Geometria 2 - basic: Metodi Algebrici

1) Geometria Analitica

2) Vettori

3) Numeri Complessi

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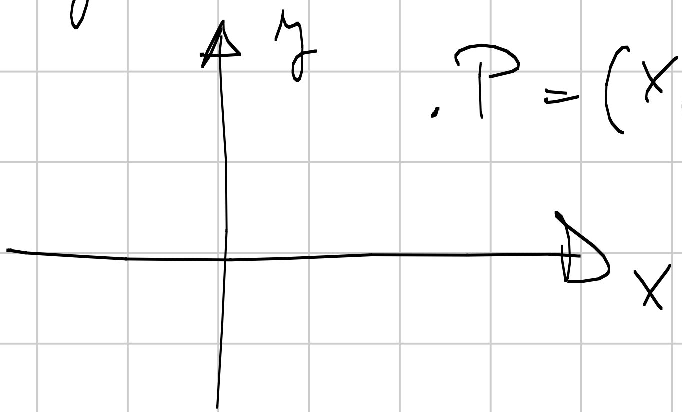
1) Geom. Analitica



$. P = (x_P, y_P)$

$$d(P,Q) = \sqrt{(x_P - x_Q)^2 + (y_P - y_Q)^2}$$

retta per P, Q

$$\frac{x - x_P}{x_Q - x_P} = \frac{y - y_P}{y_Q - y_P}$$

$$ax + by + c = 0$$

$$y = mx + q$$

$$py + qx = 1$$

-) è una retta perché è lineare

-) ha come "radici" P, Q

→ posso avere la stessa retta con diversi coeff.

