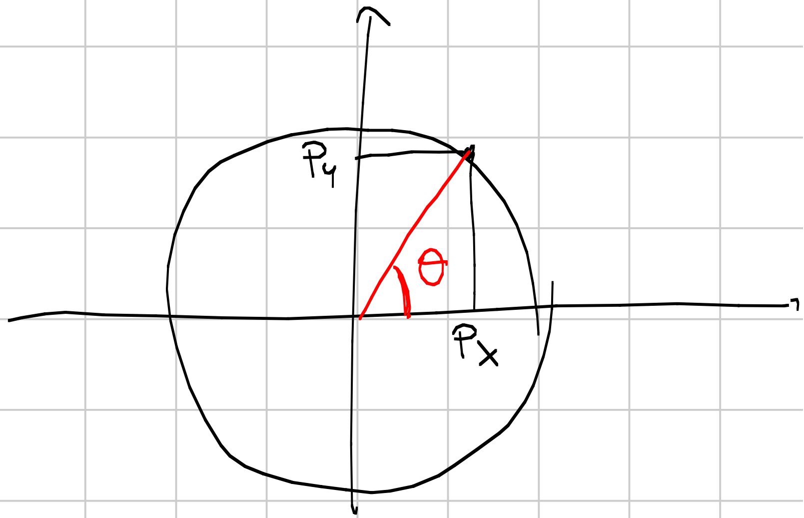
G 1 TRIGONOMETRIA BASIC

Titolo nota 07/09/2009

$P = (P_x, P_y)$

$P_x^2 + P_y^2 = 1$



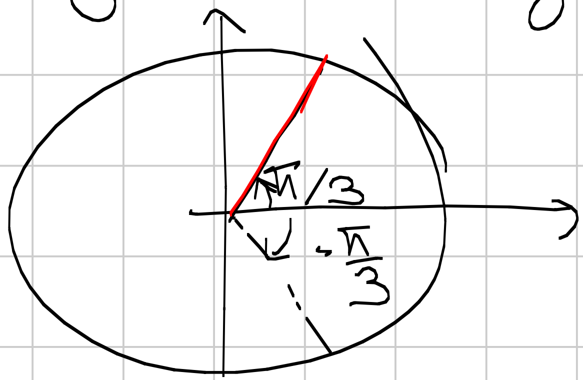
$P_x = \cos\theta \qquad P_y = \sin\theta$

Gradi : radianti

$360 : 2\pi = \theta° : \theta$

$\cos^2\theta + \sin^2\theta = 1$ (RELAZIONE FONDAMENTALE)

Angoli con segno



$\frac{\pi}{3} + 2\pi = \frac{7}{3}\pi$

$\sin(\theta + 2\pi) = \sin\theta$

$\cos(\theta + 2\pi) = \cos\theta$

$\theta \;,\; \pi - \theta \;,\; \frac{\pi}{2} + \theta \;,\; \frac{\pi}{2} - \theta$

$\sin\theta$

$\cos\theta$

$\sin(\pi - \theta) = \sin\theta$

$\cos(\pi - \theta) = -\cos\theta$

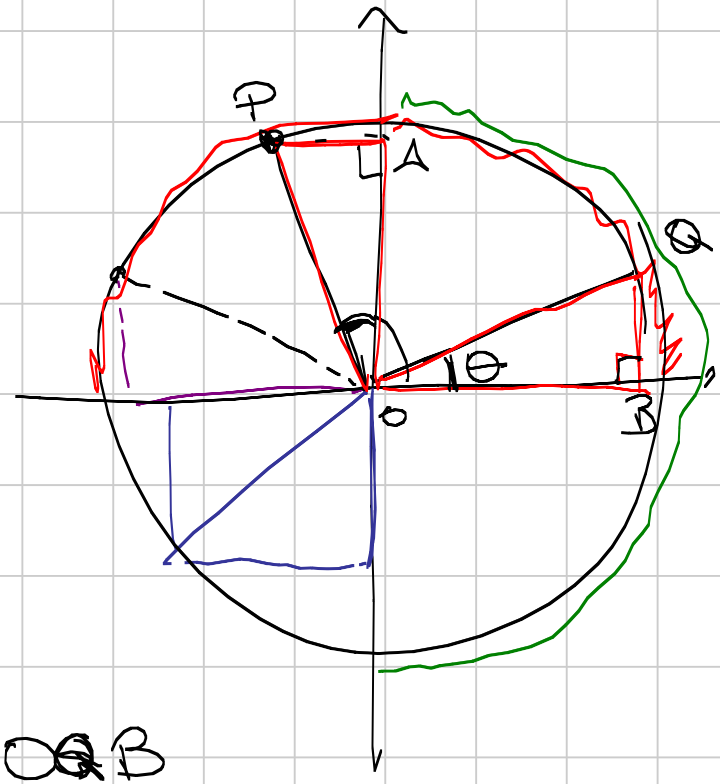
$\sin(\frac{\pi}{2} + \theta) = \cos\theta$

$\cos(\frac{\pi}{2} + \theta) = -\sin\theta$

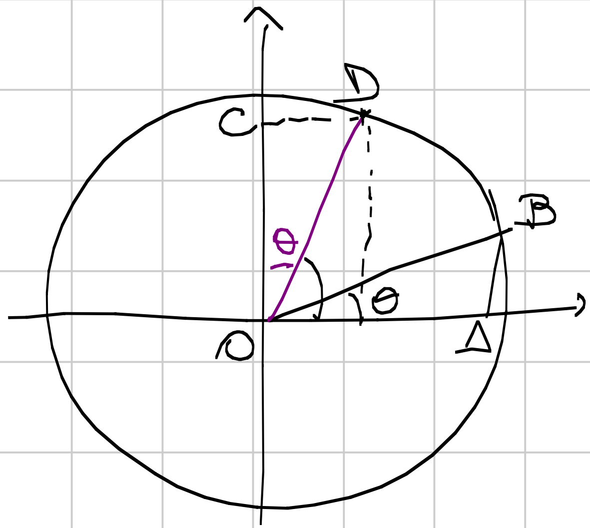
$\sin(\frac{\pi}{2} - \theta) = \cos\theta$

$\cos(\frac{\pi}{2} - \theta) = \sin\theta$

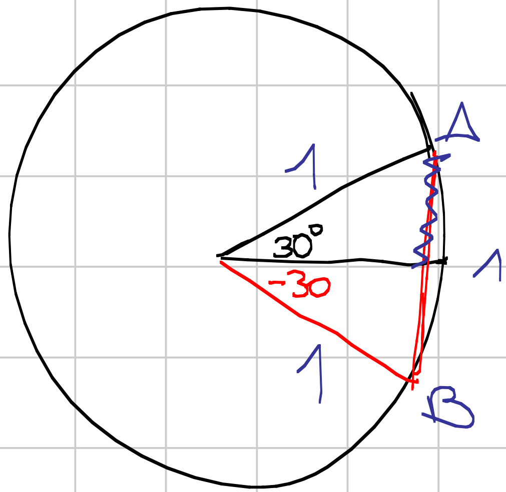
$tg\theta = \frac{\sin\theta}{\cos\theta}$



$OPA \cong OQB$



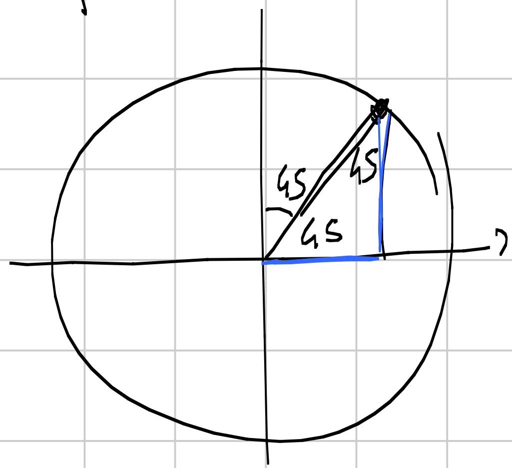
	rad	sim	cos	tg
$0°$	0	0	1	0
$30°$	$\pi/6$	$1/2$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
$45°$	$\pi/4$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
$60°$	$\pi/3$	$\frac{\sqrt{3}}{2}$	$1/2$	$\sqrt{3}$
$90°$	$\pi/2$	1	0	?
$180°$	π	0	-1	0



