G3

Titolo nota 11/09/2008

Ceva - Men.

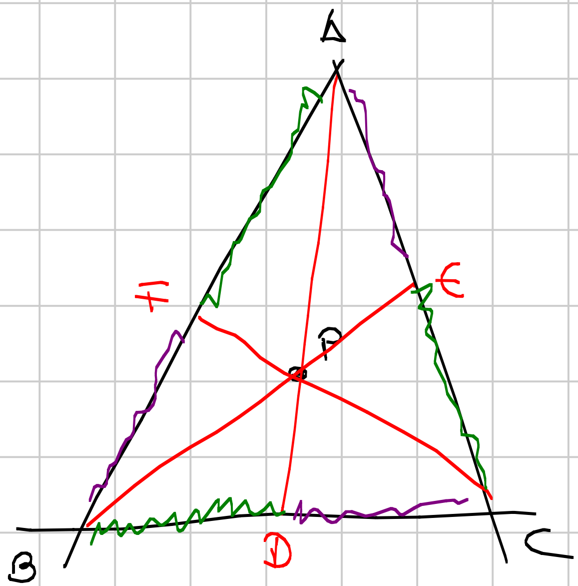
Potenze

TEO. DI CEVA

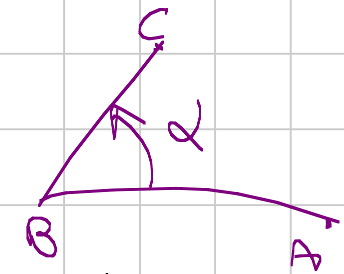
AD, BE, CF concorrano

$$\iff \frac{AF}{FB} \cdot \frac{BD}{DC} \cdot \frac{CE}{EA} = 1$$

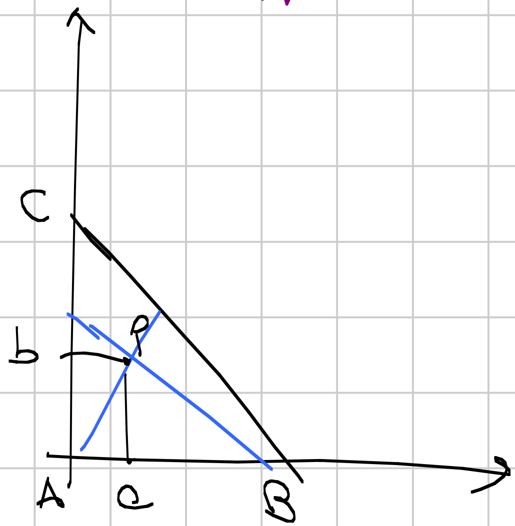
$DC = -CD$



$$C\hat{B}A = -\alpha$$



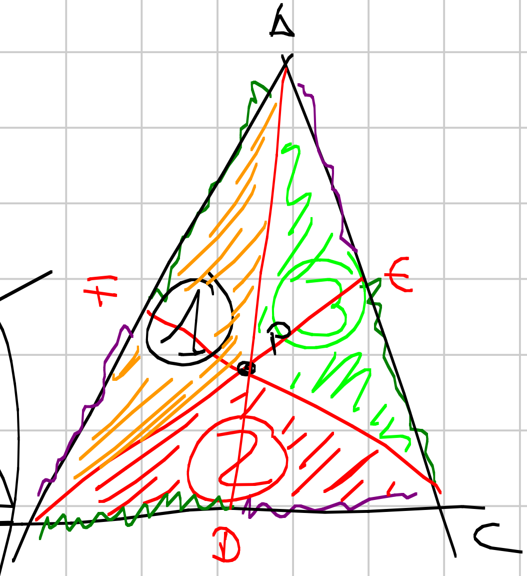
1°)



2°) $$\frac{BD}{DC} = \frac{[BDP]}{[CDP]} = \frac{[ABD]}{[ADC]}$$

$$\stackrel{?}{=} \frac{[APB]}{[APC]} = \frac{1}{3}$$

$$= \frac{[ABD]-[BPD]}{[APC]-[DCP]} = \frac{[ABD]}{[APC]} \cancel{\left(\frac{1-\frac{[BPD]}{[ABD]}}{1-\frac{[DCP]}{[APC]}} \right)}$$



$$\frac{CE}{EA} = \frac{2}{1}$$

$$\frac{AF}{FB} = \frac{3}{2}$$

$$\frac{1}{3} \cdot \frac{2}{1} \cdot \frac{3}{2} = 1$$

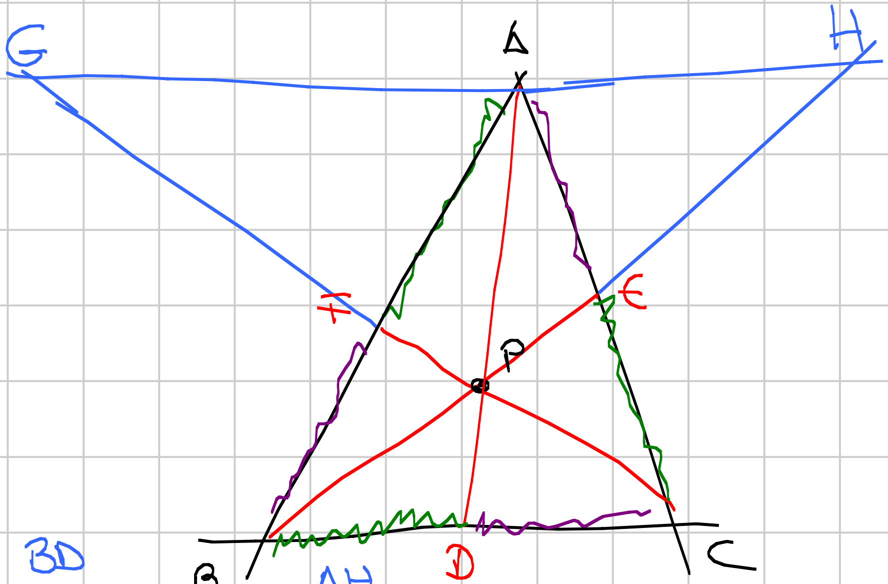
3ª dim)

$$\frac{AF}{FB} = \frac{\cancel{AG}}{\cancel{BC}}$$

$$\frac{CE}{EA} = \frac{\cancel{CB}}{\cancel{AH}}$$

$$\frac{BD}{DC} = \frac{\cancel{AH}}{\cancel{AG}}$$

$$= 1$$



$$\frac{BD}{DC} = \frac{\frac{BD}{PD}}{\frac{DC}{PD}} = \frac{\frac{AH}{\cancel{AP}}}{\frac{AG}{\cancel{AP}}}$$

$\Leftarrow)$

$$\frac{AF}{FB}\cdot\frac{BD}{DC}\cdot\frac{CE}{EA}=1$$

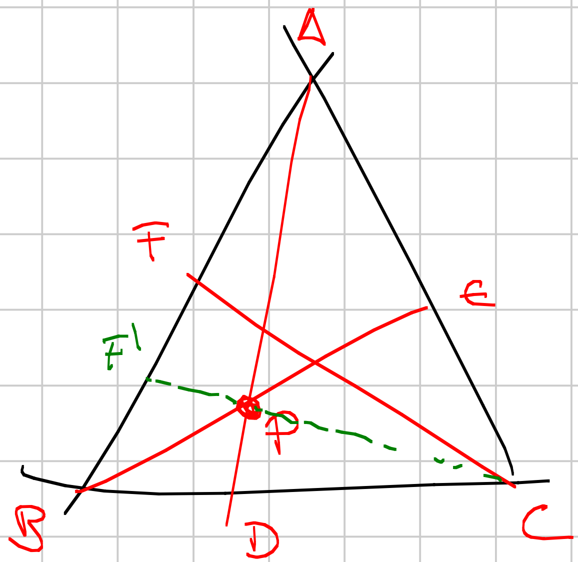
$$\frac{AF'}{F'B}\cdot\frac{BD}{DC}\cdot\frac{CE}{EA}=1$$

$$\frac{AF}{FB}\ \cancel{<}\ \frac{AF'}{F'B}$$

$$AF' > AF$$
$$FB < FB$$

$$\frac{AF}{FB}+1=\frac{AF'}{F'B}+1$$

$$\frac{AB}{FB}=\frac{AB}{F'B}$$



$a = b$

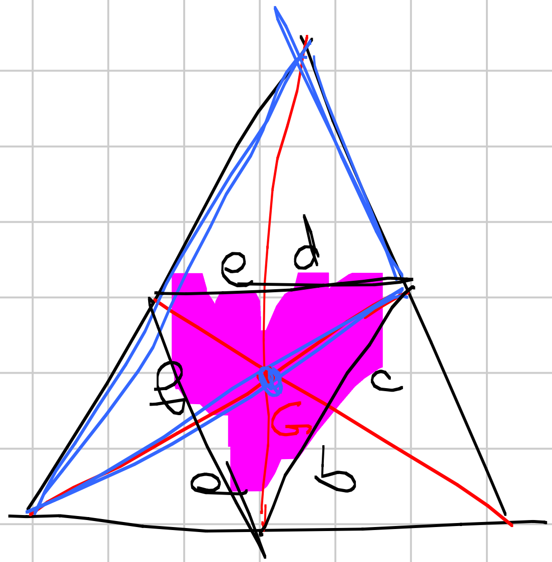
$c = d$

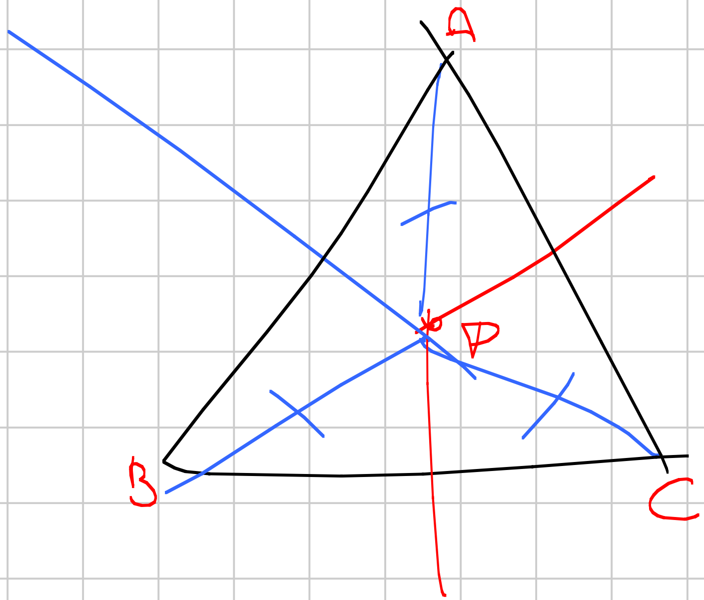
$e = f$

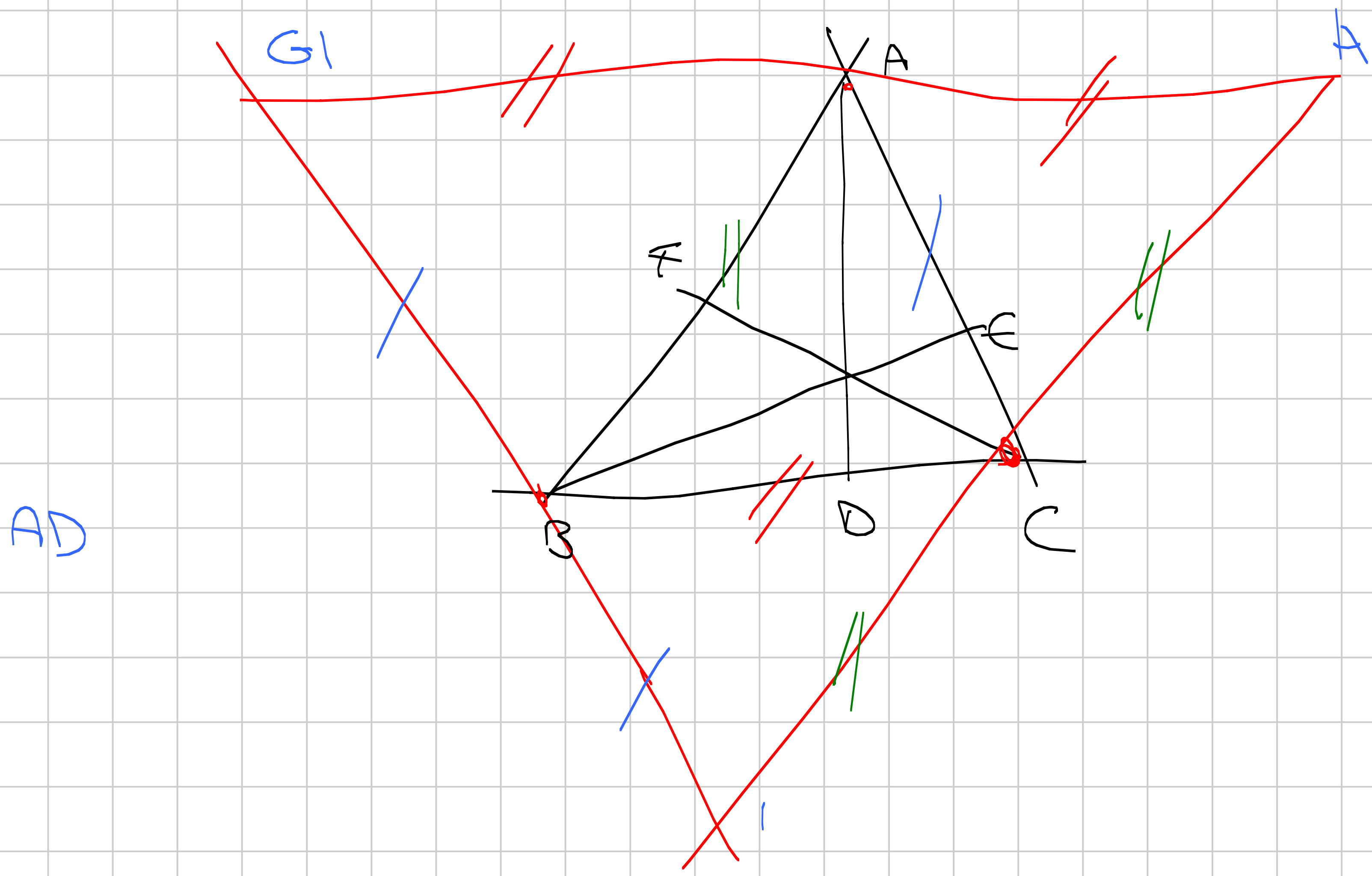
$\cancel{a} + f + e = \cancel{b} + c + d$

