$$\sin\theta = \frac{PQ}{OP} \qquad \cos\theta = \frac{OQ}{OP} \qquad \tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{RT}{OP}$$
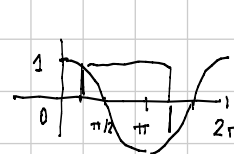

sin   cos   tan

$$f(-x) = -f(x)$$
$$\sin(\pi+x) = -\sin(x)$$

$$f(x) = f(-x)$$
$$\cos(\pi+x) = -\cos x$$

$$\beta + \gamma = \pi/2$$
$$\sin\gamma = \frac{AB}{BC} = \cos\beta$$

$$\sin\left(\frac{\pi}{2}-\beta\right) = \cos(\beta)$$
$$\sin^2\gamma + \cos^2\gamma = 1$$

F. di addizione

$$\begin{cases} \sin(x+y) = \sin x \cdot \cos y + \sin y \cdot \cos x \\ \cos(x+y) = \cos x \cdot \cos y - \sin x \cdot \sin y \\ \sin(x-y) = \sin x \cdot \cos y - \sin y \cdot \cos x \end{cases}$$

$$\cos(x-y) = \cos x \cdot \cos y + \sin x \cdot \sin y$$

$$e^{ix} = u \begin{cases} e^{ix} = \cos x + i\sin x \\ e^{-ix} = \cos x - i\sin x \end{cases}$$

$$\cos x = \frac{e^{ix}+e^{-ix}}{2} \qquad \sin x = \frac{e^{ix}-e^{-ix}}{2i}$$

Werner - prostaferesi

- $\sin(x+y) + \sin(x-y) = 2\sin x \cos y$
- $\cos(x+y) + \cos(x-y) = 2\cos x \cos y$
- $\sin(x+y) - \sin(x-y) = 2\sin y \cos x$
- $\cos(x+y) - \cos(x-y) = -2\sin x \sin y$

$$\begin{cases} x+y = u \qquad x = (u+v)/2 \\ x-y = v \qquad y = (u-v)/2 \end{cases}$$

$$\sin u + \sin v = 2\sin\frac{u+v}{2}\cos\frac{u-v}{2}$$

$$\sin(2x) = \sin(x+x) = 2\sin x \cos x$$

$$\cos(2x) = \cos(x+x) = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$$

$$\tan(2x) = \frac{2\sin x \cos x}{\cos^2 x - \sin^2 x} = \frac{2\tan x}{1 - \tan^2 x} \qquad \Big| \qquad \tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

$$\underline{\cos(n \arccos x)} \overset{?}{=} \text{polinomio !}$$
di Chebyshev del 2° tipo

$$\cos\theta = u$$
$$\cos n\theta \overset{?}{=}$$

$$T_n(x)$$
$$\partial T_n = n$$

$$\begin{cases} T_0(x) = 1 \\ T_1(x) = x \\ T_2(x) = 2x^2 - 1 \end{cases}$$

$$\boxed{T_{n+2}(x) = 2x\, T_{n+1}(x) - T_n(x)}$$
$$x = \cos y$$

$$\cos((n+2)y) = 2\cos y \cos((n+1)y) - \cos(ny) \qquad \underline{\text{Werner !}}$$

$$U_n(x) = \frac{\sin((n+1)\arccos x)}{\sin(\arccos x)}$$

$$U_0(x) = 1$$
$$U_1(x) = 2x$$
$$U_2(x) = 4x^2 - 1$$

$$\sin(\arccos x) = \sqrt{1 - x^2}$$

$$U_1(x) = \frac{2\sin(\arccos x)\cos(\arccos x)}{\sin(\arccos x)} = 2x$$

$$\sin(3\theta) = \sin(\theta + 2\theta) = \sin\theta\cos 2\theta + \cos\theta\sin 2\theta = \sin\theta\cos 2\theta + 2\sin\theta\cos^2\theta$$

$$\sin\theta\left(2\cos^2\theta - 1 + 2\cos^2\theta\right) = \sin\theta\left(4\cos^2\theta - 1\right)$$

$$U_2(x) = \frac{\sin(3\arccos x)}{\sin(\arccos x)} = \frac{\sin(\arccos x)(4x^2 - 1)}{\sin(\arccos x)}$$

$$\boxed{U_{n+2}(x) = 2x\, U_{n+1}(x) - U_n(x)} \qquad \text{Werner !}$$

Quali sono le radici di $T_n(x)$?