$$\text{sim}^2\theta + \cos^2\theta = 1$$

$$\frac{1}{4} + \cos^2\theta = 1$$

$$\cos^2\theta = \frac{3}{4}$$

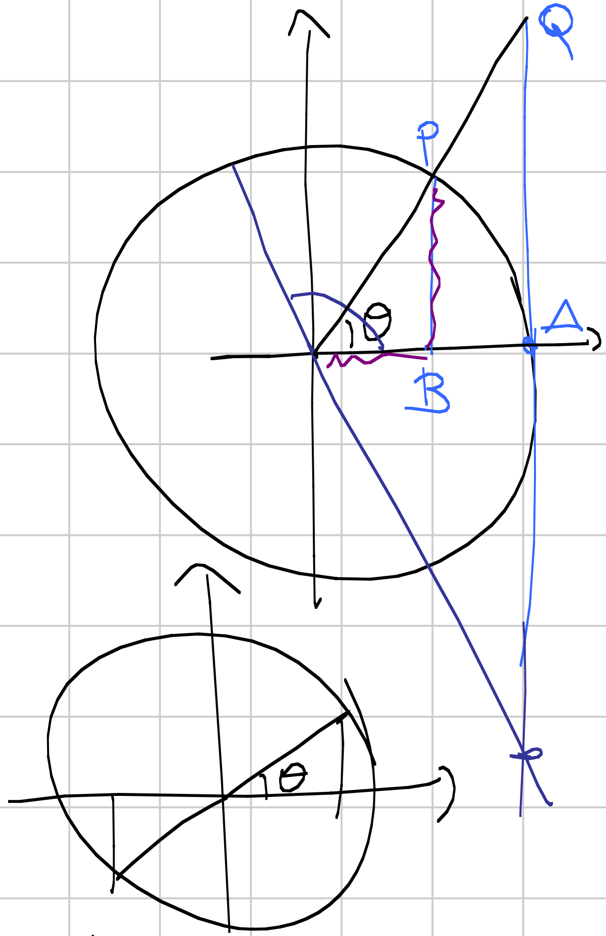


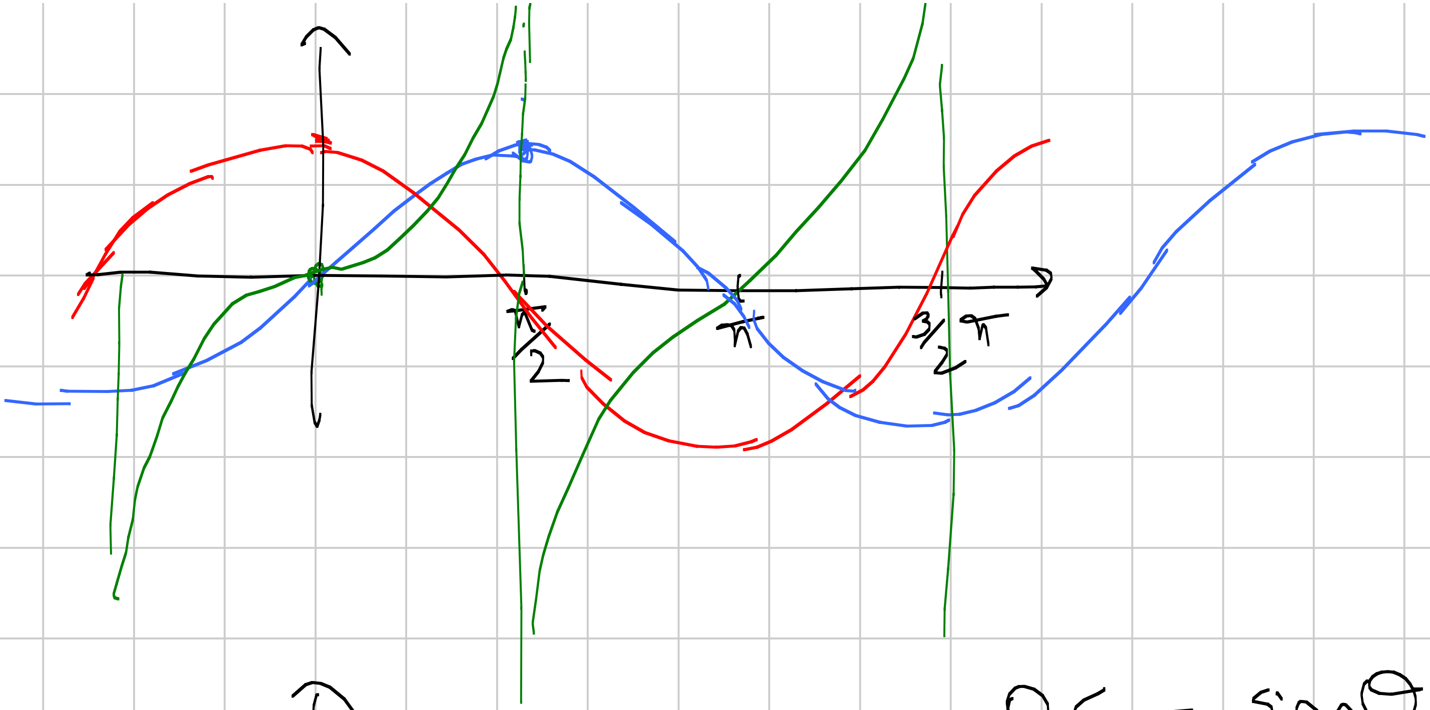
$$\mathrm{tg}\,\theta = \frac{PB}{BO} = \frac{Q\Delta}{\Delta O} = Q\Delta$$

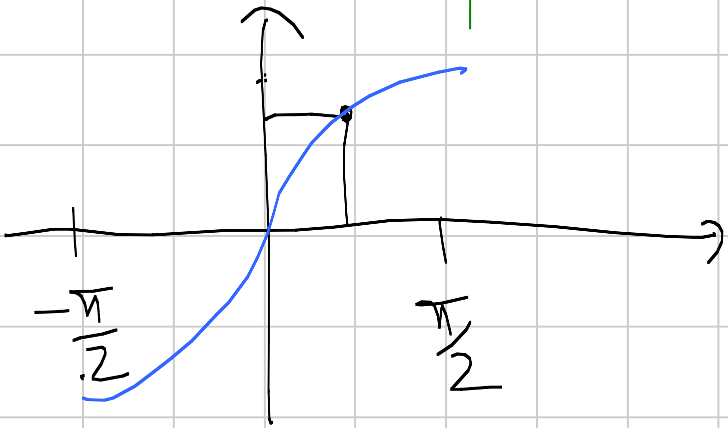
$$\mathrm{tg}\,\theta+\pi = \mathrm{tg}\,\theta$$

$$\frac{\sin\theta+\pi}{\cos\theta+\pi} = \frac{-\sin\theta}{-\cos\theta} = \mathrm{tg}\,\theta$$

$$\mathrm{tg}\left(\frac{\pi}{2}-\theta\right) = \frac{\sin\frac{\pi}{2}-\theta}{\cos\frac{\pi}{2}-\theta} = \frac{\cos\theta}{\sin\theta} = \frac{1}{\mathrm{tg}\,\theta}$$







$0{,}5 = \sin\theta$

$\arcsin 0{,}5 = \frac{\pi}{6} + 2k\pi$

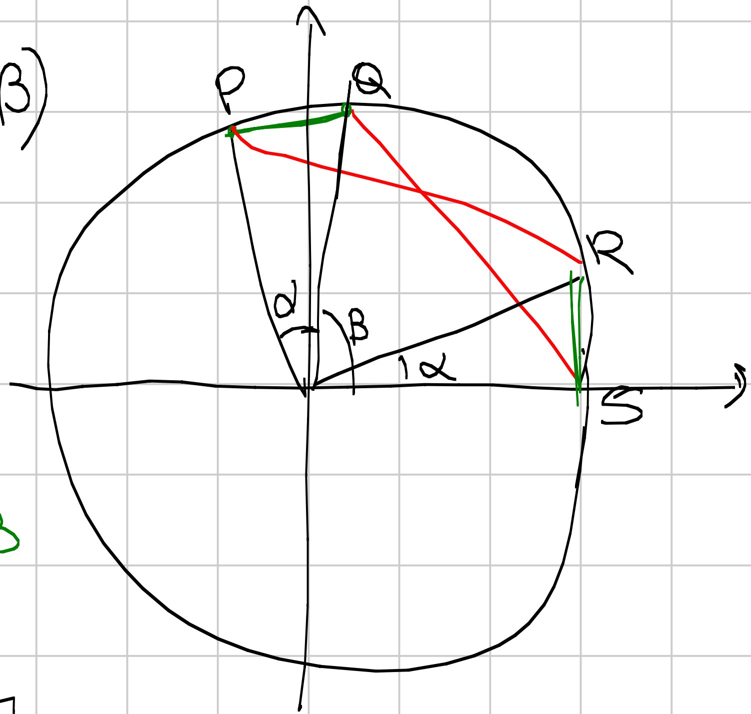
$k \in \mathbb{Z}$

FORMULE DI ADDIZIONE

$\operatorname{sim} x+y = \operatorname{sim} x \cos y + \operatorname{sim} y \cos x$