$2e = 2c$

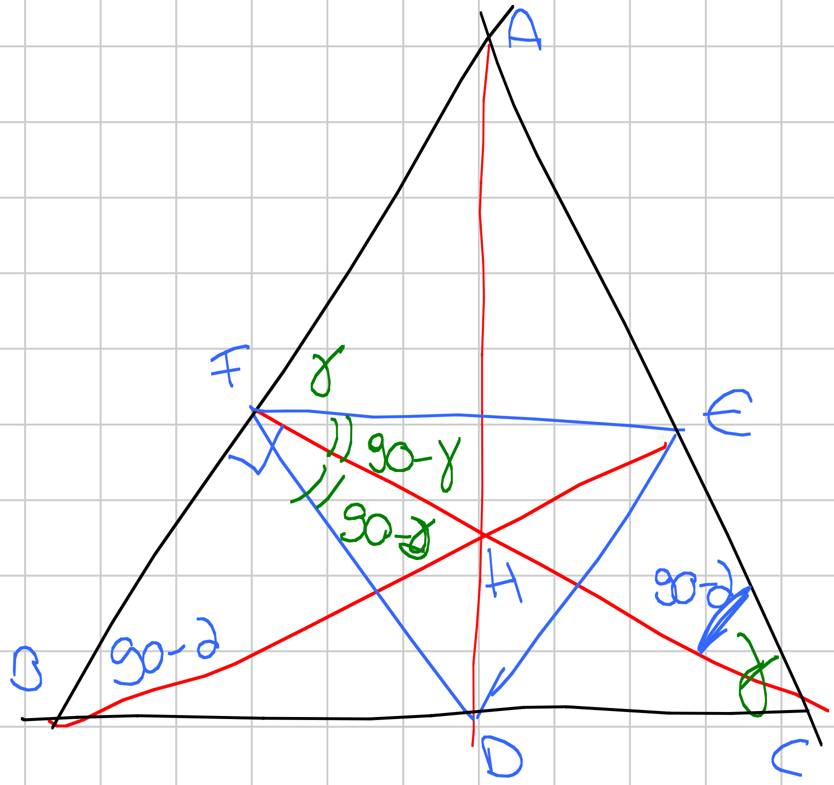
$e + f = 2d$



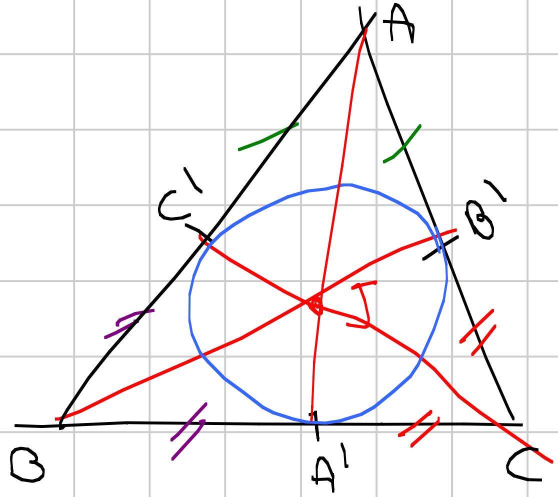



G
A
H
F
E
B
D
C
AD

$D\hat{F}C = D\hat{A}C$

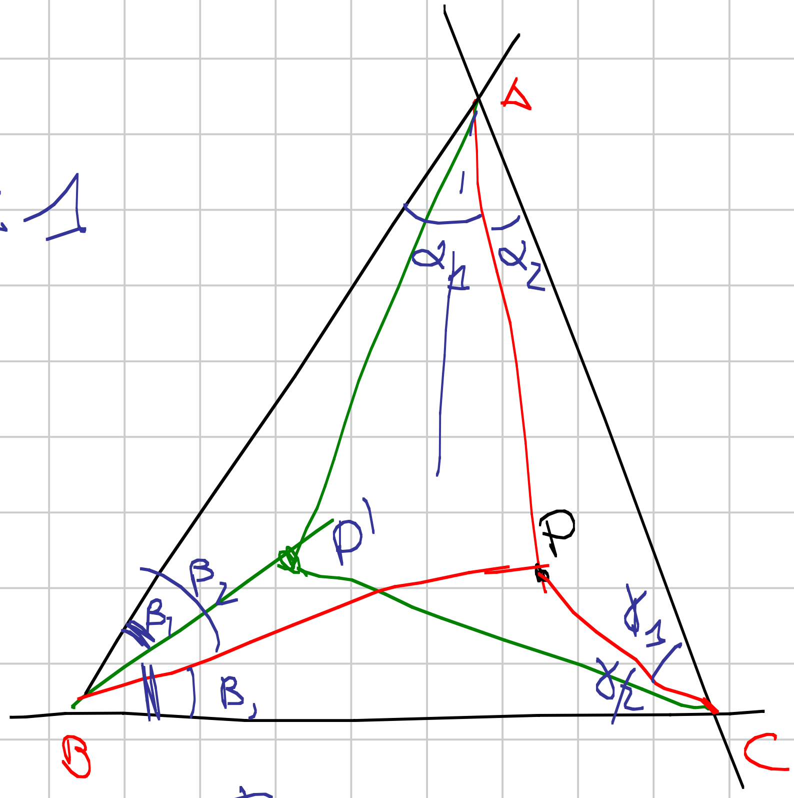


Es pto di Gergonne



~~Ceva trigo~~

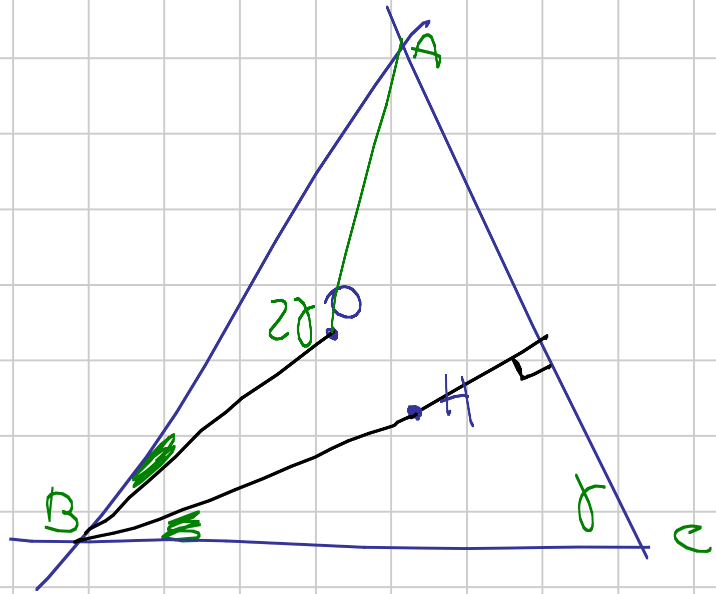
$$\frac{\sin \alpha_1}{\sin \alpha_2} \cdot \frac{\sin \beta_1}{\sin \beta_2} \cdot \frac{\sin \gamma_1}{\sin \gamma_2} = 1$$



Es: circoc = coniug is. ortocentro

$O\hat{B}A = 90 - \gamma$

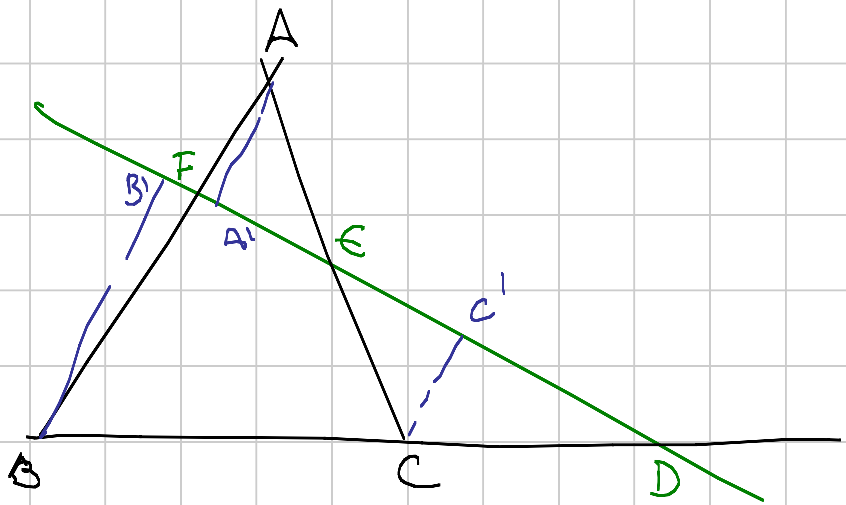
$H\hat{B}C = 90 - \gamma$



TEO DI MENELAO

D, E, F sono all

$$\Longleftrightarrow \frac{BD}{DC} \cdot \frac{CE}{EA} \cdot \frac{AF}{FB} = -1$$



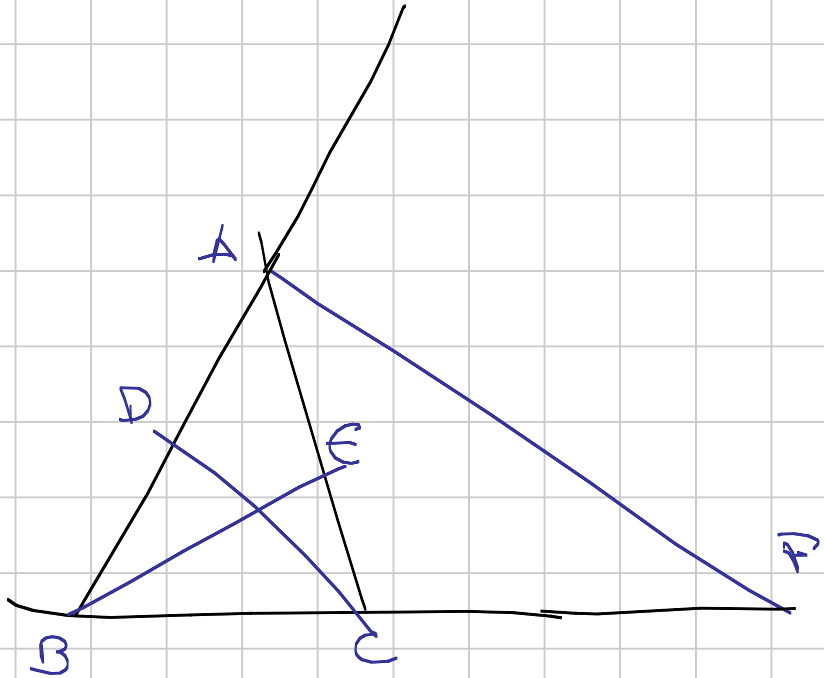
$$\frac{BD}{DC} = \frac{BB'}{CC'}$$

$$\frac{CE}{EA} = \frac{CC'}{AA'}$$

$$\frac{AF}{FB} = \frac{AA'}{BB'}$$

Es:

D, E, F allineati



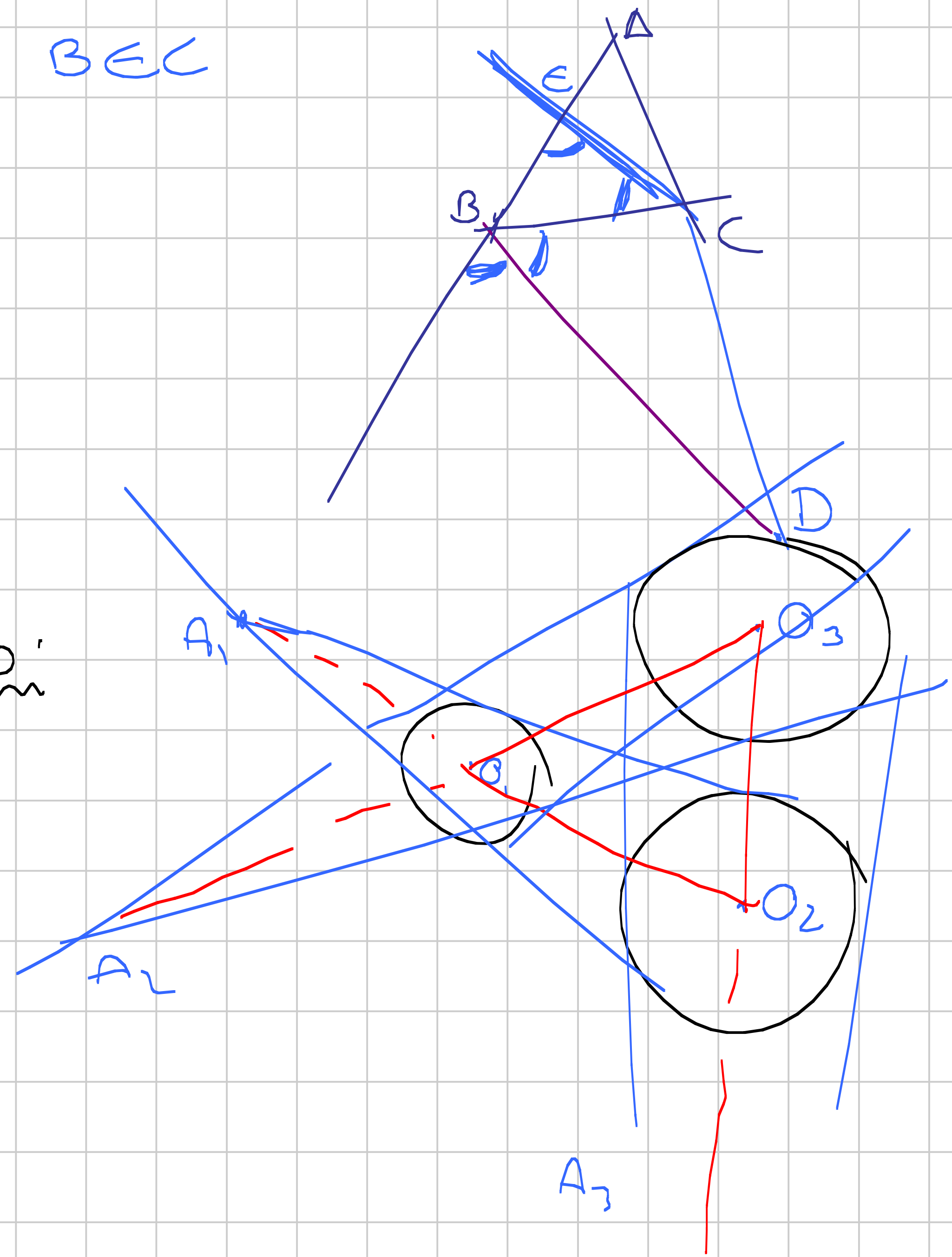
Lemma:
teo della bis

$$\frac{AD}{DB} = \frac{AC}{BC} \qquad \frac{CE}{EA} = \frac{BC}{AB} \qquad \frac{BF}{FC} = \frac{AB}{AC}$$