$$\cos(n\arccos x) = 0$$

$$x = \cos\left(\frac{\pi}{2n}\right), \cos\left(\frac{3\pi}{2n}\right), \dots \qquad \leftarrow \begin{cases} n\arccos x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \dots \\ \arccos x = \frac{\pi}{2n}, \frac{3\pi}{2n}, \frac{5\pi}{2n}, \dots \end{cases}$$

Quali sono le radici di $U_n(x)$? $\quad \sin\left(\frac{\pi}{2n}\right), \dots$

$$\sum_{n=1}^{10^3} \cos\left(\frac{n\pi}{15\cdot7^8}\right) = \qquad\qquad \sum_{n=1}^{10^3} e^{i\frac{n\pi}{15\cdot7^8}} = \sum_{n=1}^{10^3} (u)^n$$

$$\cos x = \frac{e^{ix} + e^{-ix}}{2} \qquad\qquad u \doteq e^{i\frac{\pi}{15\cdot7^8}}$$

$$\qquad\qquad\qquad\qquad\qquad \longrightarrow \left|\,\left|\frac{u^{10^3+1}-1}{u-1}\right|\,\right|$$

$$\sum_{n=1}^{5} \cos\left(\frac{(2n-1)\pi}{10}\right) \overset{?}{=}$$

le radici di T_5

$$\sum_{n=1}^{5} \cos^2\left(\frac{(2n-1)\pi}{10}\right)$$

la somma delle radici $\underset{\longrightarrow}{\text{Viète}}$ i coeff. del polinomio

$\qquad\qquad\qquad\qquad\qquad\qquad$ f. sym delle radici

$$\overset{\|}{\underset{}{\frac{T_5(0)}{2^5}}} = 0 \qquad\qquad \overset{x}{2x^2-1}$$

$$\qquad\qquad\qquad\qquad 4x^3 - 3x$$

$$\qquad\qquad\qquad\qquad 8x^4 - 8x^2 + 1$$

$$\qquad\qquad\qquad\qquad 16x^5 - 16x^3 + 2x - 4x^3 + 6x$$

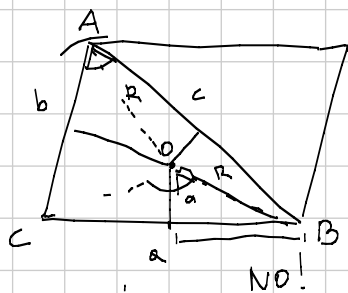
Bisezione

$$\cos 2x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$$

$$\cos^2 x = \frac{\cos 2x + 1}{2} \qquad \sin^2 x = \frac{1 - \cos 2x}{2} \left.\right\} \quad \cos x = \sqrt{\frac{1 + \cos 2x}{2}} \qquad \sin x = \sqrt{\frac{1 - \cos 2x}{2}}$$

$$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \cos\frac{x}{2} = \sqrt{\frac{1 + \cos x}{2}} \qquad \sin\frac{x}{2} = \sqrt{\frac{1 - \cos x}{2}}$$

$$\tan\frac{x}{2} = \sqrt{\frac{1 - \cos x}{1 + \cos x}} = \sqrt{\frac{1 - \cos^2 x}{(1 + \cos x)^2}} = \frac{\sin x}{1 + \cos x} \,\|$$

NO!