$S=(1,0) \qquad R=(\cos\alpha, \operatorname{sim}\alpha)$

$Q=(\cos\beta, \operatorname{sim}\beta) \qquad P=(\cos\alpha+\beta, \operatorname{sim}\alpha+\beta)$



$$(\cos\alpha+\beta - \cos\beta)^2 + (\operatorname{sim}\alpha+\beta - \operatorname{sim}\beta)^2$$
$$= (\cos\alpha - 1)^2 + \operatorname{sim}^2\alpha$$

$$\cancel{1}+\cancel{1} - 2\cos\alpha+\beta \cos\beta - 2\operatorname{sim}\alpha+\beta \operatorname{sim}\beta$$
$$= \cancel{1}+\cancel{1} - 2\cos\alpha$$

$$-\cos\alpha \quad \boxed{\cos\alpha+\beta \cos\beta + \operatorname{sim}\alpha+\beta \operatorname{sim}\beta = \cos\alpha}$$

$$(\cos\alpha+\beta - \cos\alpha)^2 + (\sin\alpha+\beta - \sin\alpha)^2 = (\cos\beta - 1)^2 + \sin^2\beta$$

$$\cos\beta \quad \boxed{\cos\alpha+\beta\,\cos\alpha + \sin\alpha+\beta\,\sin\alpha = \cos\beta}$$

$$\sin\alpha+\beta\,(\sin\alpha\cos\beta - \sin\beta\cos\alpha) = \cos^2\beta - \cos^2\alpha$$

$$\sin\alpha+\beta = \frac{\boxed{\cos^2\beta} - \cos^2\alpha + \cos^2\beta\cos^2\alpha \boxed{- \cos^2\beta\cos^2\alpha}}{\sin\alpha\cos\beta - \sin\beta\cos\alpha}$$

$$= \frac{\cos^2\beta\sin^2\alpha - \cos^2\alpha\cdot\sin^2\beta}{\sin\alpha\cos\beta - \sin\beta\cos\alpha}$$

$$= \sin\alpha\cos\beta + \sin\beta\cos\alpha$$

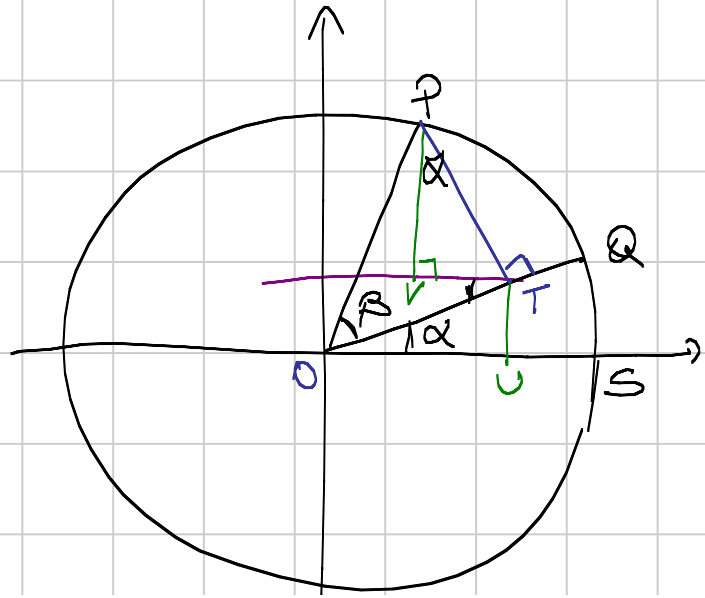
DIM 2:

$\sin\alpha+\beta = PV + TU$

$V\hat{P}T =$

$PV = PT\cos\alpha = \sin\beta\cos\alpha$

$TU = OT\cdot\sin\alpha = \cos\beta\sin\alpha$



FORMULE DI ADDIZIONE

$\sin x+y = \sin x \cos y + \sin y \cos x$

$\sin x-y = \sin x \cos y - \sin y \cos x$

$\cos x+y = \cos x \cos y - \sin x \sin y$

$\cos x-y = \cos x \cos y + \sin x \sin y$

$\sin x-y = \sin(x+(-y)) = \sin x \cdot \cos y + \cos x \cdot \sin(-y))$

$\cos(x+y) = \sin\left(\left(\frac{\pi}{2}-x\right)-y\right) = \sin\left(\frac{\pi}{2}-x\right)\cos y - \cos\left(\frac{\pi}{2}-x\right) \cdot \sin y$

FORMULE DI ADDIZIONE

$$\sin(x+y) = \sin x \cos y + \sin y \cos x$$

$$\sin(x-y) = \sin x \cos y - \sin y \cos x$$

$$\cos(x+y) = \cos x \cos y - \sin x \sin y$$

$$\cos(x-y) = \cos x \cos y + \sin x \sin y$$

FORMULE DI DUPLICAZIONE

$$\sin 2x = 2 \sin x \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x = 1 - 2\sin^2 x = 2\cos^2 x - 1$$

FORMULE DI BISEZIONE

$$2x = y$$

$$\cos y = 1 - 2\sin^2 \frac{y}{2}$$

$$\sin^2 \frac{y}{2} = \frac{1-\cos y}{2}$$

$$\sin \frac{y}{2} = \pm\sqrt{\frac{1-\cos y}{2}}$$

$$\cos^2 \frac{y}{2} = \frac{1+\cos y}{2}$$

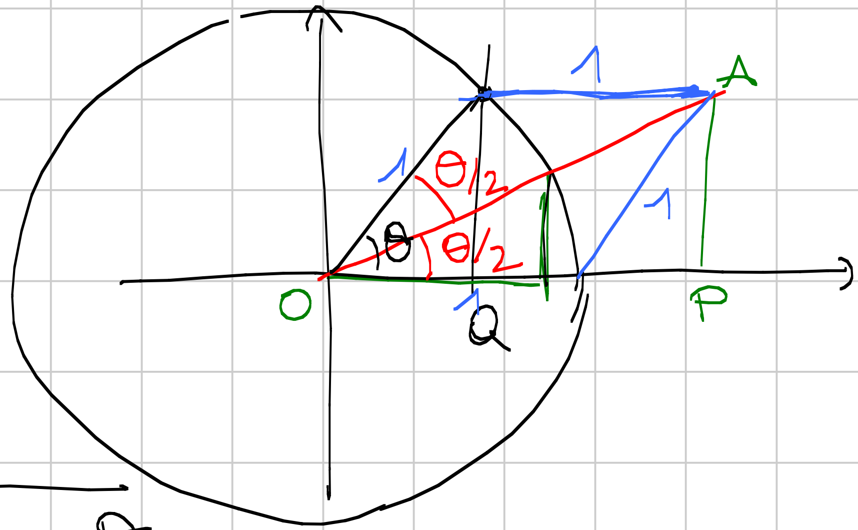
$$\mathrm{tg}\, x+y = \frac{\sin x+y}{\cos x+y} = \frac{\sin x \cos y + \sin y \cos x}{\cos x \cos y - \sin x \sin y}$$

$$= \frac{\mathrm{tg}x + \mathrm{tg}y}{1 - \mathrm{tg}x\, \mathrm{tg}y}$$

Esempio

$$\mathrm{tg}\frac{\theta}{2} = \frac{AP}{PO}$$

$$= \frac{\sin\theta}{1+\cos\theta}$$



$$\mathrm{tg}\frac{\theta}{2} = \frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}} = \sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$$

$$= \sqrt{\frac{1-\cos^2\theta}{(1+\cos\theta)^2}} = \sqrt{\frac{\sin^2\theta}{(1+\cos\theta)^2}}$$

Esercizio

$\cos 3x = \cos(2x + x)$

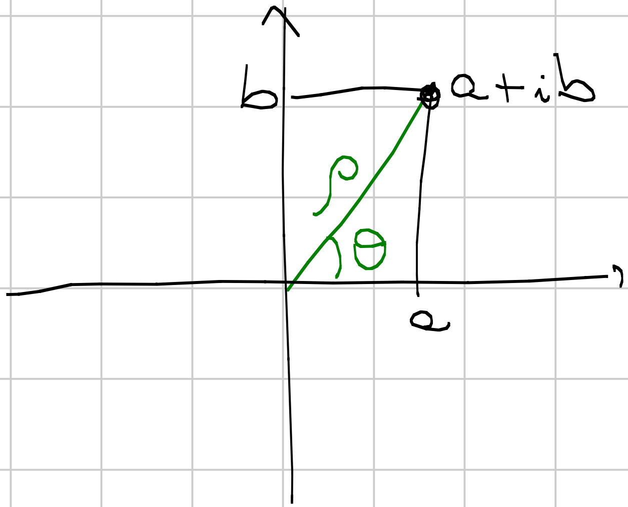
Esercizio!

$8\cos^4\alpha = \cos 4\alpha + 8\cos^2\alpha \pm 1$

$\cos 4\alpha = \cos(2\alpha + 2\alpha) = 2\cos^2 2\alpha - 1$

$= 2(2\cos^2\alpha - 1)^2 - 1$

$= 8\cos^4\alpha - 8\cos^2\alpha + 2 - 1$



$a + ib$

$\rho \cdot e^{i\theta}$

$e^{a+ib} = e^a(\cos b + i \sin b)$

$a + ib = \rho\cos\theta + i\rho\sin\theta = \rho e^{i\theta}$

$$\rho_1 e^{i\theta_1} \cdot \rho_2 e^{i\theta_2} = \rho_1 \rho_2 \underbrace{e^{i\theta_1} \cdot e^{i\theta_2}}_{\shortparallel \atop e^{i\theta_1+\theta_2}}$$

$$(\cos\theta_1 + i\sin\theta_1)(\cos\theta_2 + i\sin\theta_2) =$$

$$\cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2 + i(\sin\theta_1\cos\theta_2 + \sin\theta_2\cos\theta_1)$$