$$\frac{AD}{DC} = \frac{AB}{AC}$$

BEC

$$\frac{AD}{DC} = \frac{AB}{BE} = \frac{AB}{BC}$$



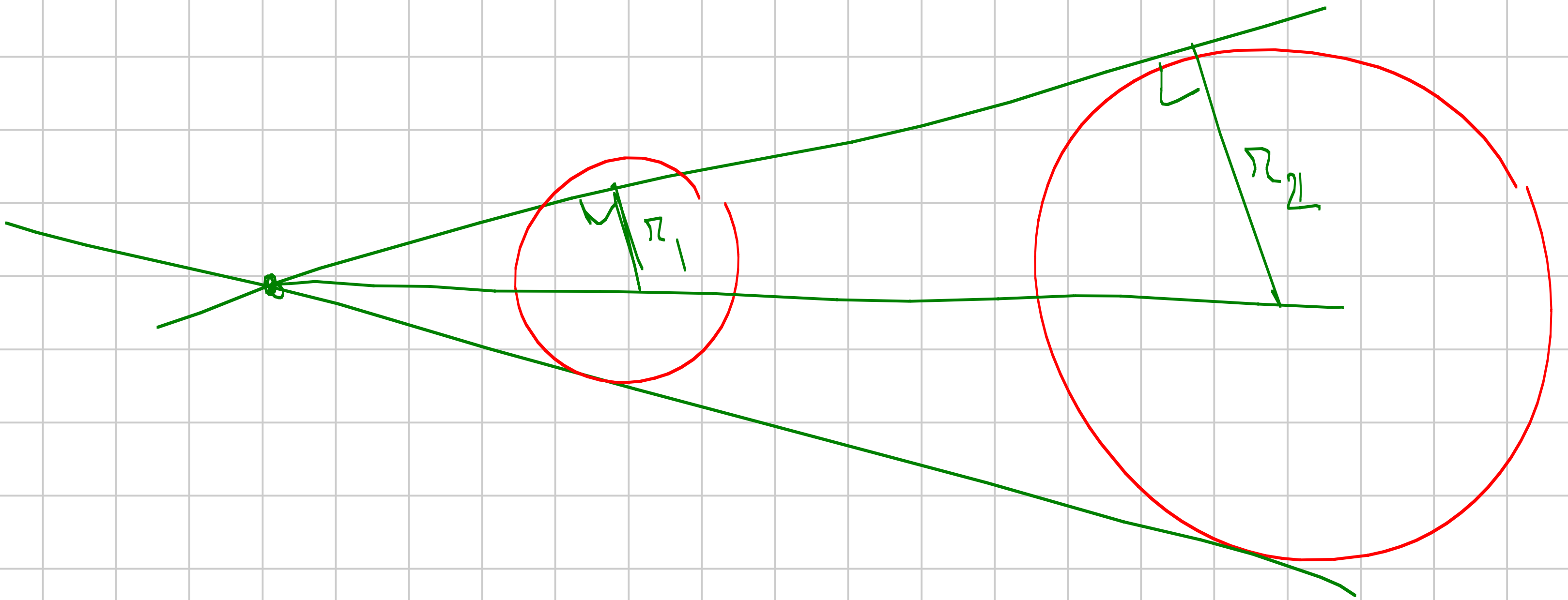
ES

3 cerchi

$$\frac{O_2A_1}{A_1O_1} = \frac{r_2}{r_1}$$

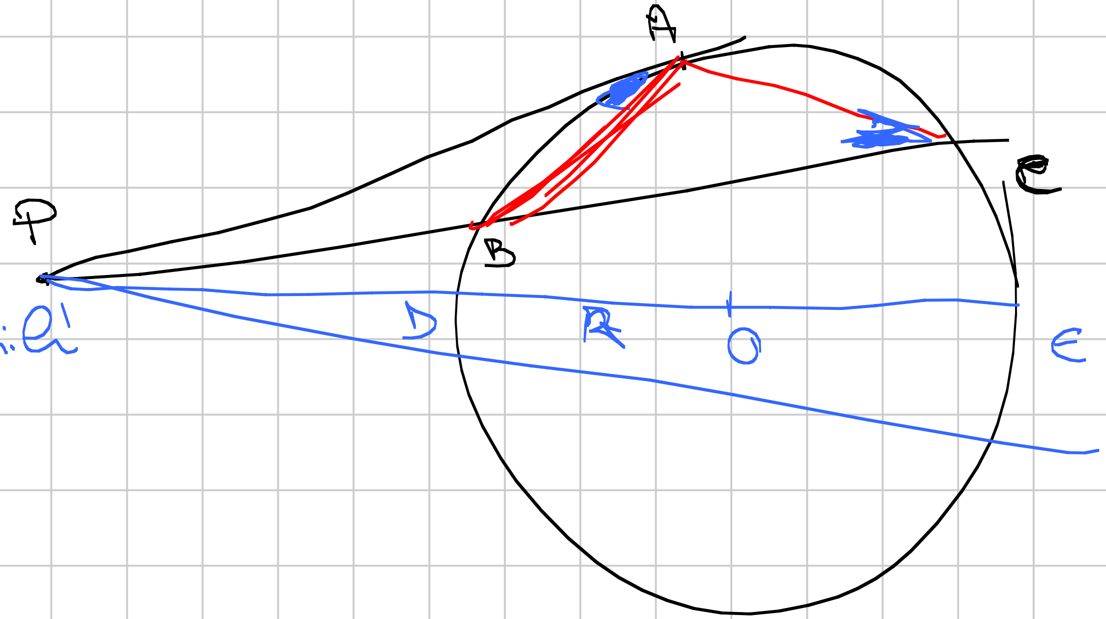
$$\frac{O_3A_2}{A_2O_1} = \frac{r_3}{r_1}$$