T. del seno

$$\triangle = \frac{1}{2} ab \sin\gamma = \frac{1}{2} ac \sin\beta$$

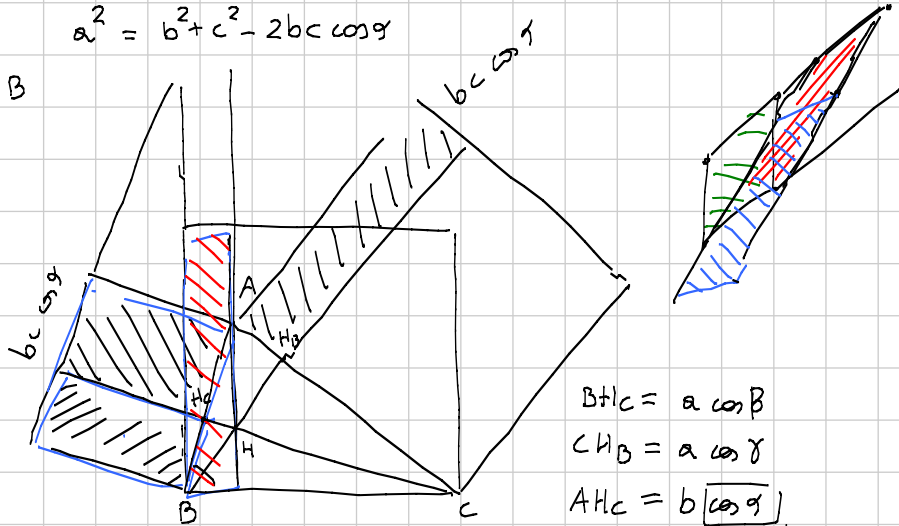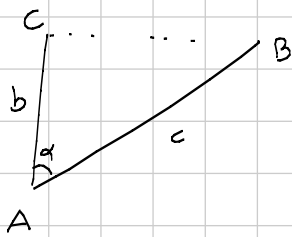$$\frac{\sin\gamma}{c} = \frac{\sin\beta}{b} = \frac{\sin\alpha}{a} = \frac{1}{2R}$$

$$R\sin\alpha = \frac{a}{2}$$

$$a = 2R\sin\alpha \qquad \frac{a}{\sin\alpha} = 2R$$
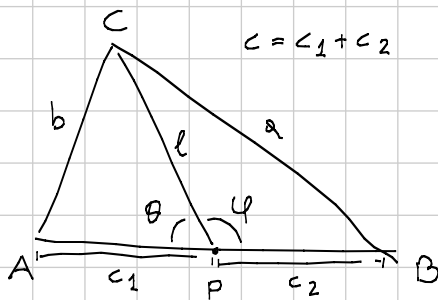
T. del coseno

$$a^2 = b^2 + c^2 - 2bc\cos\varphi$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$BH_c = a\cos\beta$$
$$CH_B = a\cos\gamma$$
$$AH_c = b\boxed{\cos\varphi}$$
$$AH_B = c\boxed{\cos\varphi}$$

$$(c^2 - bc\cos\varphi) + (b^2 - bc\cos\varphi)$$
$$= a^2$$

T. Stewart.

$$c = c_1 + c_2$$

$$\cos\theta = \frac{c_1^2 + \ell^2 - b^2}{2\ell c_1}$$

$$\cos\varphi = \frac{c_2^2 + \ell^2 - a^2}{2\ell c_2}$$

$$c_2(c_1^2 + \ell^2 - b^2) + c_1(c_2^2 + \ell^2 - a^2) = 0$$

$$c_2 c_1 c + c\ell^2 = c_2 b^2 + c_1 a^2$$

$$c(c_2 c_1 + \ell^2) = c_2 b^2 + c_1 a^2$$

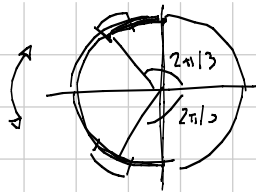$$c_2 +_1 + \ell^2 = \frac{c_2 b^2 + c_1 a^2}{c}$$

$$\left| \ell^2 = \frac{c_2 b^2 + c_1 a^2}{c} - c_1 c_2 \right|$$

2) $\cos(2^n\theta) < 0 \quad \forall n \in \mathbb{N}$

$\theta = \dfrac{2\pi}{3}, \dfrac{4\pi}{3}$

$\underset{\overset{\|}{a_0}}{\cos\theta} \qquad \underset{\overset{\|}{a_1}}{\cos 2\theta}$

