y = nx \qquad \boxed{f((n+1)x) = f(x) + f(nx)}$

Lemma:

$$f(nx) = n\, f(x) \qquad \forall n \in \mathbb{N},\ x \in \mathbb{Q}$$

$\underline{n = 1}$

$n \to n+1$

$$f((n+1)x) = f(x) + n\, f(x)$$

$$= (n+1)\, f(x)$$

$$f(n) = n \cdot c = c \cdot n \qquad c = f(1)$$

$$f\left(n \tfrac{3}{2}\right) = c \cdot n \qquad c = f\left(\tfrac{3}{2}\right)$$

$$f\left(\frac{m}{n}\right) = c\,\frac{m}{n} \qquad\qquad c = f(1)$$

$$\cancel{x = \frac{1}{n} \quad f\left(\frac{m}{m}\right) = m\, f\left(\frac{1}{m}\right)}$$

$$x = \frac{m}{n} \qquad\qquad f\left(\cancel{n} \cdot \frac{m}{\cancel{n}}\right) = n\, f\left(\frac{m}{n}\right)$$

$$f\left(\frac{m}{n}\right) = \frac{f(m)}{n} = \frac{c \cdot m}{n}$$

$$f(q) = c \cdot q.$$

$f: \mathbb{R} \to \mathbb{R} \qquad f(x+y) = f(x) + f(y)$

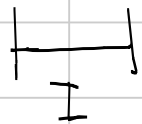
$f(x) = ax$

(1) Monotonia di f

(2) Continuità di f

(3) f sia limitata sup o inf

$f(I)$



$f: \mathbb{Q} \to \mathbb{R} \qquad f(x) \cdot f(y) = f(x+y)$

$\ln f(x) + \ln f(y) = \ln f(x+y)$

$g = \ln \circ f \qquad g(x) + g(y) = g(x+y) \qquad g(x) = ax$

$$g(x) = ax = \ln f(x)$$

$$e^{ax} = f(x)$$

$$x = y = \frac{z}{2} \qquad f\left(\frac{z}{2}\right)^2 = f(z) \geq 0 \quad \forall z$$

$$f(x_0) = 0 \quad 0 = f(x_0) \cdot f(y) = f(\underbrace{x_0 + y}_{z})$$

$$f(z) = 0 \quad \forall z$$

Esempio: f monotone $\mathbb{R} \to \mathbb{R}$

$$f(x + f(y)) = f(x) + y$$

$x = 0 \qquad f(f(y)) = y + f(0) \Rightarrow f$ bigettiva

$$y = 0 \quad \cancel{f}(f(0)) = \cancel{f}(0)$$

$$y = f(y)$$

$$f(x + y + \underset{\overset{\shortparallel}{0}}{f(0)}) = f(x) + f(y)$$

$f(x) = ax$

Esercizi: $f : \mathbb{Q} \to \mathbb{R}$

1. $f(x+y) - yf(x) - xf(y) = f(x)f(y) - x - y + xy$
2. $f(x) = 2f(x+1) + 2 + x$
3. $f(1) + 1 > 0$

$$f(x+y) + x + y = f(x)f(y) + yf(x) + xf(y) + xy$$
$$= (f(x)+x)(f(y)+y)$$

$g(x) = f(x) + x$

$$g(x+y) = g(x) \cdot g(y)$$

$$g(x) = 2g(x+1)$$
$$g(1) > 0$$

$g(x) = a^x$ ⓐ

$a^x = 2a^{x+1} \Rightarrow 1 = 2a \quad a = \frac{1}{2}$

$g(x) = \left(\frac{1}{2}\right)^x$

$f(x) = \frac{1}{2^x} - x$

GUARDARE I PTI FISSI

$f: \mathbb{R}^+ \to \mathbb{R}^+$

① $f(x f(y)) = y f(x)$

② $\lim_{x \to \infty} f(x) = 0$

$f(f(x) \cdot f(y)) = xy$

$x \Rightarrow f(x)$

f

$x = 1 \qquad f(f(y)) = y \cdot f(1)$

f è biiettiva

$x = y$

$f(\underbrace{x f(x)}) = \underbrace{x f(x)}$

$x f(x) = 1$

$S = \{ x \mid f(x) = x \}$

a, b

$x = a \quad y = b$

$f(a \cdot f(b)) = b\, f(a)$

$f(ab) = ba$

① $f(x f(y)) = y f(x)$

② $\lim_{x \to \infty} f(x) = 0$

$a \in S$ $\quad \boxed{a \neq 1}$

$a^2 \quad a^n$ è pto fisso

$f(a^n) = a^n$

$\boxed{\text{se } a > 1}$

$1 \in S$?

$$\begin{cases} x = 1 \\ y = 1 \end{cases} \qquad f(f(1)) = f(1)$$

$1 \in S$

$a < 1$

$a < 1$ è pto fisso

$\Rightarrow \frac{1}{a}$ è pto fisso

$\boxed{\begin{array}{l} 1 \quad f(x f(y)) = y f(x) \\ 2 \quad \lim_{n \to \infty} f(x) = 0 \end{array}}$

$x = \frac{1}{a} \qquad y = a$

$$f\left(\frac{1}{\cancel{a}} \cdot \cancel{f(a)}\right) = a \cdot f\left(\frac{1}{a}\right)$$

$$f\left(\frac{1}{a}\right) = \frac{1}{a} \Rightarrow \frac{1}{a} \in S$$

1. f biettiva
2. S
3. $1 \in S$
4. $a, b \in S \to ab \in S$
5. $a \in S \to \frac{1}{a} \in S$

a^n $\left(\frac{1}{a}\right)^n$