$$= r_3$$



γ
r_1
r_2

Potenze



$\left\{\begin{array}{l} PAB \\ PCA \end{array}\right.$ sono simili

$$\frac{PA}{PB} = \frac{PC}{PA}$$

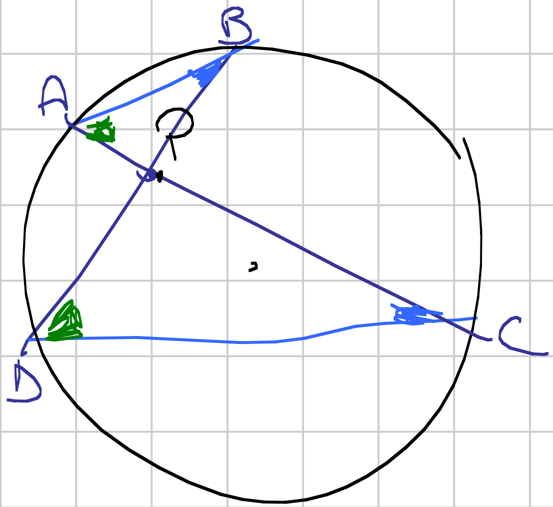
$$PA^2 = PB \cdot PC$$

$$PD \cdot PE = (d-R)(d+R) = Pow_{\Gamma}(P) = d^2 - R^2$$

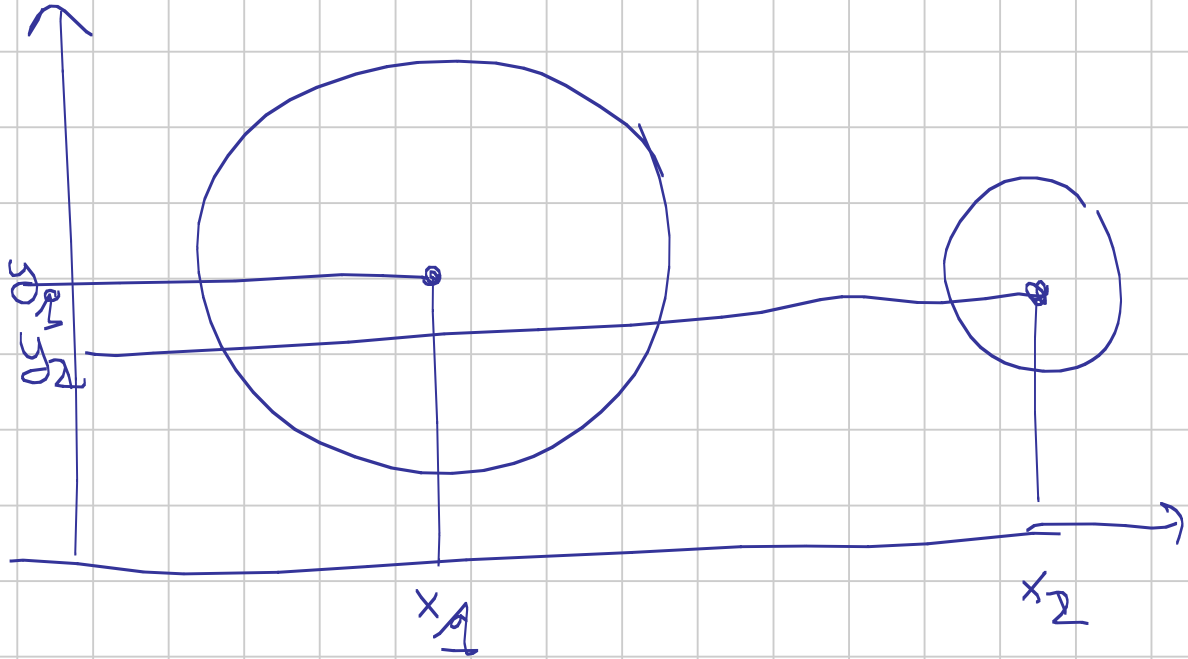
$PA \cdot PC = PB \cdot PD$

$\hat{APB} \sim \hat{PDC}$

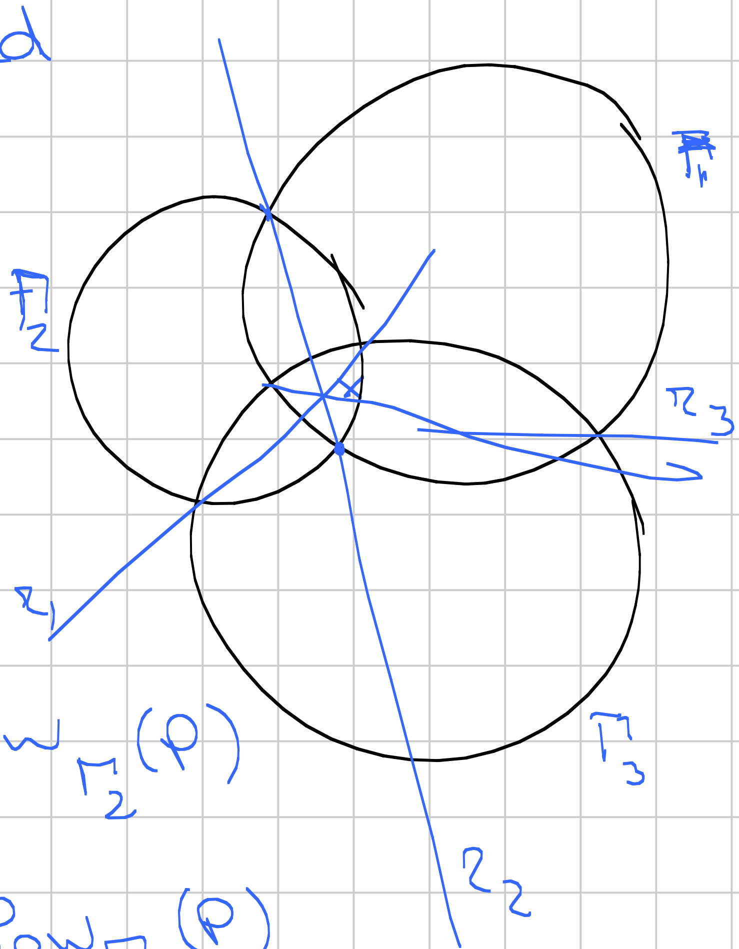
$Pow_{\Gamma}(P) = d^2 - R^2$



ASSI RADICALI



Es: gli assi rad concorrono



r_1: P $\quad Pow_{\Gamma_3}(P) = Pow_{\Gamma_2}(P)$

$\quad\quad\quad\quad ||$

r_2 $\quad Pow_{\Gamma_3}(P) = Pow_{\Gamma_1}(P)$

$X = r_1 \cap r_2$ $\quad\quad Pow_{\Gamma_1}(X) = Pow_{\Gamma_2}(X)$

ES IMO 1

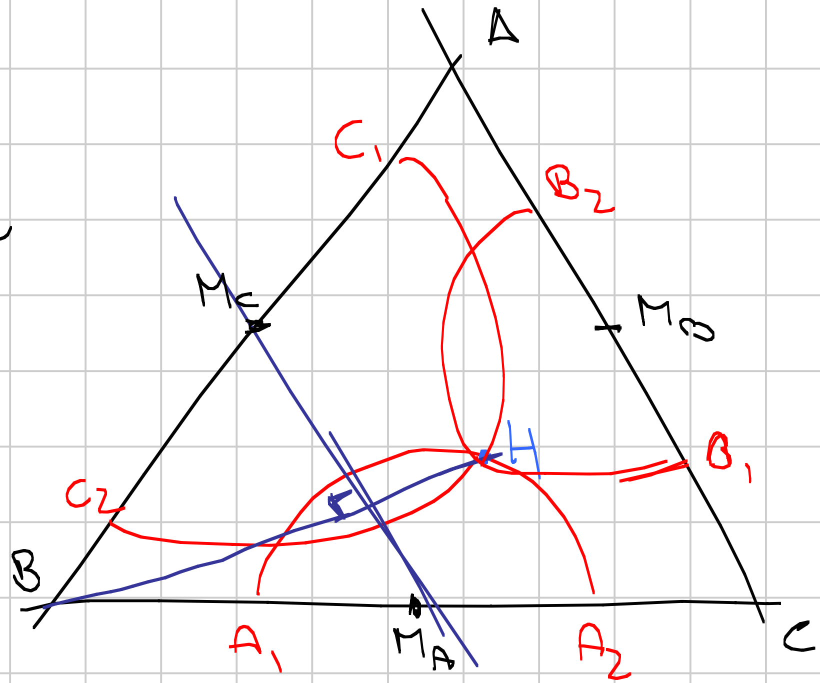
1° ass: O sarebbe il centro

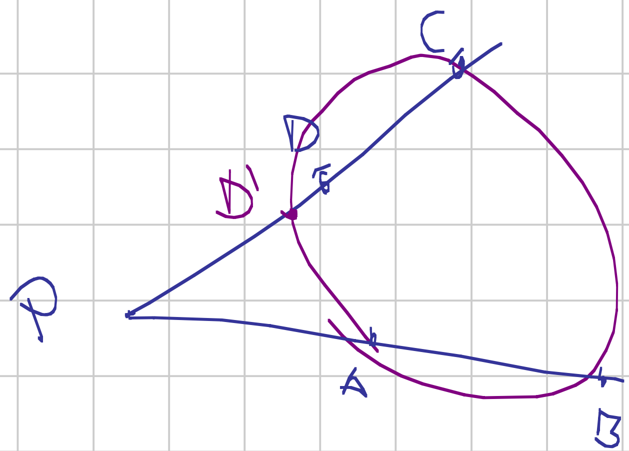
BH è asse rad Γ_C e Γ_A

$BA_1 \cdot BA_2 = BC_2 \cdot BC_1$

$PD \cdot PC = PA \cdot PB$

$PD' \cdot PC = PA \cdot PB$





Es: Simson line

A_1, B_1, C_1 son all

$\Longleftrightarrow$ $P \in$ circocirc

$A\hat{B}_1C_1 = C_1\hat{P}A =$

$= 90 - C_1\hat{A}P$

$A_1\hat{B}_1C = A_1\hat{P}C = 90 - P\hat{C}A_1$

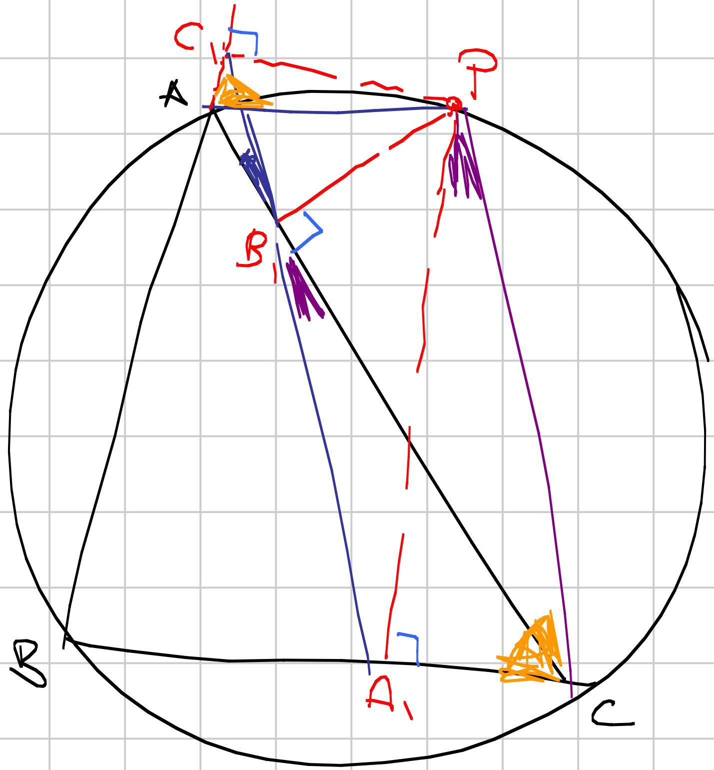
$A\hat{B}_1C_1 = A_1\hat{B}_1C \Longleftrightarrow 90 - C_1\hat{A}P = 90 - P\hat{C}A_1$

$\Downarrow$

$\Rightarrow$

$C_1\hat{A}P = P\hat{C}A_1$

$\Longleftrightarrow$ ABCP è cicl.



INSULTALO!!!

PAC_1B_1

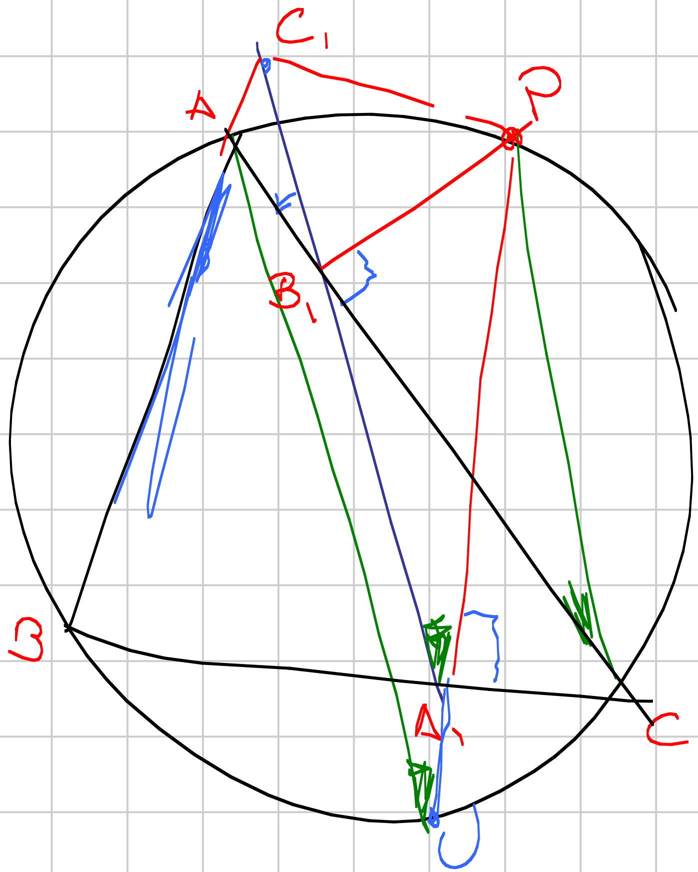
$AU \parallel A_1C_1$

$P\hat{U}A = P\hat{C}A$

$= P\hat{A}_1B_1$

PB_1A_1C

$=$ ciclico

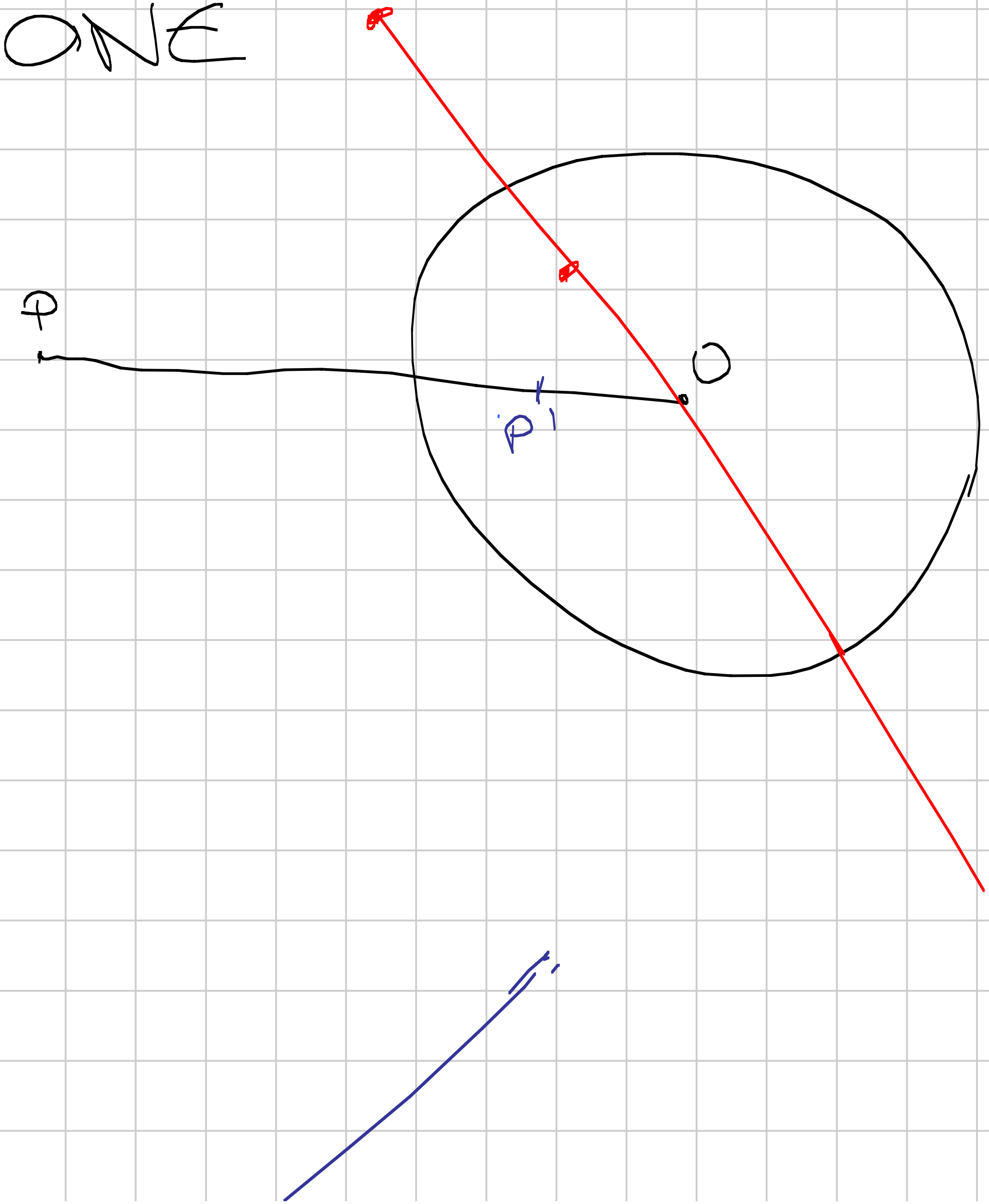


Il popolo chiede Van Obel!