$\underset{x}{\|} \qquad \underset{2x^2-1}{\|}$

$a_{n+1} = 2a_n^2 - 1$

$\theta = \pi\left(1 + \dfrac{1}{2} - \dfrac{1}{4} + \dfrac{2}{8} - \cdots\right) =$

$\pi \displaystyle\sum_{n=0}^{+\infty} 2^{-n} = \pi \cdot \dfrac{1}{1+\frac{1}{2}} = \dfrac{2\pi}{3}$

il pt fisso di
$2x^2 - 1 = x$
è iperbolico
(repulsivo)

$f(x) = ax^2 + bx + c$

$f^{(n)}(x)$ si scrive in termini di f, elementari
se e solo se $\exists\, \alpha, \beta$

$f(x) = \dfrac{2(\alpha x + \beta)^2 - 1 - \beta}{\alpha}$

Cor. $x^2 + 2$, $x^2 - 1/2$

$g(x) = 2x^2 - 1 = T_2(\cos\theta) \qquad x \doteq \cos\theta$

$g^{(n)}(x) = T_{2^n}(\cos\theta) = \cos(2^n \arccos\theta)$

1- Dividiamo per $4R^2$

$\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$

$1 - \cos^2\alpha + 1 - \cos^2\beta + 1 - \cos^2\gamma = 2$

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\dfrac{\cos 2\alpha + 1}{2} + \dfrac{\cos 2\beta + 1}{2} + \dfrac{\cos 2\gamma + 1}{2} = 1$

$\displaystyle\sum_{cyc} \cos 2\alpha = -1 \qquad \cos 2\alpha + \cos 2\beta = \cos\big((\alpha+\beta)+(\alpha-\beta)\big) + \cos\big((\alpha+\beta)-(\alpha-\beta)\big)$

$\overset{\|}{}$

$2\cos(\alpha+\beta)\cos(\alpha-\beta) = -2\cos\gamma\cos(\alpha-\beta)$

$\cos 2\gamma = 2\cos^2\gamma - 1$
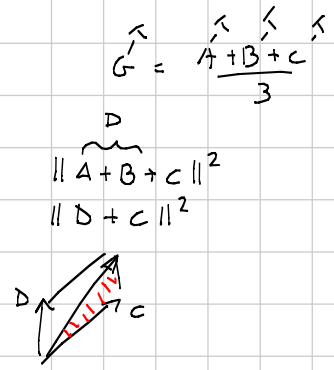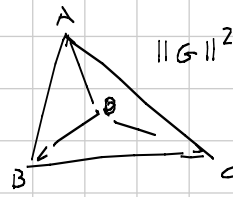
$\cos\gamma\big(\cos\gamma - \cos(\alpha-\beta)\big) = 0$

$\cos\gamma\big(\cos(\alpha+\beta) + \cos(\alpha-\beta)\big) = 0$

$\displaystyle\prod_{cyc} \cos\alpha = 0$

$$\sin u \; \sin v \; \sin w$$

$$= \frac{1}{8}\left(\sin(u+v-w) + \sin(u-v+w) + \sin(-u+v+w) - \sin(\overbrace{u+v+w})\right)$$

$$OH^2 = 9R^2 - (a^2+b^2+c^2)$$

$$\left.\begin{array}{c} H \in \overline{\lceil_{ABC}} \\ \overline{U_{AB}}(H) \in \overline{\lceil_{ABC}} \end{array}\right\} \begin{array}{c} H \in \text{lati} \\ \downarrow \\ H \in \text{vertici} \\ \downarrow \\ ABC \perp \end{array}$$

$$O \equiv H \qquad \text{assi} \equiv \text{altezze} \quad \text{su ogni lato } ABC \text{ è isoscele} \longrightarrow ABC \text{ equilatero}$$

$$\widehat{G} = \frac{\widehat{A}+\widehat{B}+\widehat{C}}{3}$$

$$\|G\|^2$$

$$\|\overbrace{A+B}+C\|^2$$

$$\|D+C\|^2$$

1.