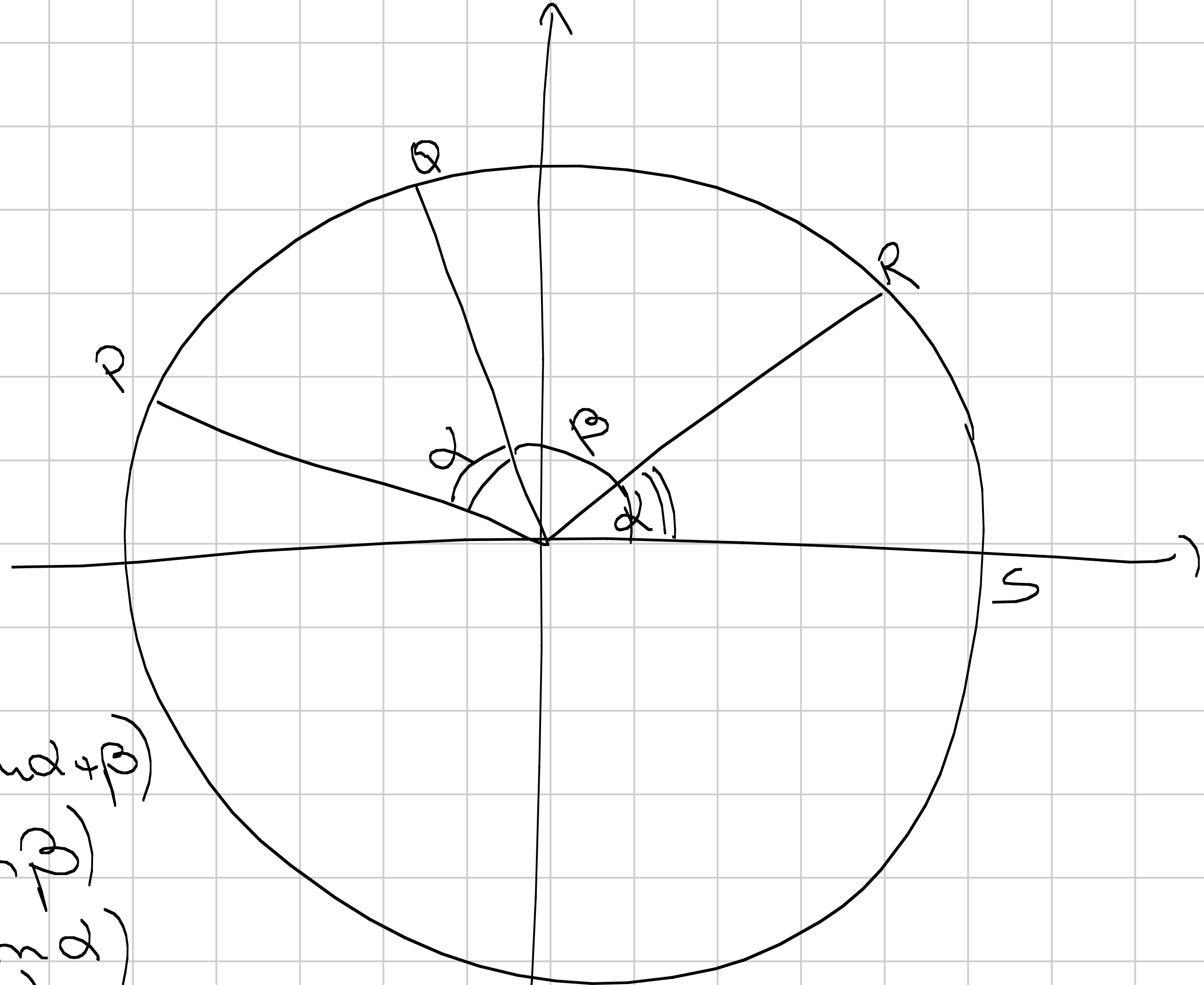
$$= \cos\theta_1+\theta_2 + i\sin\theta_1+\theta_2$$

$$e^{i\theta_1} \cdot e^{i\theta_2} = e^{i(\theta_1+\theta_2)}$$

$$\cos 6\alpha + i\sin 6\alpha = e^{i6\alpha} = (e^{i\alpha})^6$$

$$= (\cos\alpha + i\sin\alpha)^6 =$$

$$\underbrace{Re(e^{i\alpha})^6}_{\shortparallel \atop \cos 6\alpha} = \cos^6\alpha - \binom{6}{2}\cos^4\alpha\sin^2\alpha + \binom{6}{4}\cos^2\alpha\sin^4\alpha - \sin^6\alpha$$



$P = (\cos \alpha+\beta, \sin \alpha+\beta)$

$Q = (\cos\beta, \sin\beta)$

$R = (\cos\alpha, \sin\alpha)$

$$|PR|^2 = (\cos\alpha+\beta - \cos\alpha)^2 + (\sin\alpha+\beta - \sin\alpha)^2 = (\cos\beta - 1)^2 + \sin^2\beta$$

$$2 - 2\cos\alpha+\beta\cos\alpha - 2\sin\alpha+\beta\sin\alpha = 2 - 2\cos\beta$$

$$\cos\beta\left(\cos\alpha+\beta\,\cos\alpha+\sin\alpha+\beta\,\sin\alpha=\cos\beta\right)$$

$$\cos\alpha\left(\cos\alpha+\beta\,\cos\beta+\sin\alpha+\beta\,\sin\beta=\cos\alpha\right)$$

$$\sin\alpha+\beta\left(\cos\alpha\sin\beta-\sin\alpha\cos\beta\right)=\cos^2\alpha-\cos^2\beta$$

$$\sin\alpha+\beta=\frac{\cos^2\alpha-\cos^2\beta}{\cos\alpha\sin\beta-\sin\alpha\cos\beta}\overset{?}{=}\cos\alpha\sin\beta+\cos\beta\sin\alpha$$

$$\cos^2\alpha-\cos^2\beta=\cos^2\alpha\underbrace{\sin^2\beta}_{1-\cos^2\beta}-\underbrace{\sin^2\alpha}_{1-\cos^2\alpha}\cos^2\beta$$

Esercizio: $\cot x - \tan x = 2$

$$\cot x = \frac{1}{\tan x}$$

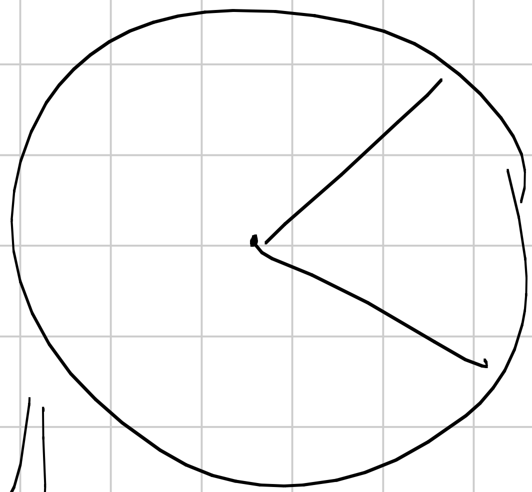
1 $$\frac{\cos x}{\sin x} - \frac{\sin x}{\cos x} = 2$$

$$\cos^2 x - \sin^2 x = 2 \sin x \cos x$$

$$\underbrace{\cos^2 x - \sin^2 x}_{=\cos 2x}$$

$$\cos 2x = \sin 2x = \cos \frac{\pi}{2} - 2x$$

$$\begin{cases} 2x = \frac{\pi}{2} - 2x + 2k\pi \\ -2x = \frac{\pi}{2} - 2x + 2k\pi \end{cases}$$



$$\Longleftrightarrow \begin{cases} x = \frac{\pi}{8} + \frac{k\pi}{2} \\ \text{NO SOL} \end{cases}$$

$$\operatorname{tg} 2x = 1$$

$$2x = \frac{\pi}{4} + k\pi$$

$$x = \frac{\pi}{8} + \frac{k\pi}{2}$$

$$x = \boxed{\frac{\pi}{8}}, \frac{\pi}{8} + \frac{\pi}{2}, \dots$$

$$\boxed{\frac{\pi}{8} - \frac{\pi}{2}},$$

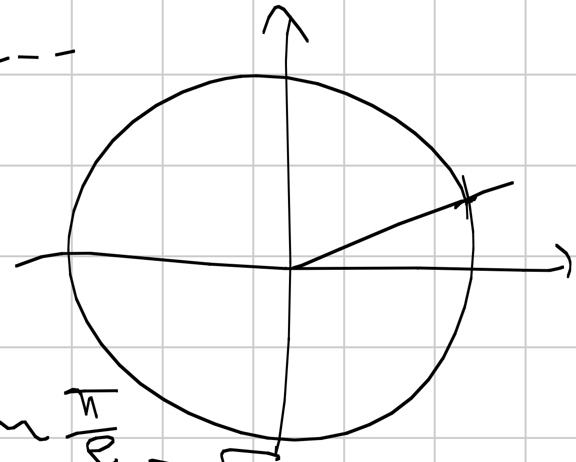
2° sol) $t = \tan x$

$$\frac{1}{t} - t = 2$$

$$t^2 + 2t - 1 = 0$$

$$t = -1 \pm \sqrt{2}$$

$$x = \operatorname{arctg} -1 \pm \sqrt{2} + k\pi$$



$$\tan\frac{\pi}{8} = \sqrt{2} - 1$$

$$\tan\frac{\pi}{8} - \frac{\pi}{2} = -\sqrt{2} - 1$$

FORMULE PARAMETRICHE: $t = \operatorname{tg}\frac{\theta}{2}$

$$\sin\theta = \frac{2t}{1+t^2} \qquad \cos\theta = \frac{1-t^2}{1+t^2}$$

$$5\sin\theta + 2\cos\theta = 1 \qquad \theta = 2\operatorname{arctg} t$$

$$5 \cdot \frac{2t}{1+t^2} + 2\,\frac{1-t^2}{1+t^2} = 1$$

$$\sin\theta = \frac{2\,\frac{\sin\theta}{1+\cos\theta}}{1+\frac{\sin^2\theta}{(1+\cos\theta)^2}} \qquad \operatorname{tg}\frac{\theta}{2} = \frac{\sin\theta}{1+\cos\theta}$$