||

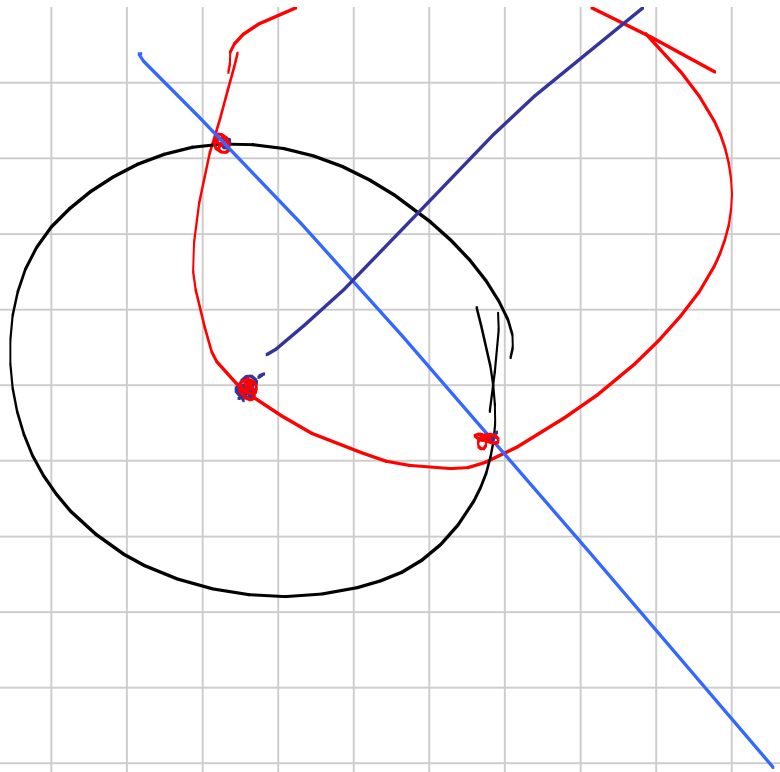
Jack

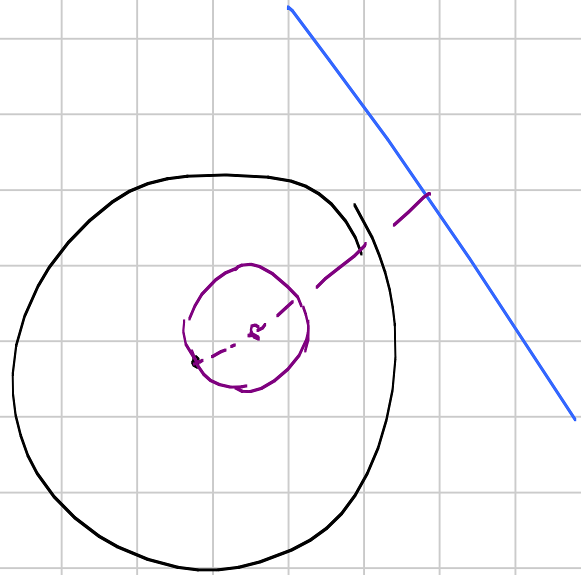
INVERSIONE

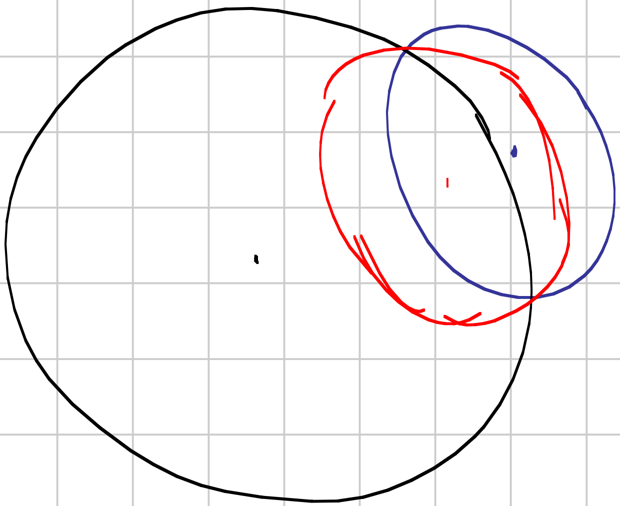


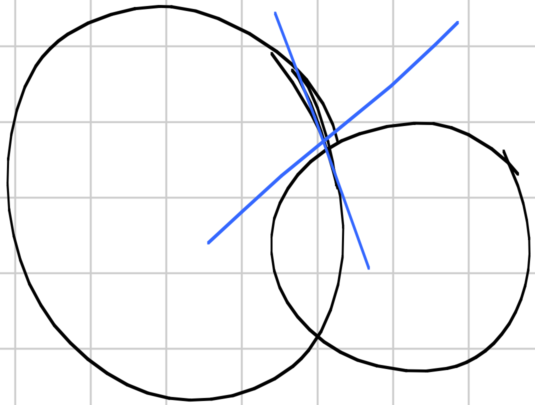
$OP' \cdot OP = r^2$

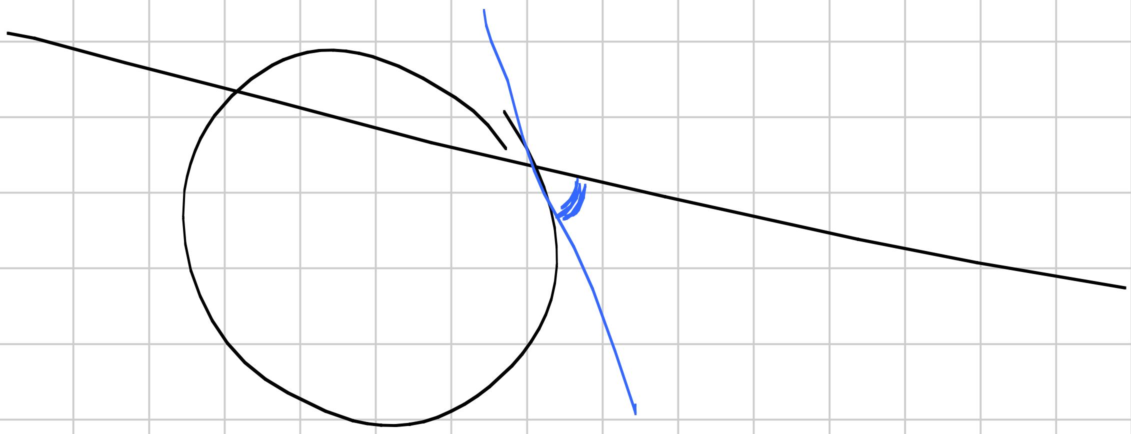
circ fissa

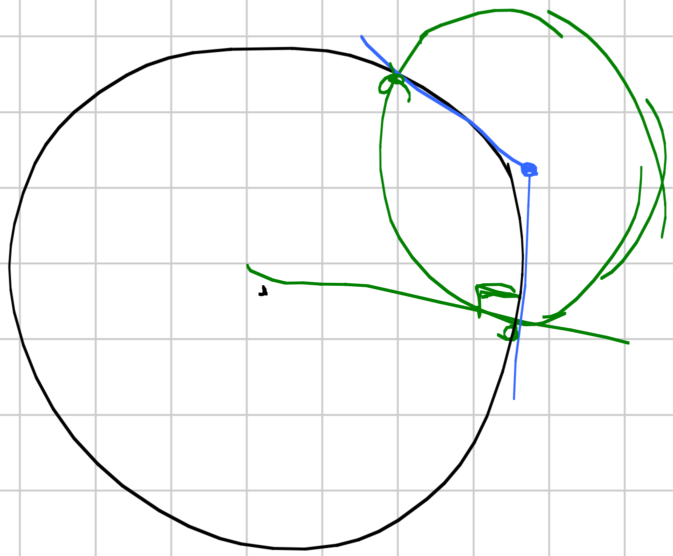






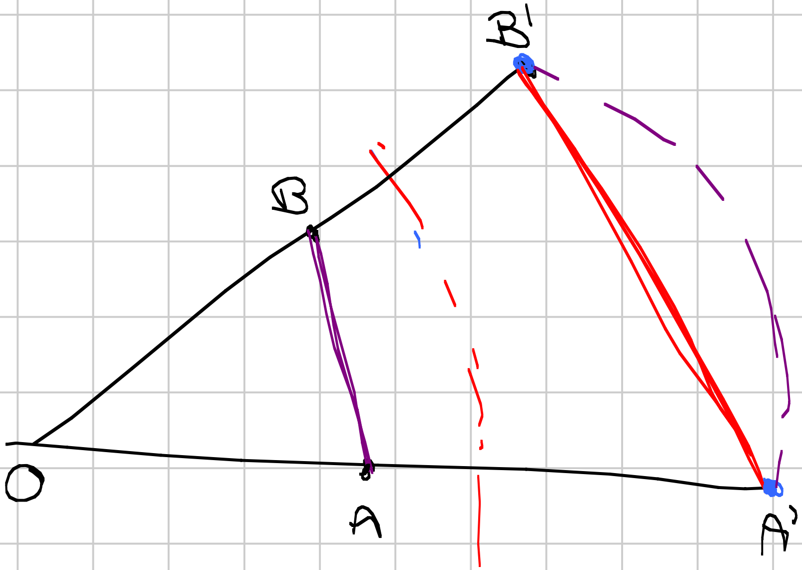






r

$AB \longrightarrow A'B'$



$OA \cdot OA' = r^2 = OB \cdot OB'$

$\frac{OA}{OB} = \frac{OB'}{OA'}$ $\triangle OAB \sim \triangle OB'A'$

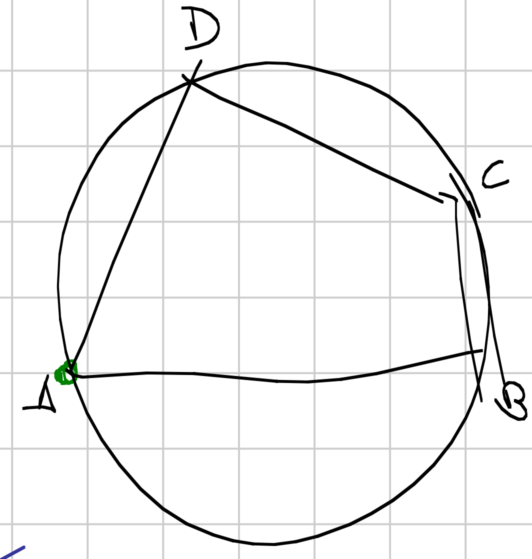
$$\frac{A'B'}{AB} = \frac{OA'}{OB} = \frac{\frac{r^2}{OA}}{OB}$$

$$\Longrightarrow A'B' = AB \cdot \frac{r^2}{OA \cdot OB}$$

Es: TEO DI TOLOMEO

$AB\cdot CD + BC\cdot AD \geq AC\cdot BD$

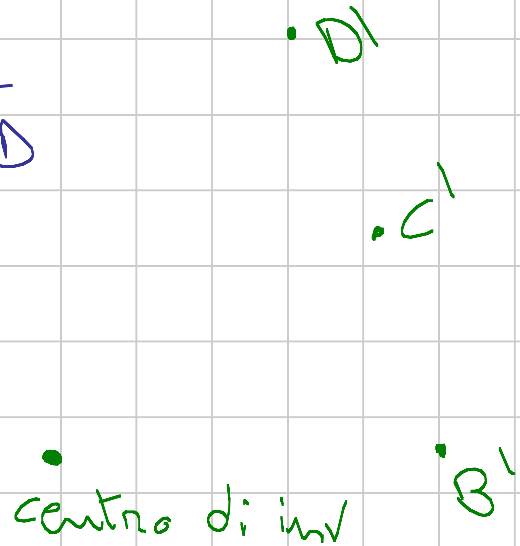
con = $\Longleftrightarrow$ ciclico ABCD



$\overline{B'C'} + \overline{C'D'} \geq \overline{B'D'}$

$BC\cdot \frac{\cancel{r^2}}{AB\cdot AC} + CD\cdot \frac{\cancel{r^2}}{AC\cdot AD} \geq BD\,\frac{\cancel{r^2}}{AB\cdot AD}$

$BC\cdot AD + CD\cdot AB \geq BD\cdot AC$

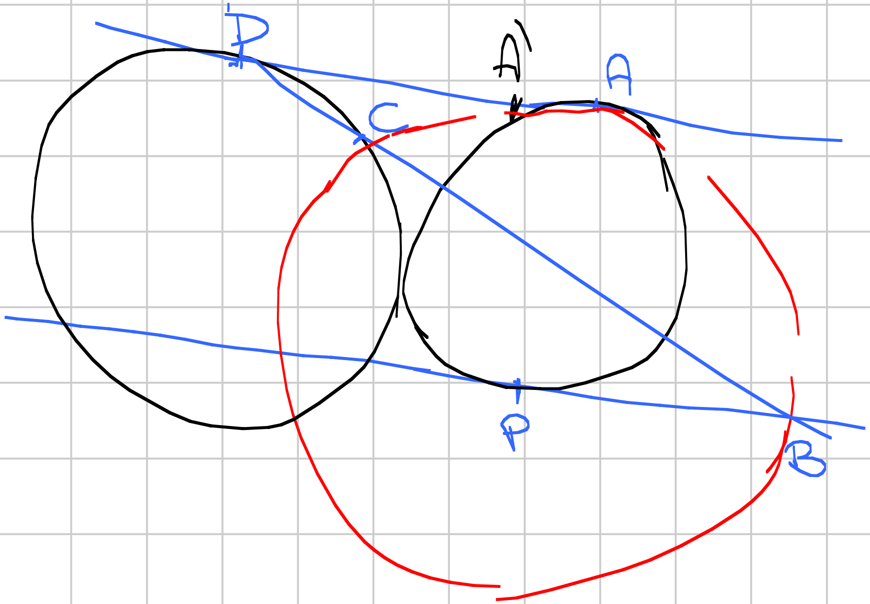


ES Romania

AD è tangente
al circocerchio di ABC

$$DA^2 = DB \cdot DC$$

$$DA \cdot DA' = DB \cdot DC$$

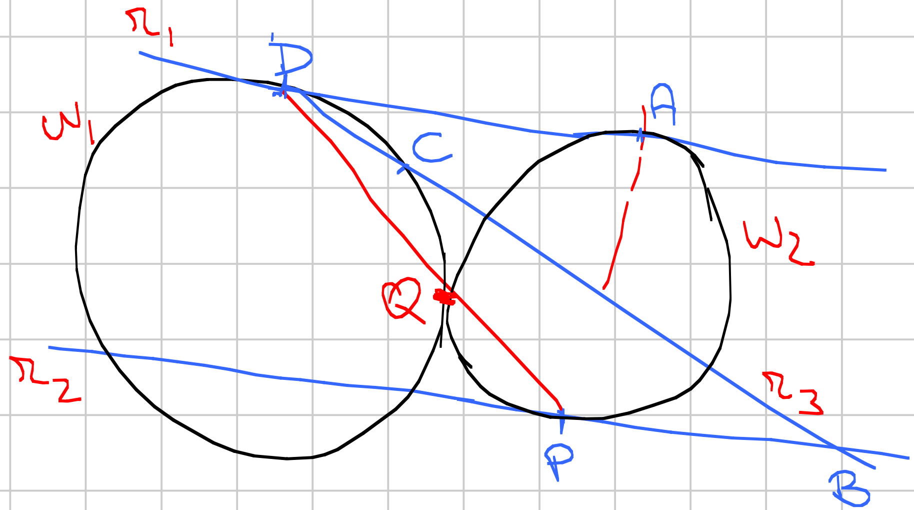


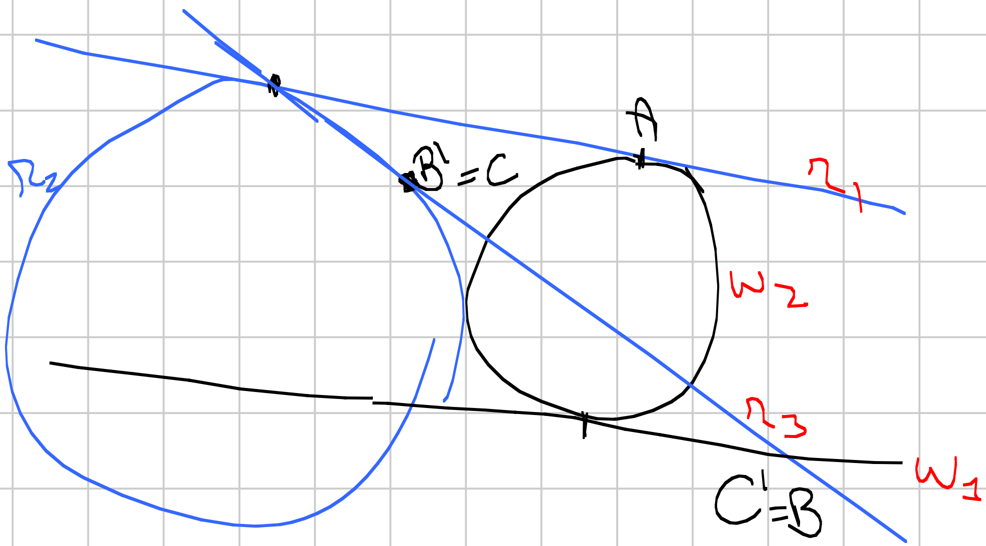
$DA^2 = DC \cdot DB$

inv D, raggio DA

$\omega_1 \to$ retta $\parallel r_1$
tangente alla
circ piccola

$DC \cdot DB = DA^2$



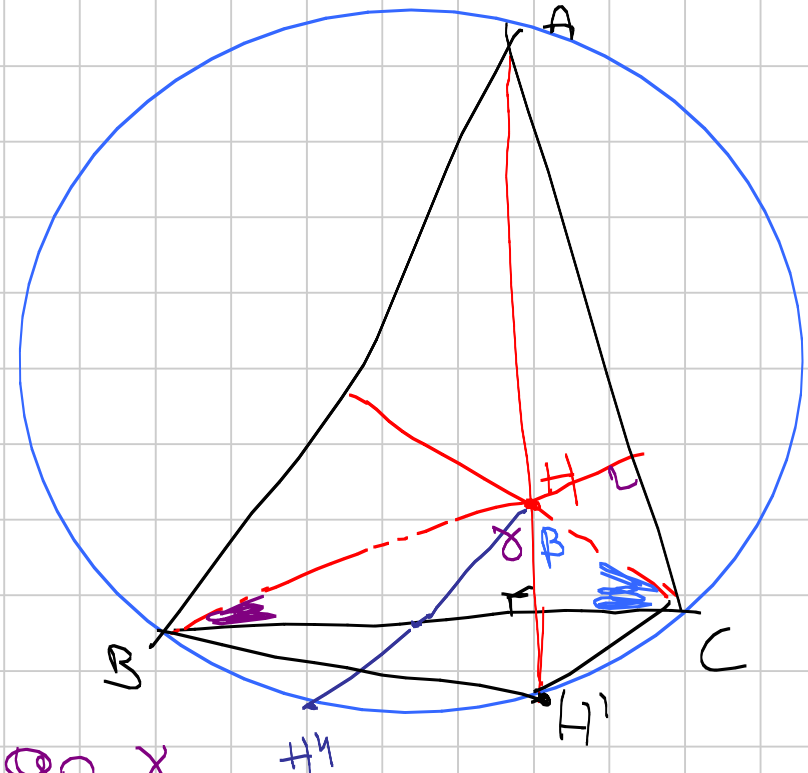


LEMMA DEL SIMM DELL'ORTOCENTRO

le simm dell'ortoc rispetto a un lato o a un pto medio sta sul circocerchio.