$$\underbrace{\sin\alpha \cdot \sin\alpha} + \underbrace{\sin\beta \cdot \sin\beta} = \sin\alpha \cos\beta + \sin\beta \cos\alpha \qquad 0 < \alpha, \beta < \pi/2$$

$$\underline{\sin\alpha \; \sin\left(\frac{\pi}{2}-\beta\right)} + \underline{\sin\beta \; \sin\left(\frac{\pi}{2}-\alpha\right)}$$

$$\left|\begin{array}{c} \alpha+\beta > \pi/2 \\ \alpha > \pi/2 - \beta \\ \beta > \pi/2 - \alpha \end{array}\right|$$

$$\alpha+\beta = \pi/2 \; .$$

__Erone__

$$\triangle = \frac{1}{2} ab \sin\gamma \qquad \triangle^2 = \frac{1}{4} a^2 b^2 (1+\cos\gamma)(1-\cos\gamma)$$
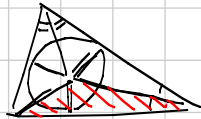
$$\cos\gamma = \frac{a^2+b^2-c^2}{2ab} \qquad 1+\cos\gamma = \frac{(a+b)^2 - c^2}{2ab} = \frac{(a+b+c)(a+b-c)}{2ab}$$

$$p = \frac{a+b+c}{2}$$

$$\triangle^2 = p(p-a)(p-b)(p-c)$$

$$\triangle = \frac{abc}{4R} = \frac{1}{2}ab\left(\frac{c}{2R}\right) = \frac{1}{2}ab\,\sin\gamma$$
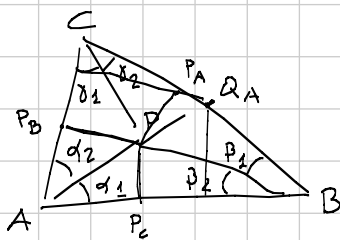
$$\triangle = p\,r$$

$$1 + \cos g = 2\cos^2\frac{g}{2}$$

Dalle f. di birezione regue

$$\sin\frac{g}{2} = \sqrt{\frac{(P-b)(P-c)}{bc}} \qquad \cos\frac{g}{2} = \sqrt{\frac{P(P-a)}{bc}} \qquad (Briggs)$$

Nepero

$$\bigcirc \quad \frac{b+c}{b-c} = \frac{\tan\frac{\beta+\gamma}{2}}{\tan\frac{\beta-\gamma}{2}} = \frac{\sin(\beta+\gamma)}{\sin(\beta-\gamma)}\left(\frac{\cos\frac{\beta-\gamma}{2}}{\cos\frac{\beta+\gamma}{2}}\right)^2 = \frac{\sin\beta\cos\gamma + \sin\gamma\cos\beta}{\sin\beta\cos\gamma - \sin\gamma\cos\beta}\left(\frac{2\cos\frac{\beta-\gamma}{2}\sin\frac{\beta+\gamma}{2}}{2\cos\frac{\beta+\gamma}{2}\sin\frac{\beta+\gamma}{2}}\right)^2$$

$$= \frac{(a^2+b^2-c^2)+(a^2-b^2+c^2)}{(a^2+b^2-c^2)-(a^2-b^2+c^2)}\left(\frac{\sin\beta+\sin\gamma}{\sin g}\right)^2 = \frac{a^2}{b^2-c^2}\left(\frac{b+c}{a}\right)^2 \quad \bigcirc$$
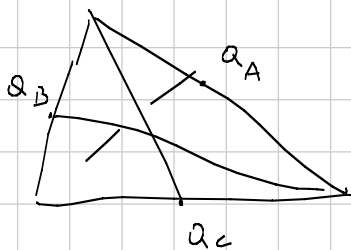

T. di Ceva

$$\frac{AP \cdot BP \cdot CP}{BP \cdot CP \cdot AP} = 1 \qquad \prod_{cyc}\sin q_1 = \prod_{cyc}\sin q_2$$
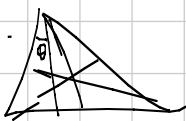
$$PAB: \quad \frac{AP}{BP} = \frac{\sin\beta_2}{\sin q_1}$$