$$= \frac{2\sin\theta\,(1+\cos\theta)}{2+2\cos\theta} = \sin\theta$$

$$\sin\theta = \frac{2t}{1+t^2} \qquad \theta/2$$

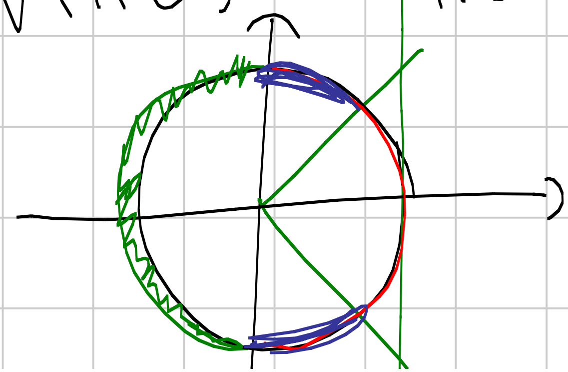
$$2\sin\frac{\theta}{2}\cos\frac{\theta}{2} \overset{?}{=} \frac{2\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}\left(\frac{1}{(\cos\theta/2)^2}\right)} \qquad 1+\operatorname{tg}^2\alpha = \frac{1}{\cos^2\alpha}$$

$$\cos^2\alpha + \sin^2\alpha = 1$$

$$\cos\theta = -\sqrt{1-\sin^2\theta} = -\sqrt{1-\frac{4t^2}{(1+t^2)^2}} = -\sqrt{\left(\frac{1-t^2}{1+t^2}\right)^2} = \frac{1-t^2}{1+t^2}$$

$$\theta \in \left[\frac{\pi}{2}, \frac{3}{2}\pi\right] \qquad t = \tan\frac{\theta}{2}$$

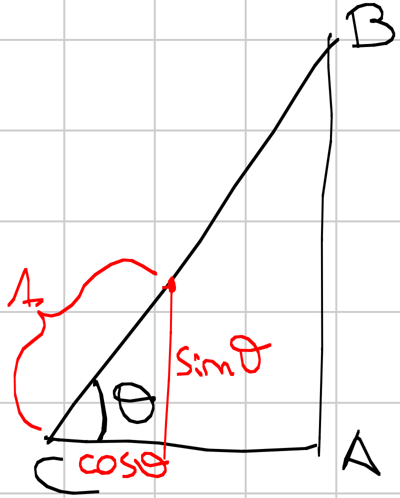
$$|t|>1$$

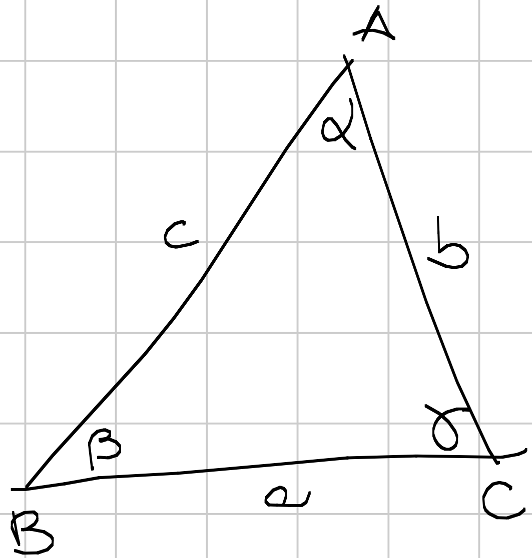


TRIANGOLI

$$\frac{AB}{AC} = tg\,\theta$$

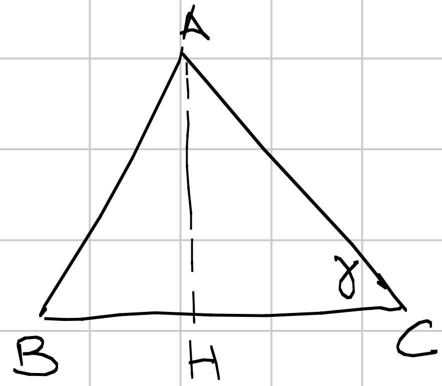
$$\frac{AC}{BC} = \cos\theta \qquad \frac{AB}{BC} = \sin\theta$$





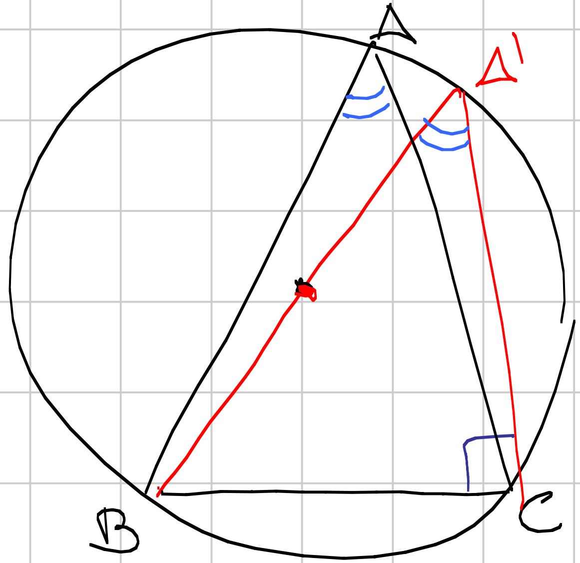
FORMULA PER L'AREA

$$\underbrace{\text{Area}}_{} = \frac{BC \cdot AH}{2} = \underbrace{\frac{1}{2} a b \cdot \sin\gamma}_{}$$



TEOREMA DEI SENI

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma} = 2R$$



$\hat{A'} = A$

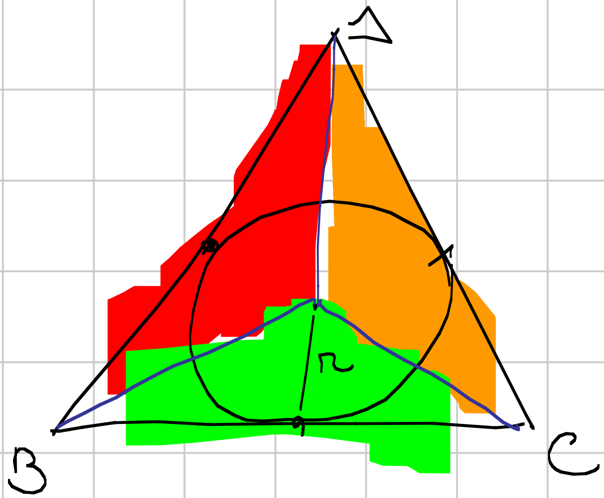
$$\frac{BC}{A'B} = \sin A' = \sin A$$

$$\frac{a}{2R} = \sin A$$

$$\text{Area} = \frac{1}{2} a \cdot b \cdot \sin\gamma \overset{\frac{c}{2R}}{=} \frac{abc}{4R}$$

$$\text{Area} = p \cdot r \qquad p = \frac{a+b+c}{2}$$

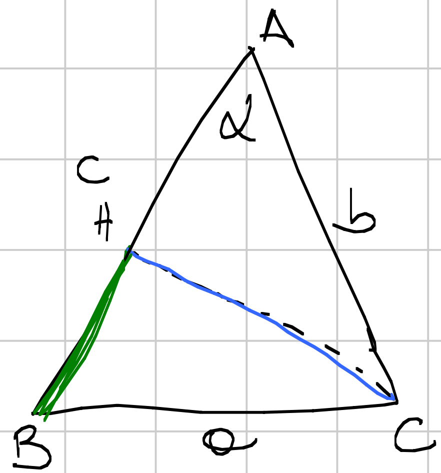
$$\frac{c\cdot r}{2}+\frac{b\cdot r}{2}+\frac{a\cdot r}{2}=p\cdot r$$



TEOREMA DI CARNOT

Noti b, c, γ.

$$a^2 = b^2 + c^2 - 2bc\cos\alpha$$



$CH = b\sin\alpha$

$BH = c - AH = c - b\cos\alpha$

Per il teo di Pitagora su $\overset{\triangle}{BHC}$:

$$BH^2 + CH^2 = a^2$$

$$a^2 = (c - b\cos\alpha)^2 + (b\sin\alpha)^2$$

$$= c^2 + \underbrace{b^2\cos^2\alpha} - 2bc\cos\alpha + \underbrace{b^2\sin^2\alpha}$$

$$= c^2 + b^2 - 2bc\cos\alpha$$

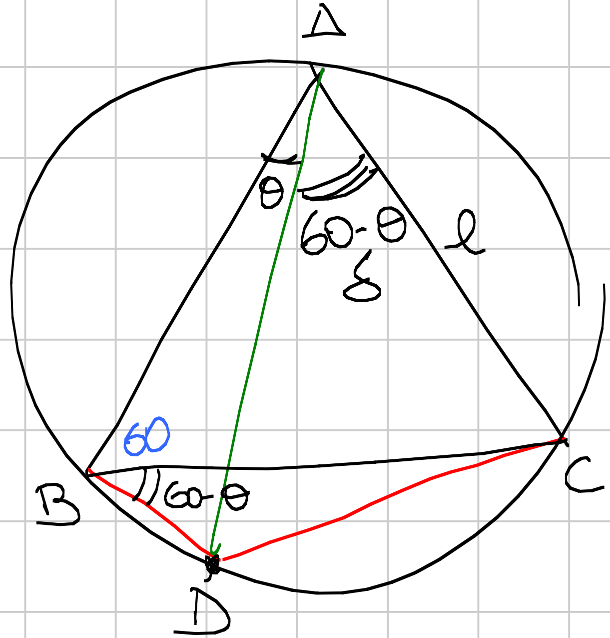
$$\ell \cdot BD + \ell \cdot DC = \ell \cdot AD$$