ABH'C è ciclico?

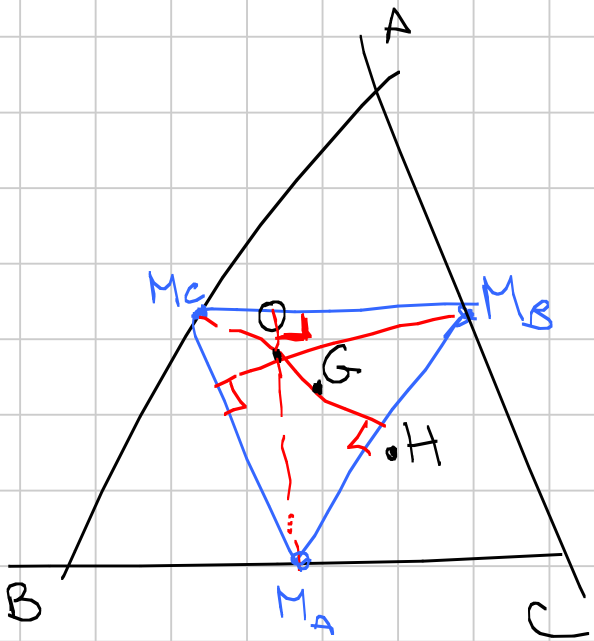
$\hat{A} + \hat{H'} = 180$



$= 90 - \gamma$

$B\hat{H}'C = B\hat{H}C = \gamma + \beta = 180 - \alpha$

RETTA DI EULERO



O, G, H sono allineati

$\overrightarrow{OG} = \frac{1}{3}\overrightarrow{OH}$

$\frac{A+B+C}{3} = \frac{1}{3}\overrightarrow{OH}$

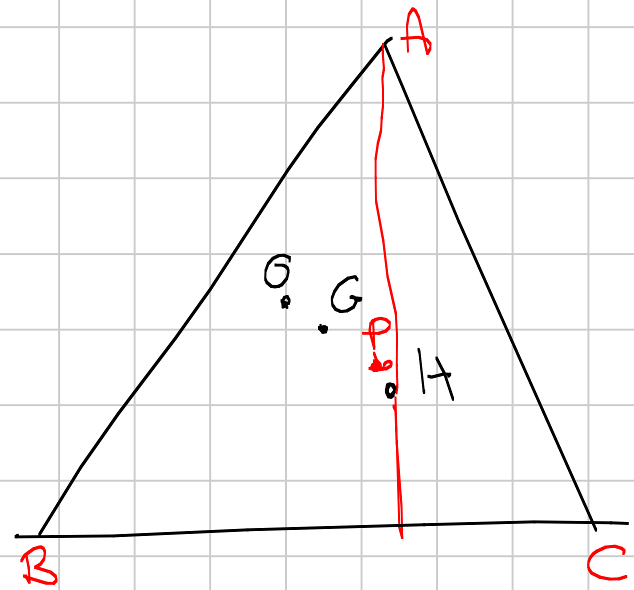
Omotetia di centro G e rapporto $-\frac{1}{2}$

$H \longrightarrow H'$ = ortocentro di $M_A M_B M_C = O$

OM_A = asse di BC = altezza triangolo $M_A M_B M_C$

$\frac{1}{2} GH = OG$

$H = A + B + C$



$P = A + B + C$

$\overrightarrow{AP} \perp \overrightarrow{BC}$

$\overrightarrow{BP} \perp \overrightarrow{AC}$

$\overrightarrow{CP} \perp \overrightarrow{AB}$

$(P - A) \cdot (C - B) \overset{?}{=} 0$

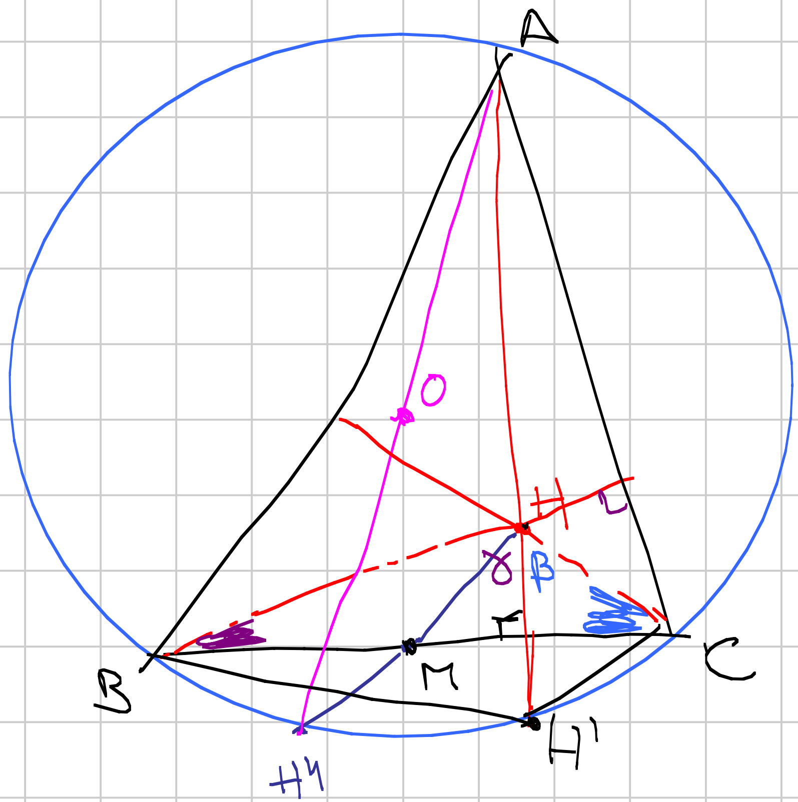
$(B + C) \cdot (C - B) = (B + C) \cdot C - (B + C) \cdot B$

$= |C|^2 - |B|^2 = R^2 - R^2 = 0 \qquad C \cdot C = |C|^2$

$H = A + B + C$

il simm di H rispetto ad M (pto media) sta sulla circonferenza.

$|H''|^2 = R^2$



$M = \dfrac{B+C}{2}$

$H'' = 2M - H = B + C - (A+B+C) = -A$

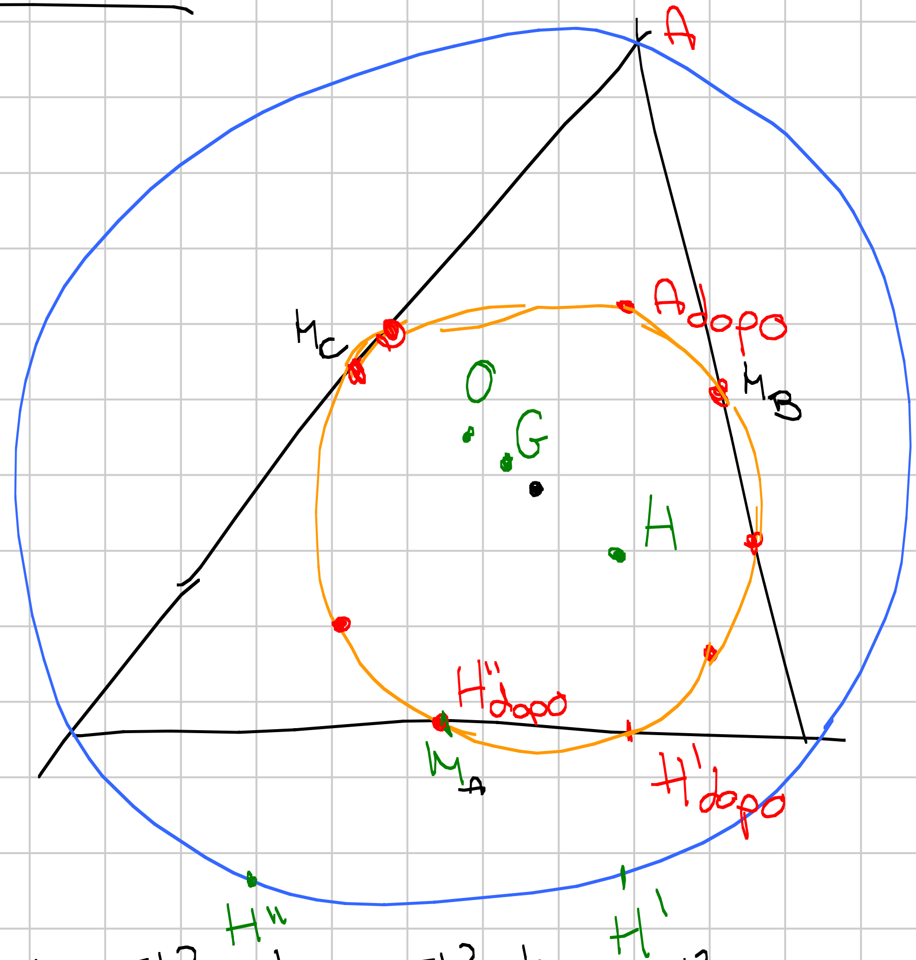
$|H''|^2 = |-A|^2 = R^2$

Circonf di Feuerbach

omotetia di centro H e rapporto $\frac{1}{2}$

A_{dopo} = pto medio AH

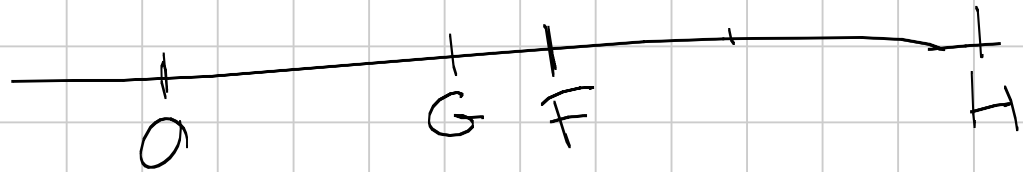
- piedi delle altezze
- pti medi dei lati
- i pti medi delle cong vertice - ortocentro.

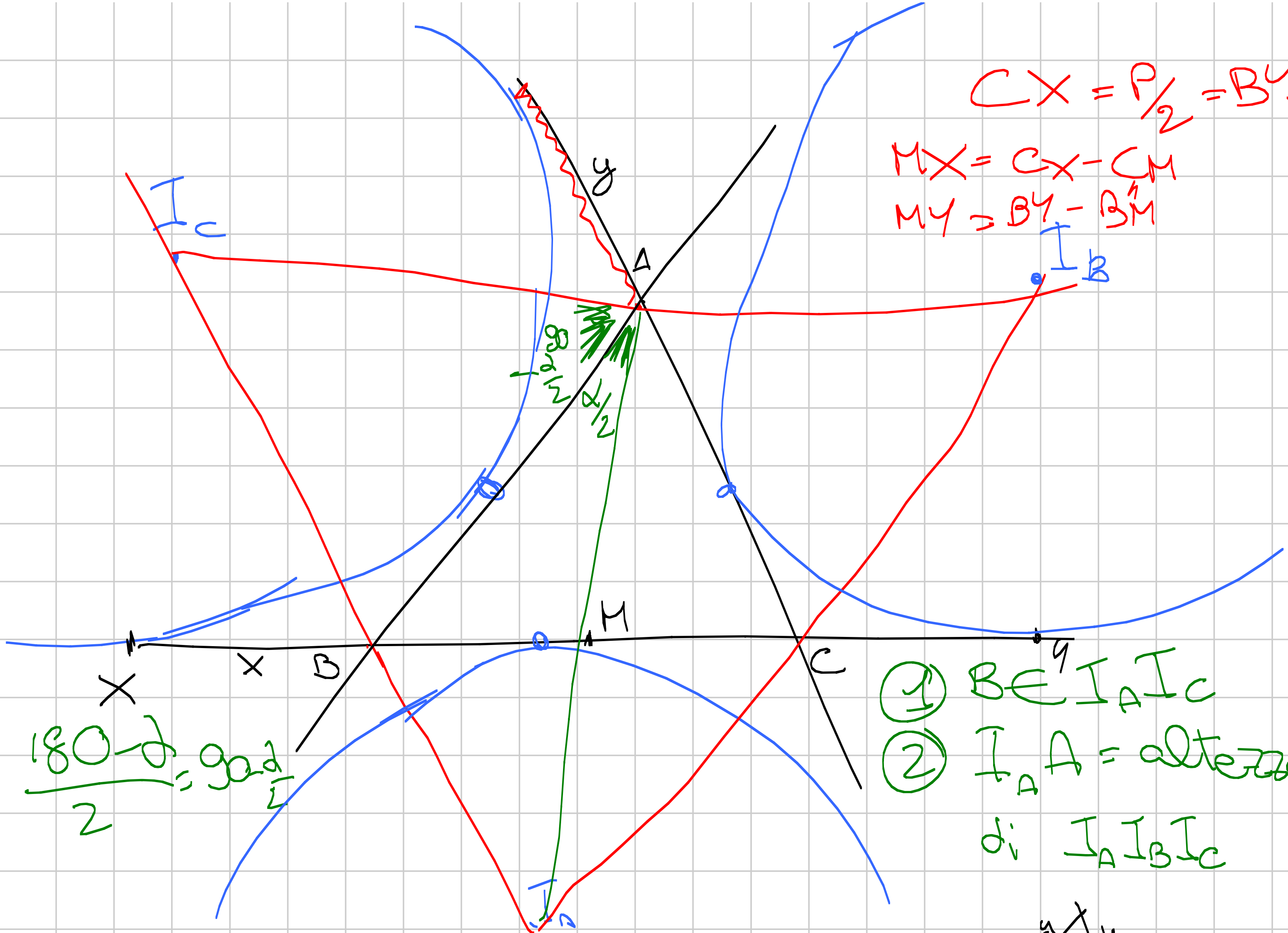


$F = \frac{A+B+C}{2}$ $M_C = \frac{A+B}{2}$ $M_A = \frac{B+C}{2}$ $M_B = \frac{C+A}{2}$

$|M_A - F|^2 = |M_B - F|^2 = |M_C - F|^2$

- Feuerbach tange la circ inscritta e i 3 excerchi.