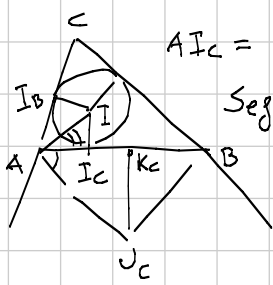$$\left(\frac{PP_A \cdot PP_B \cdot PP_C}{PP_B \cdot PP_C \cdot PP_A} = 1\right)$$

$$\boxed{\frac{BQ_A \cdot CQ_B \cdot AQ_C}{CQ_A \cdot AQ_B \cdot BQ_C}} = 1$$

Q_B Q_A

Q_C

CQ_C, BQ_B, AQ_A concorrono se e solo se vale una tra

4. (T. Carnot)

$AI_c = \dfrac{b+c-a}{2} = p-a$ $\widehat{K_cAJ_c} = \dfrac{\pi-\gamma}{2}$

Segm. di Soddy $r_c = J_cK_c$ $\dfrac{J_cK_c}{AK_c} = \dfrac{1}{\tan\frac{\gamma}{2}} = \dfrac{1+\cos\gamma}{\sin\gamma}$
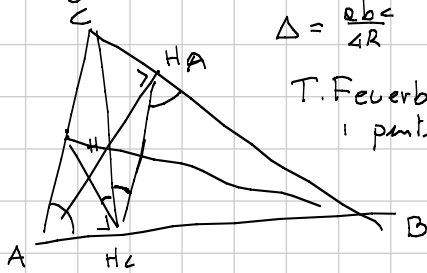
$AK_c = \dfrac{a+c-b}{2}$

$\dfrac{abc}{4R} = \Delta$ $R = \dfrac{abc}{4\Delta}$

$$abc = \left(\sum_{cyc} p(p-b)(p-c)\right) - \prod_{cyc}(p-a) = g(a,b,c)$$

$$\begin{cases} (a \mid g(a,b,c) \leftarrow g(0,b,c)=0)_{cyc} \\ g(1,1,1)=1 \end{cases}$$

$$p^2(p-c) + p^2(p-b) = p^2(2p-b-c) = ap^2 = 0.$$

5. (Disug. Eulero $R \geq 2r$)

$\Delta = \dfrac{abc}{4R}$

T. Feuerbach
i punti medi dei lati e i piedi delle alt. stanno sulla stessa Γ

Il raggio della circ. circoscritta a $H_AH_BH_c = R/2$

$BH_A \cdot BC = BH_c \cdot BA$

BH_AH_c e BCA sono simili

$\dfrac{H_cH_A}{b} = \dfrac{BH_A}{c} = \dfrac{c\cos\beta}{c} = \cos\beta$

$H_AH_c = b\cos\beta$

$[H_AH_BH_c] = \dfrac{abc \prod_{cyc}\cos\gamma}{2R}$ $[ABC] = \dfrac{abc}{4R}$

$$\prod_{cyc} \cos\alpha \leq \frac{1}{8}$$

$\updownarrow$

$$\sum \cos 2\alpha \geq k$$

$\updownarrow$

$$\sum \cos^2\alpha \geq k$$

$\updownarrow$

$$\sum \sin^2\alpha \geq k \quad \xrightarrow{\cdot R^2}$$

$$ABC$$
$$[I_A I_B I_C] = 2p \cdot R$$

$$R \geq 2r$$

$$9R^2 - (a^2+b^2+c^2) \geq 0$$
$$\overset{\shortparallel}{OH^2}$$

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6. $a, b, c \in \mathbb{R}^+$
   $A, B, C \in \mathbb{R}^+$

   $\begin{cases} a = B+C \\ b = A+C \\ c = A+B \end{cases}$

   $(a+b > c)_{cyc}$ $a+b+c > 2 \operatorname{Max}(a,b,c)$

   $\left(\sqrt{\frac{1}{2}a^2 + \frac{1}{2}b^2 - \frac{1}{4}c^2} \right)_{cyc}$

   le mediane formano un triangolo !

   $$4M + 3P = 6P$$
   $$M = \frac{3}{4}P.$$