Esempio: ABC equilatero

$D \in \overset{\frown}{BC}$

$$AD = BD + DC$$

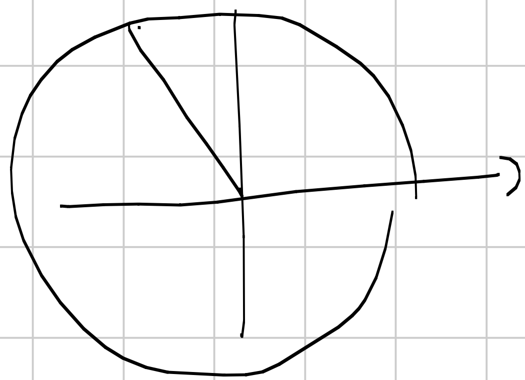
$$\frac{BD}{\sin\theta} = 2R$$



$$BD = 2R\sin\theta$$

$$DC = 2R\sin 60-\theta$$

$$AD = 2R\sin 120-\theta$$



$$\sin 120-\theta \overset{?}{=} \sin 60-\theta + \sin\theta$$

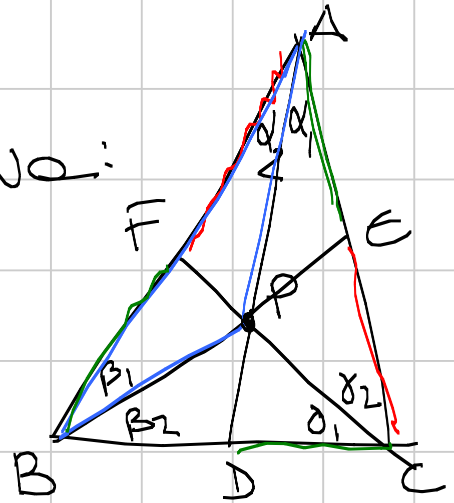
$$\sin 120 \cdot \cos\theta - \sin\theta\cos 120$$

$$\sin 120-\theta = \frac{\sqrt{3}}{2} \cdot \cos\theta + \frac{\sin\theta}{2}$$

$\sin(60-\theta) + \sin\theta = \frac{\sqrt{3}}{2}\cos\theta - \frac{\sin\theta}{2} + \sin\theta$

Esempio CEVA TRIGONOMETRICO

Teo di Ceva:



$$\frac{AF}{FB} \cdot \frac{BD}{DC} \cdot \frac{CE}{EA} = 1$$

Teo di Ceva trigo

$$\frac{\sin\alpha_1}{\sin\alpha_2} \cdot \frac{\sin\beta_1}{\sin\beta_2} \cdot \frac{\sin\gamma_1}{\sin\gamma_2} = 1$$

$$\frac{BP}{\sin\gamma_1} = \frac{PC}{\sin\beta_2}$$

$$\frac{BP}{\sin\alpha_2} = \frac{AP}{\sin\beta_1}$$

Dim

Considiamo $\hat{APB}$: $\frac{\sin\beta_1}{\sin\alpha_2} = \frac{AP}{BP}$

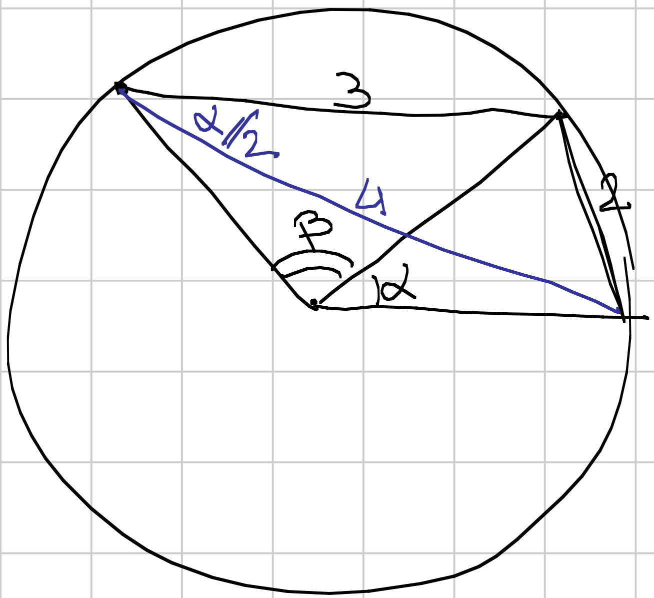
$\hat{BPC}$: $\frac{\sin\gamma_1}{\sin\beta_2} = \frac{BP}{CP}$

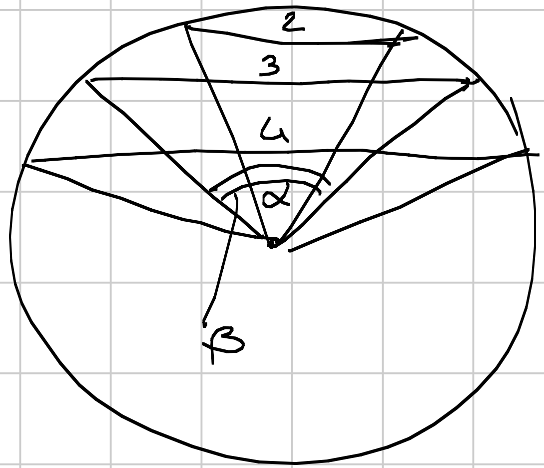
$$\frac{\sin \alpha_1}{\sin \gamma_2} = \frac{CP}{\Delta P}$$

Esercizio fascicolo:

3 corde parallele lunghe 2, 3, 4 che insistono su archi lunghi α, β, $\alpha+\beta$.

Trovare $\cos\alpha$





$\cos \frac{\alpha}{2}$

$$2^2 = 3^2 + 4^2 - 2\cdot 3\cdot 4 \cos\frac{\alpha}{2}$$

$$2 \cdot 3 \cdot 4 \cos\frac{\alpha}{2} = 21$$

$$\cos\frac{\alpha}{2} = \frac{7}{8}$$

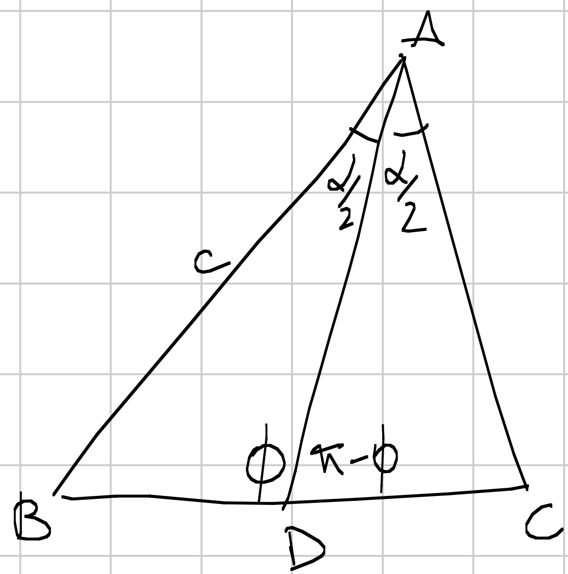
$$\cos^2\frac{\alpha}{2} = \frac{1+\cos\alpha}{2}$$

$$2 \cdot \frac{49}{64} = 1 + \cos\alpha$$

$$\cos\alpha = \frac{49}{32} - 1$$

Teorema della bisettrice:

$$\frac{BD}{DC} = \frac{AB}{AC}$$



Ricaviamo BD

teo seni su $\triangle ABD$ $\qquad \dfrac{BD}{\sin\alpha/2} = \dfrac{c}{\sin\phi}$

teo seni su ΔACD $\quad \dfrac{CD}{\sin \frac{\alpha}{2}} = \dfrac{b}{\sin(\pi - \phi)}$

$$\sin(\pi-\phi) = \sin\phi$$

$$\frac{BD}{CD} = \frac{c}{b}$$

Esercizio $\quad ABC$ è triangolo acutangolo

$$\operatorname{tg} A + \operatorname{tg} B + \operatorname{tg} C = \operatorname{tg} A \operatorname{tg} B \operatorname{tg} C \geq 3\sqrt{3}$$

$$\hat{A} + \hat{B} + \hat{C} = 180^\circ$$

$$\underline{\sin A \cos B \cos C + \sin B \cos A \cos C} + \sin C \cos A \cos B \overset{?}{=} \sin A \sin B \sin C$$

$$C = \pi - \hat{A} - \hat{B}$$

$$\cos C \,(\sin \underbrace{A+B}_{\pi - C}) = \sin C \,(-\cos \underbrace{A+B}_{\pi - C})$$