$CX = P/2 = BY$

$MX = CX - CM$

$MY = BY - BM$

$$\frac{180-\alpha}{2} = 90 - \frac{\alpha}{2}$$

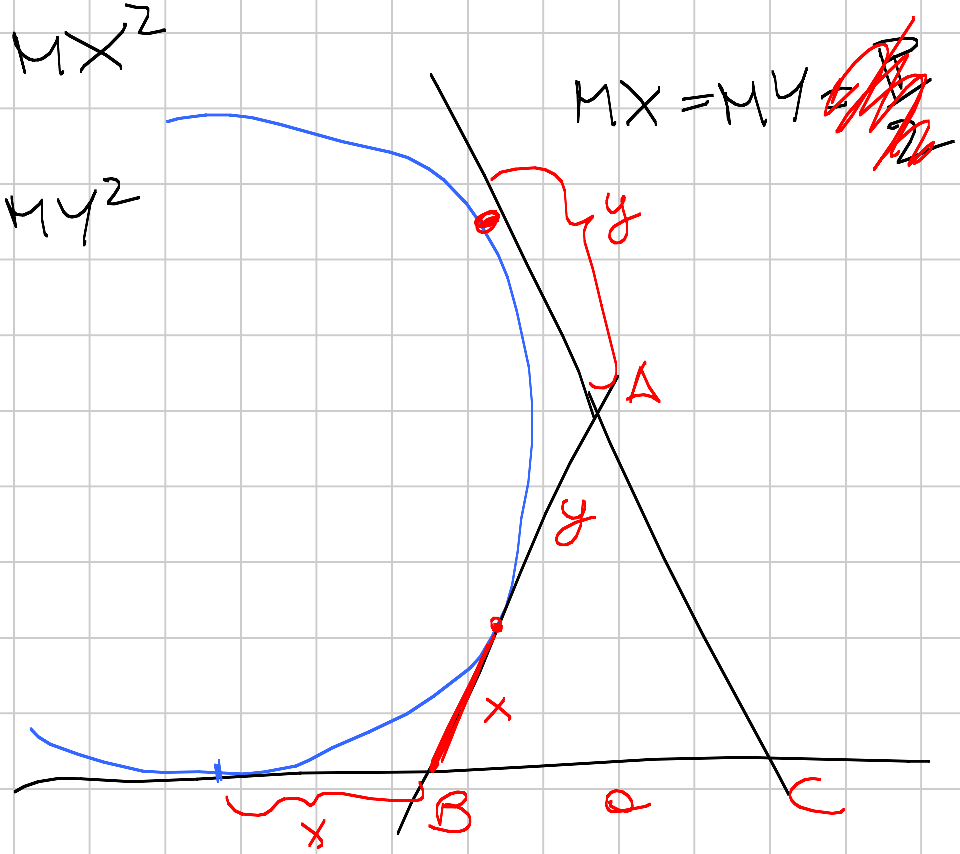
(1) $B \in I_A I_C$

(2) $I_A A$ = altezza di $I_A I_B I_C$

ASSE radicale di 2 ex-cerchi
passa per il pto medio del lato

$$Pow_{\Gamma_C}(M) = MX^2$$

$$Pow_{\Gamma_B}(M) = MY^2$$



$$x+y=c$$

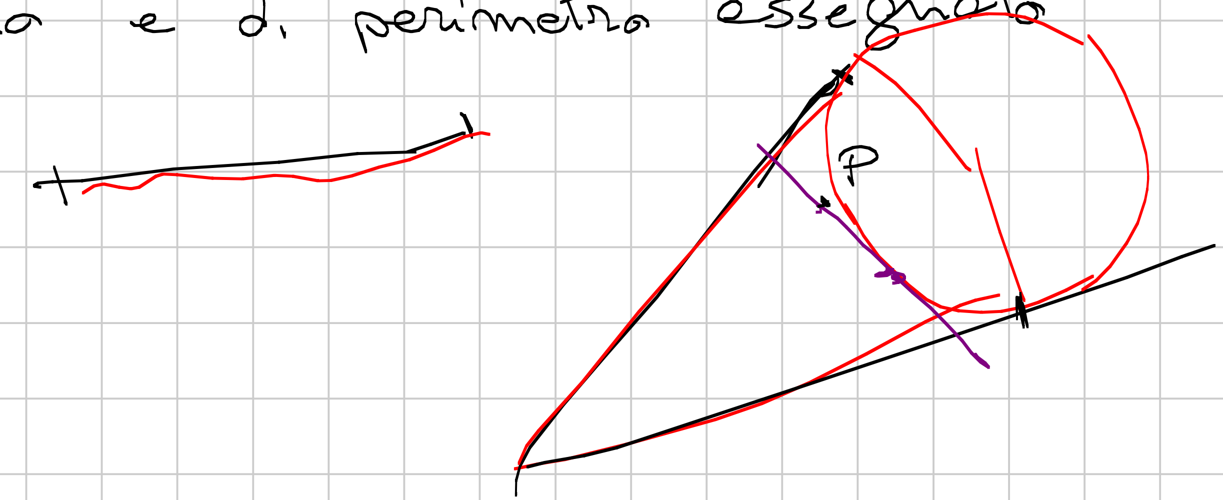
$$b+y=a+x$$

$$x+\cancel{y}+a+x=c+b+\cancel{y} \Rightarrow x=\frac{c+b-a}{2}$$

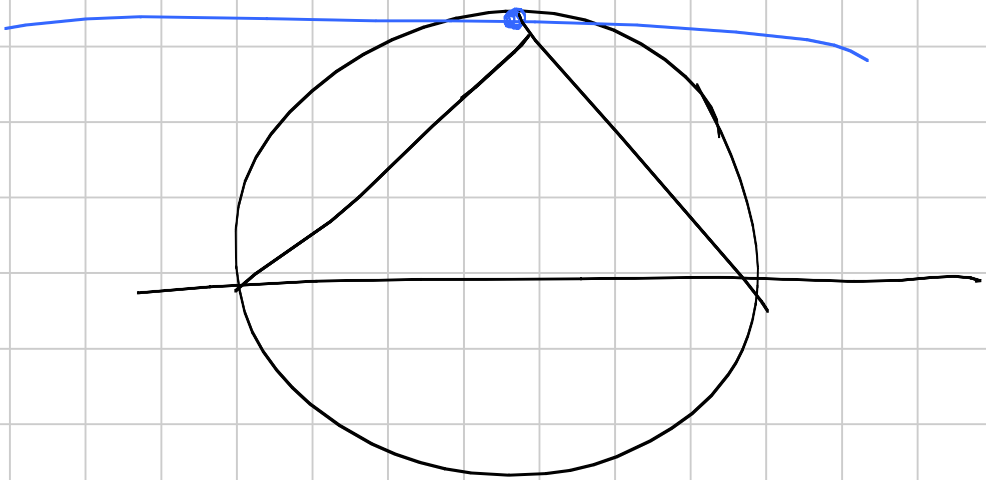
$$BE+x=\frac{c+b-a}{2}+a=\frac{a+b+c}{2}=P/2$$

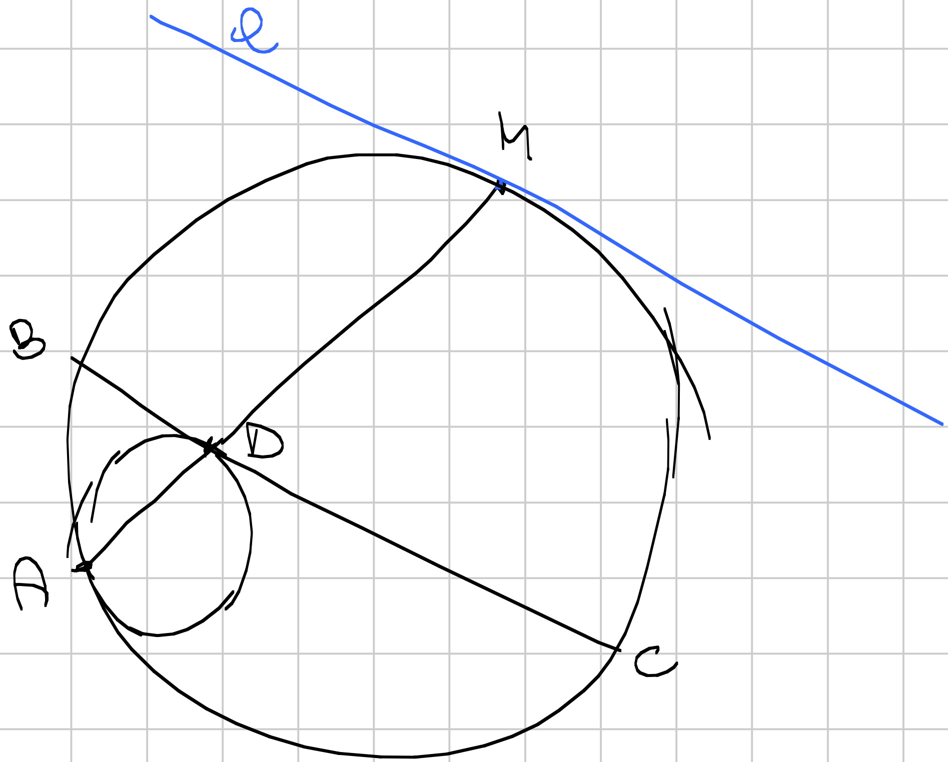
ES dato un angolo ~~acuto~~ e pto P interno

Trovare un triangolo che abbia 2 lati sull'angolo e di perimetro assegnato



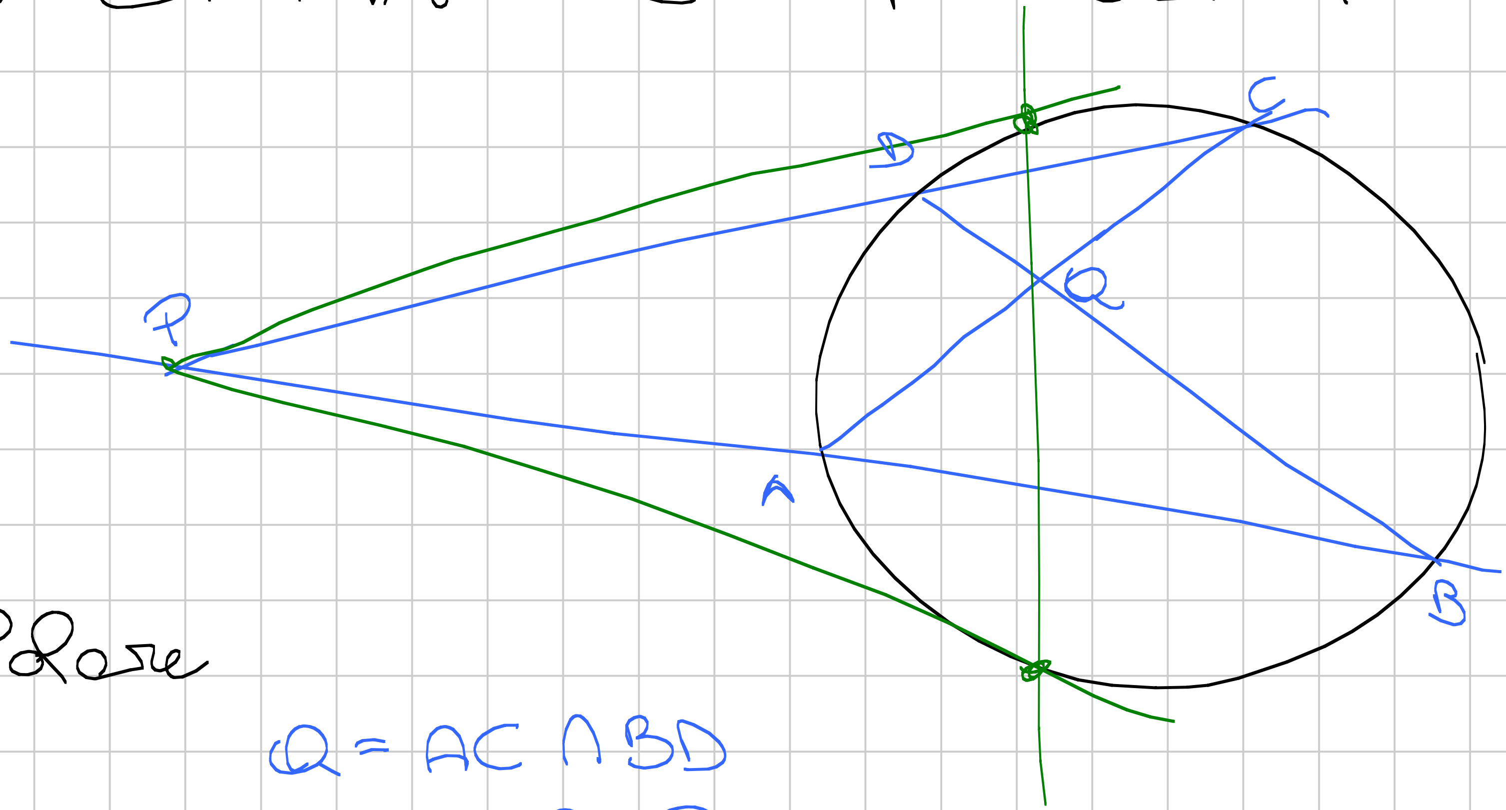
$\overset{\frown}{BM} = \overset{\frown}{MC}$





$\overset{\frown}{MB} = \overset{\frown}{MC}$

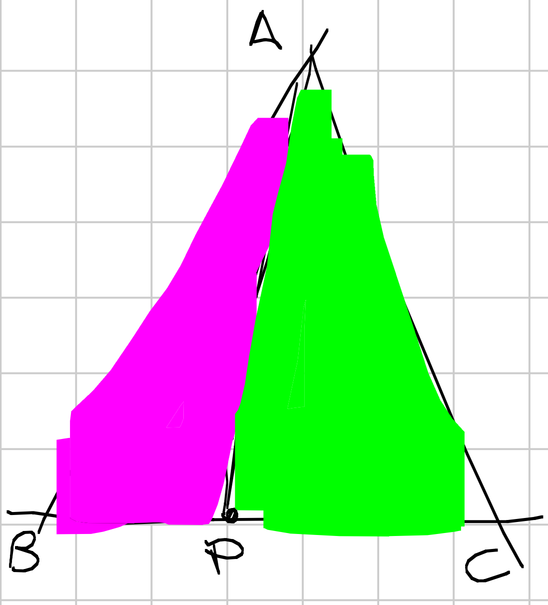
LEMMA DELLA POLARE



Polare

$Q = AC \cap BD \in pol\, P$

Lemma 1: $\dfrac{PB}{PC} = \dfrac{AB}{AC} \cdot \dfrac{\sin \widehat{PAB}}{\sin \widehat{PAC}}$



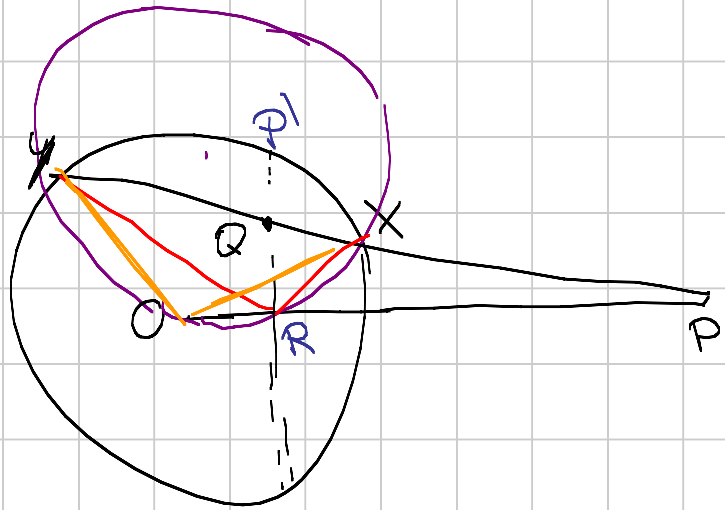
$$\frac{PB}{\sin \widehat{PAB}} = \frac{AB}{\sin \hat{P}}$$

$$\frac{PC}{\sin \widehat{PAC}} = \frac{AC}{\sin \hat{P}}$$

$$\frac{PB}{PC} \cdot \frac{\sin PAC}{\sin PAB} = \frac{AB}{AC}$$

Lemma 2

$$\frac{YP}{PX} = \frac{YQ}{QX}$$



$YORX$ sono conciclici

$$PO \cdot PR = PP'^2 = pow_\Gamma(P) = PX \cdot PY$$

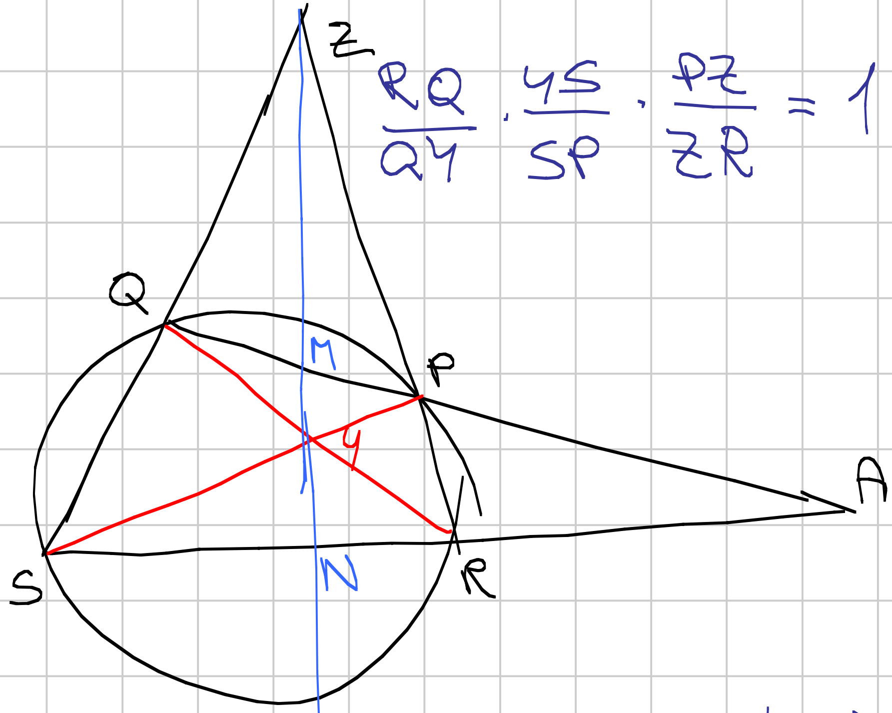
RQ è bisettrice di $Y\hat{R}X$

$Y\hat{R}O = X\hat{R}P$ $\qquad X\hat{R}P = 180 - X\hat{R}O = X\hat{Y}O = Y\hat{R}O$

$Y\hat{R}O =$

$$\frac{YQ}{QX} = \frac{YR}{RX} = \frac{YP}{PX}$$

$$\frac{QA}{PA} = \frac{QM}{MP}$$



$$\frac{RQ}{QY} \cdot \frac{YS}{SP} \cdot \frac{PZ}{ZR} = 1$$

PQS con pto A

$$\frac{QA}{PA} = \frac{SQ}{PS} \cdot \frac{\sin ASQ}{\sin ASP}$$

YQS con pto R

$$\frac{QR}{YR} = \frac{QS}{SY} \cdot \frac{\sin Q\hat{S}R}{\sin Y\hat{S}R}$$

$$\frac{QA}{PA} = \frac{\cancel{SQ}}{PS} \cdot \frac{QR}{YR} \cdot \frac{SY}{\cancel{QS}}$$

$$\frac{QR}{\cancel{YR}} \cdot \frac{SY}{PS} = \frac{QY}{\cancel{YR}} \cdot \frac{ZR}{ZP}$$

QPZ con pto M

$$\frac{QM}{MP} = \frac{ZQ}{ZP} \cdot \frac{\sin QZM}{\sin MZP}$$

ZQR con pto Y

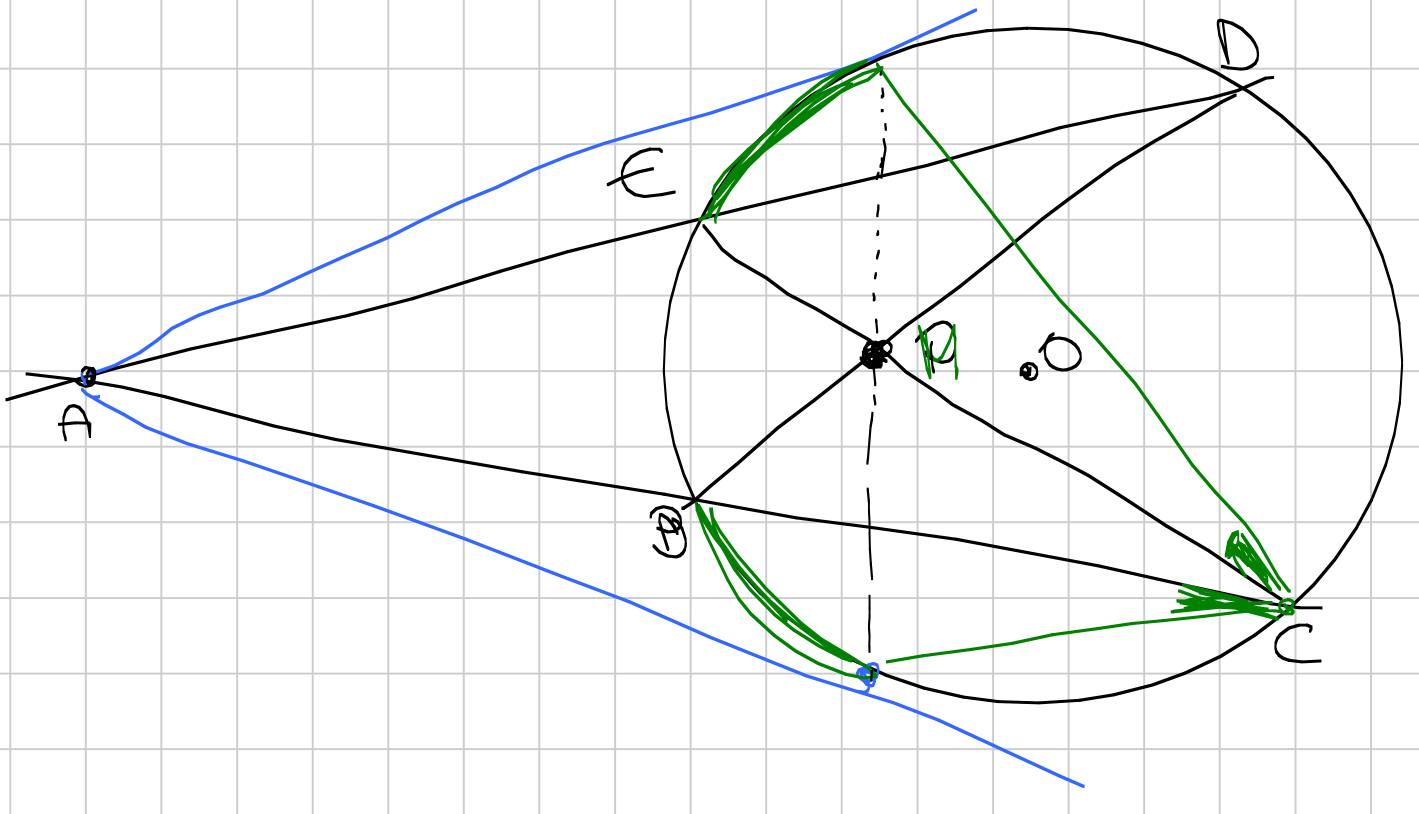
$$\frac{QY}{YR} = \frac{QZ}{ZR} \cdot \frac{\sin QZY}{\sin YZR}$$

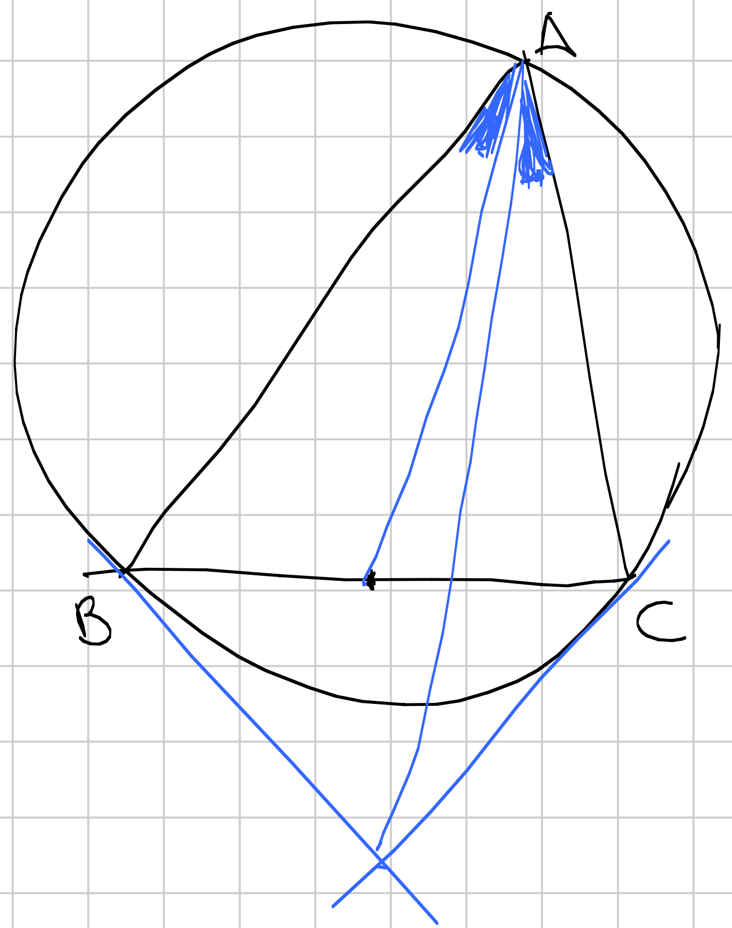
$$\frac{QM}{MP} = \frac{\cancel{ZQ}}{ZP} \cdot \frac{QY}{YR} \cdot \frac{ZR}{\cancel{QZ}}$$

$$\frac{QR}{QY} \cdot \frac{SY}{PS} \cdot \frac{ZP}{ZR} = 1$$

$$\frac{RQ}{QY} \cdot \frac{YS}{SP} \cdot \frac{PZ}{ZR} = 1$$

LEMMA DELLA SIMMEDIANA




A
B
C