$$\cos C \sin C = \sin C\,(-1)\,(-\cos C)$$

IMO 09 / 06

$\triangle ABC$ isoscele

D piede

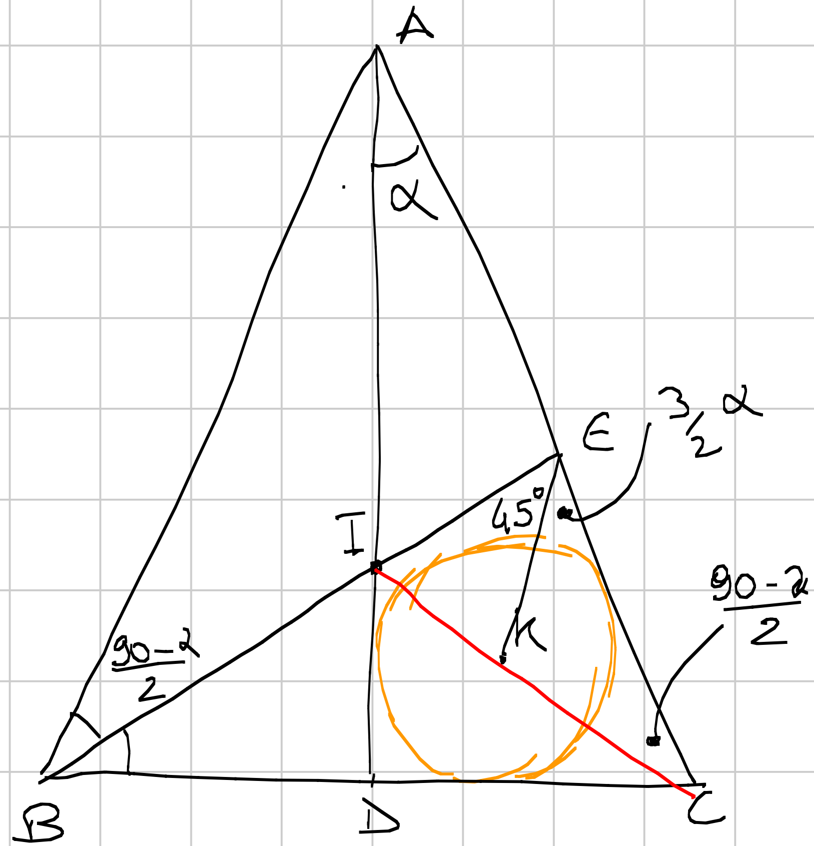
K centro incirconf ACD

I incentro

BE bisettr

$I\hat{E}K = 45°$

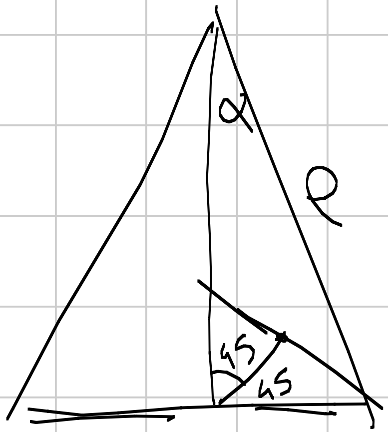
$B\hat{A}C = ?$

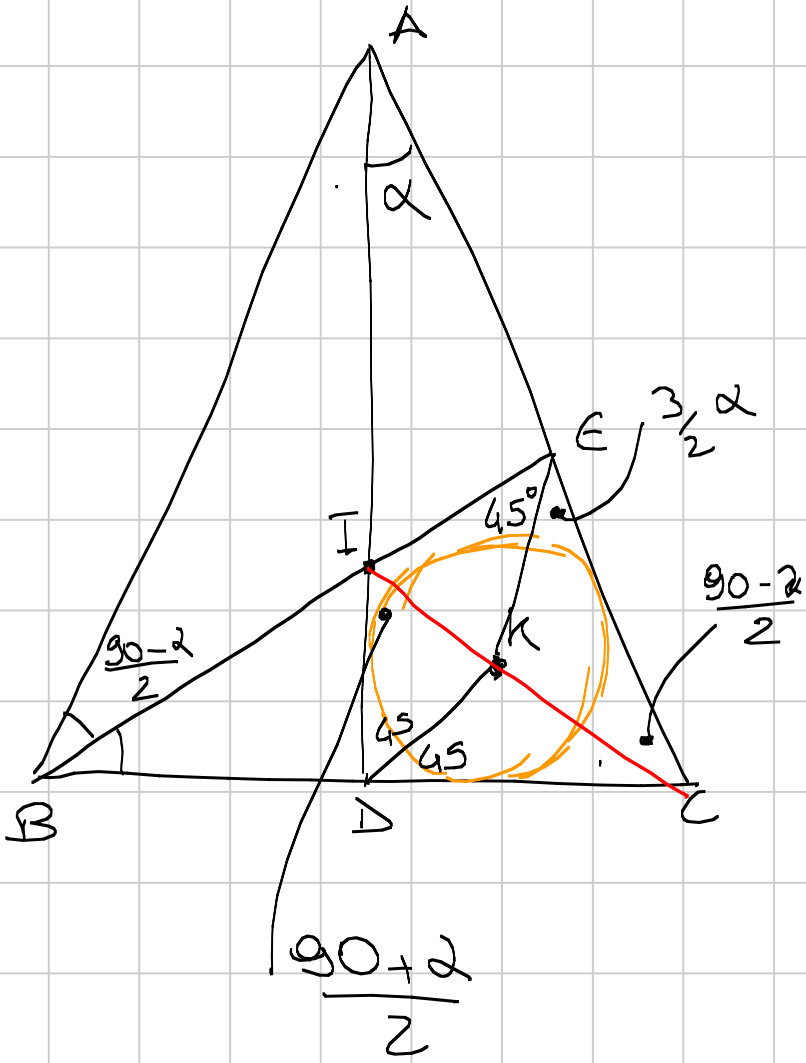


I, K, C sono allineati.

$$B\hat{E}C = 180 - \frac{90-\alpha}{2} - (90-\alpha) = 45 + \frac{3}{2}\alpha$$

$$E\hat{I}K = 180 - 45 - \frac{3}{2}\alpha - \frac{90-\alpha}{2} = 90 - \alpha$$





$$\frac{IK}{KC}$$

~~tg seni sw IKD~~

$$\cancel{\frac{IK}{\sin 45} = \frac{KD}{\sin \frac{90+\alpha}{2}}}$$

$$\frac{IK}{KC} = \frac{ID}{DC} = tg\ \frac{90-\alpha}{2}$$

teo seni su IKE

$$\frac{IK}{\sin 45} = \frac{KE}{\sin E\hat{I}K} = \frac{KE}{\sin(90-\alpha)} = \frac{KE}{\cos\alpha}$$

teo seni su KEC

$$\frac{KC}{\sin\frac{3}{2}\alpha} = \frac{KE}{\sin\frac{90-\alpha}{2}}$$

$$\Rightarrow \quad \frac{IK}{KC} = \frac{\sin 45}{\cos\alpha} \cdot \frac{\sin\frac{90-\alpha}{2}}{\sin\frac{3}{2}\alpha}$$

$$\operatorname{tg}\frac{90-\alpha}{2} = \frac{\sin 45}{\cos\alpha} \cdot \frac{\sin\frac{90-\alpha}{2}}{\sin\frac{3}{2}\alpha} \qquad \beta = \frac{\alpha}{2}$$

$$\frac{\cancel{\sin(45-\beta)}}{\cos(45-\beta)} = \frac{\sin 45}{\cos 2\beta} \cdot \frac{\cancel{\sin(45-\beta)}}{\sin 3\beta}$$

$$\sin 3\beta \cdot \cos 2\beta = \sin 45 \cdot \cos(45-\beta)$$

FORMULE SUM → PRODUCT

② PRODUCT → SUM

FORMULE DI ADDIZIONE

① $\sin x+y = \sin x \cos y + \sin y \cos x$

② $\sin x-y = \sin x \cos y - \sin y \cos x$

③ $\cos x+y = \cos x \cos y - \sin x \sin y$

④ $\cos x-y = \cos x \cos y + \sin x \sin y$

PRODUCT → SUM

② $\cos x \cos y = \dfrac{\cos x+y + \cos x-y}{2} = \dfrac{③+④}{2}$

$\cos x \sin y = \dfrac{\sin x+y - \sin x-y}{2}$

$\sin x \sin y = \dfrac{\cos x-y - \cos x+y}{2}$

SUM → PRODUCT

$$\cos\alpha + \cos\beta = 2\cos x \cos y \qquad x+y=\alpha \qquad x-y=\beta \qquad x=\frac{\alpha+\beta}{2} \qquad y=\frac{\alpha-\beta}{2}$$

$$= 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\sin\alpha + \sin\beta =$$

$$\cos x \sin y = \frac{\sin \underbrace{x+y}_{=\alpha} + \sin \underbrace{y-x}_{=\beta}}{2}$$

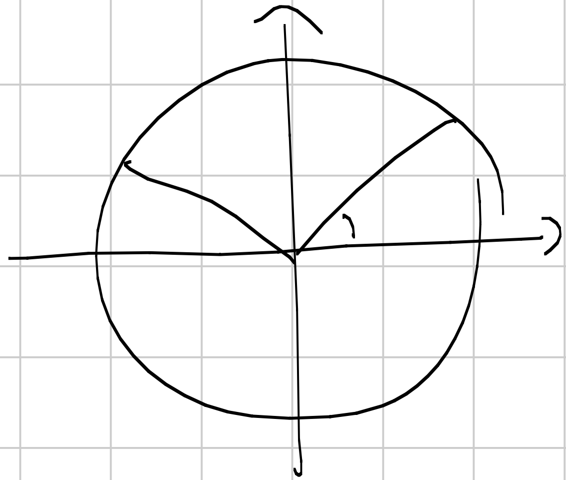
$$\sin\alpha + \sin\beta = 2\cos\frac{\alpha-\beta}{2}\cdot\sin\frac{\alpha+\beta}{2}$$

$$\sin 3\beta \cdot \cos 2\beta = \sin 45 \cdot \cos 45-\beta$$

$$\frac{\sin 5\beta + \cancel{\sin\beta}}{2} = \frac{\sin 90-\beta + \cancel{\sin\beta}}{2}$$

$$\sin 5\beta = \sin 90 - \beta$$

$$\begin{cases} 5\beta = \dfrac{\pi}{2} - \beta + 2\pi \cdot k \\ \text{oppure} \\ \pi - 5\beta = \dfrac{\pi}{2} - \beta + 2k\pi \end{cases}$$



$$6\beta = \frac{\pi}{2} + 2\pi \cdot \cancel{k}$$

$$\beta = \frac{\pi}{2 \cdot 6}$$

angolo al vertice

$$\frac{\pi}{3} = 60^\circ$$

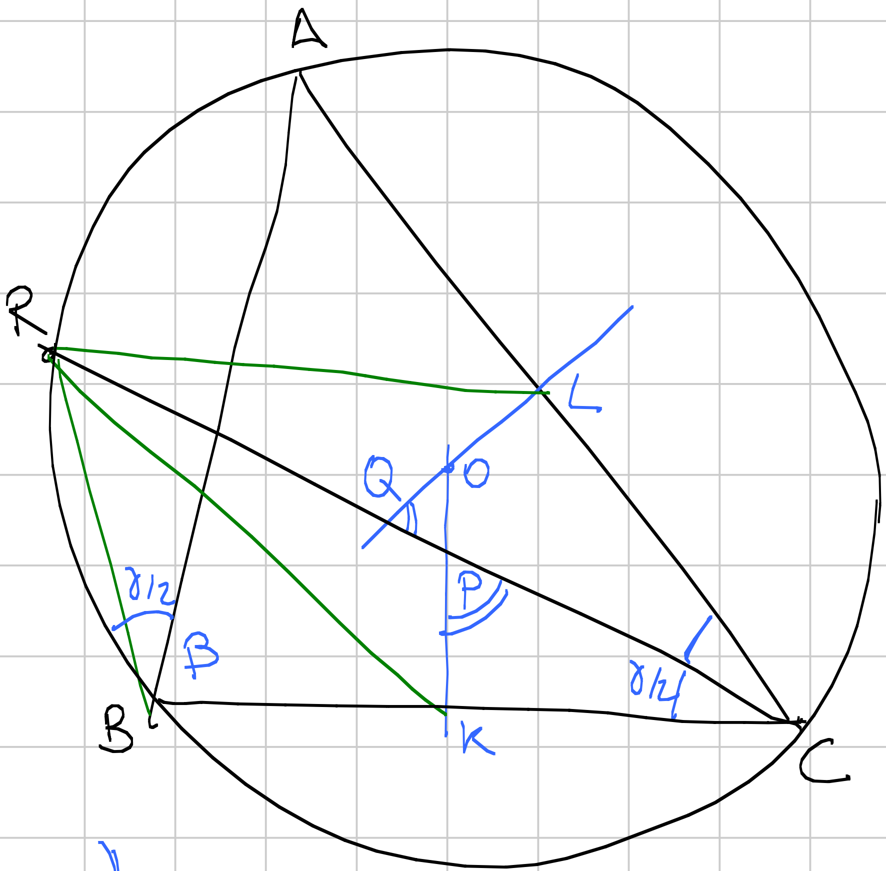
$$+4\beta = +\frac{\pi}{2} - 2k\pi$$

$$\beta = \frac{\pi}{2 \cdot 4}$$

angolo $\frac{\pi}{2} = 90^\circ$

IMO 07 / 4

tesi: area RPK =
area RQL



Lemma: OPQ è isoscele.

$) = 90 - \frac{\gamma}{2}$

area $PRK = \frac{1}{2} PR \cdot PK \cdot \sin 90 + \frac{\gamma}{2}$

$= \frac{1}{2} PR \cdot PK \cos \frac{\gamma}{2}$

$\frac{PK}{KC} = tg \frac{\gamma}{2}$ $\qquad PK = \frac{a}{2} tg \frac{\gamma}{2}$

$PR = CR - PC = CR - \frac{KC}{\cos\gamma/2} = CR - \frac{a}{2\cos\gamma/2}$

$\frac{CR}{\sin(\frac{\gamma}{2}+\beta)} = 2R$

$$PR = 2R\sin\left(\frac{\gamma}{2}+\beta\right) - \frac{a}{2\cos\gamma/2}$$

$$\text{area } PRK = \frac{1}{2}\,\frac{a}{2}\,\frac{\operatorname{tg}\gamma/2}{\cancel{\cos\gamma/2}}\left(\frac{4R\sin\frac{\gamma}{2}+\beta\,\cos\frac{\gamma}{2}-a}{2\cancel{\cos\gamma/2}}\right)$$

$$= \frac{1}{8}a\operatorname{tg}\gamma/2\left(\cancel{4}^{2}R\,\frac{\sin\alpha+\sin\beta}{\cancel{2}} - a\right)$$

$$\sin\frac{\gamma}{2}+\beta\,\cos\frac{\gamma}{2} = \frac{\sin\beta+\gamma + \sin\beta}{2}$$

$$\cancel{\sin y\cos x = \frac{\sin x+y + \sin y-x}{2}}$$

$$\cancel{y = \frac{\gamma}{2}+\beta \qquad x = \frac{\gamma}{2}}$$

$$= \frac{1}{8}a\operatorname{tg}\gamma/2\,(\cancel{a}+b-\cancel{a}) = \frac{1}{8}\operatorname{tg}\gamma/2\; ab$$

Dim euclidea

$\overset{\triangle}{RDQ}$ e $\overset{\triangle}{OCP}$ sono congruenti.

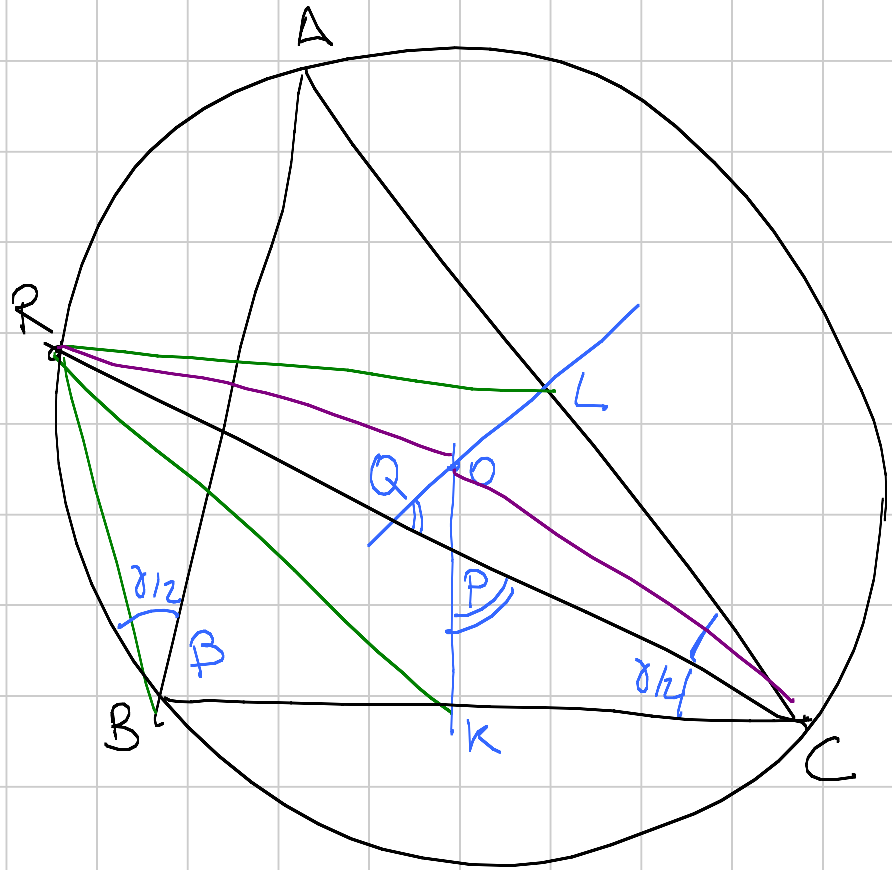
$$\left\{\begin{array}{l} OQ = OP \\ R\hat{Q}O = O\hat{P}C \\ OR = OC \end{array}\right. \Rightarrow \;)$$

$\frac{1}{2} RQ \cdot QL \sin)) = $ Area RQL

$\frac{1}{2} RP \cdot PK \sin)) = $ Area RPK

$RQ \cdot QL \overset{?}{=} RP \cdot PK$

$PC \cdot QL \overset{?}{=} QC \cdot PK$



$$\frac{PC}{PK} = \frac{1}{\sin \gamma/2} \qquad \frac{QC}{QL} = \frac{1}{\sin \gamma/2}$$