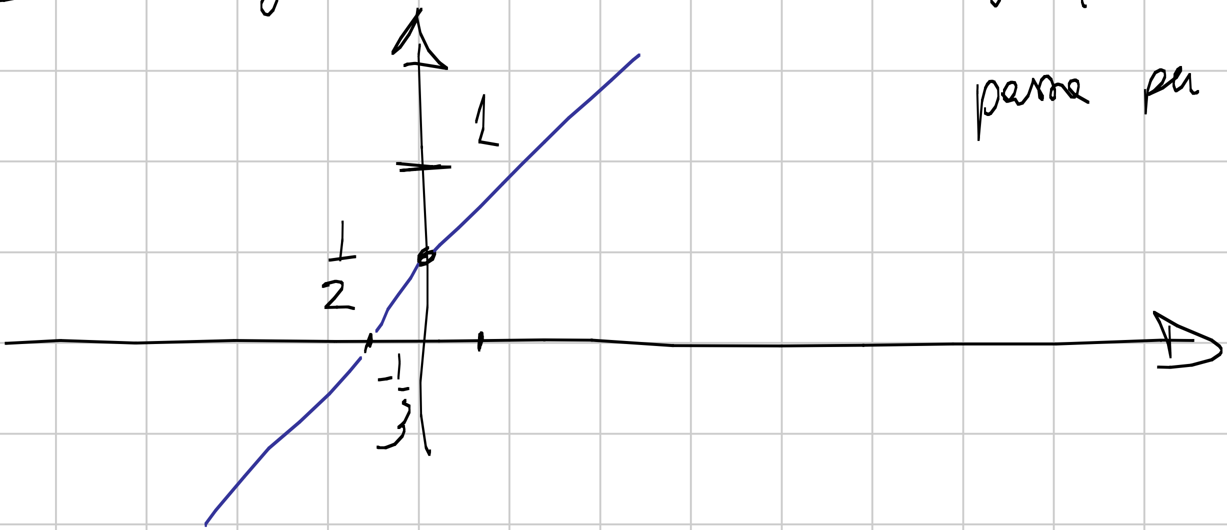
→ non ho tutte le rette
manca $x = k$

→ non ho tutte le rette
manca $y = mx$

<u>Es</u>: $2y - 3x = 1$

$py + qx = 1$

passa per $\left(0, \frac{1}{p}\right)$ $\left(\frac{1}{q}, 0\right)$

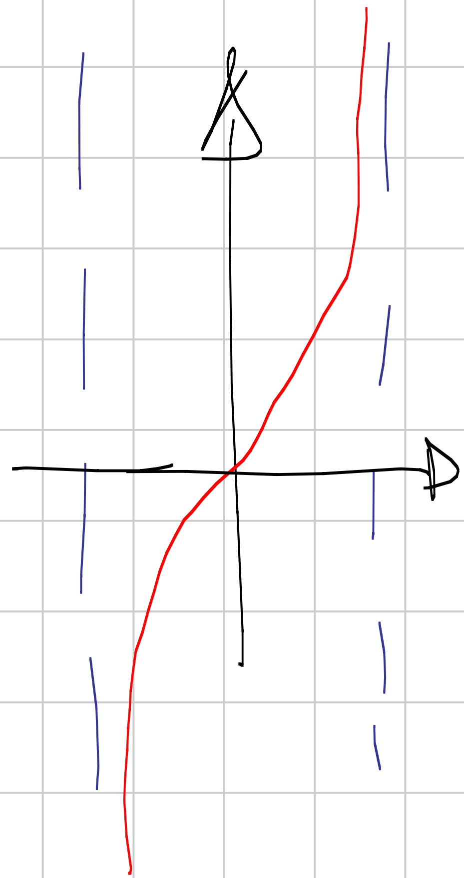


$$y = mx + p \qquad y = nx + r$$

parall se $m = n$ $(q \neq r)$

perp. se $mn = -1$

$$m = \operatorname{Tg} \alpha \qquad n = \operatorname{Tg} \beta \qquad |\beta - \alpha| = \frac{\pi}{2}$$



$$\beta - \alpha = \operatorname{arctg} n - \operatorname{arctg} m$$

$$\operatorname{Tg} \pm\frac{\pi}{2} = \operatorname{Tg}(\beta - \alpha) = \operatorname{Tg}(\operatorname{arctg} n - \operatorname{arctg} m) =$$

$$= \frac{n - m}{1 + nm}$$

ops!

$$0 = \operatorname{ctg}(\beta - \alpha) = \frac{1 + mm}{m - m} \qquad 1 + mm = 0 \qquad mm = -1$$

<u>Luoghi di punti.</u>

$$\{ P \text{ t.c. } PA = PB \} = \text{asse}$$

$$PA^2 = PB^2$$

$$(x - x_A)^2 + (y - y_A)^2 = (x - x_B)^2 + (y - y_A)^2$$

$$-2xx_A - 2yy_A + 2xx_B + 2yy_B + x_A^2 - x_B^2 + y_A^2 - y_B^2 = 0$$

$$x(x_A - x_B) + y(y_A - y_B) = \frac{1}{2}(x_A^2 - x_B^2 + y_A^2 - y_B^2)$$

Pb: E se $PA = 2PB$?

1) Non vanno via i quadrati

($\overset{?}{\Longrightarrow}$ retta $\qquad y^2+2xy+x^2=0$

le rette possono nascondersi nel 2° grado $\qquad (x+2)^2=0$

nel 3°? senza cubi? $\qquad (x+2)(x^2+1)=0$

$$x^3+2x^2+x+2=0$$

2) È proprio una conica. Sarà una circonferenza?

$$\{ P \mid PA = k\,PB \} \qquad k \neq 1, 0 \qquad k > 0$$

$$x^2 + x_A^2 - 2xx_A + y^2 + y_A^2 - 2yy_A =$$

$$= k^2x^2 + k^2x_B^2 - 2k^2xx_B + k^2y^2 + k^2y_B^2 - 2k^2yy_B$$

$$x^2(1-k^2) + y^2(1-k^2) - 2x(x_A - k^2x_B) - 2y(y_A - k^2y_B) =$$

$$= k^2x_B^2 - x_A^2 + k^2y_B^2 - y_A^2$$

Cos'è?

Proviamo con le circonferenze

$$\mathcal{C} = \{ P \mid PC^2 = R^2 \}$$

$$x^2 + y^2 - 2xx_C - 2yy_C = R^2 - x_C^2 - y_C^2$$

$$x^2 + y^2 + 2\alpha x + 2\beta y = \gamma$$

$$2\alpha = -2x_c \qquad x_c = -\alpha$$

$$2\beta = -2y_c \qquad y_c = -\beta$$

$$\gamma = R^2 - x_c^2 - y_c^2 \qquad R^2 = \gamma + x_c^2 + y_c^2 =$$

$$= \gamma + \alpha^2 + \beta^2 \geq 0$$

Tornando al problema:

$$x^2(1-k^2) + y^2(1-k^2) - 2x(x_A - k^2 x_B) - 2y(y_A - k^2 y_B) =$$

$$= k^2 x_B^2 - x_A^2 + k^2 y_B^2 - y_A^2$$

$$x^2+y^2-2x\,\frac{x_A-k^2x_B}{1-k^2}-2y\,\frac{y_A-k^2y_B}{1-k^2}=\frac{k^2x_B^2-x_A^2}{1-k^2}+\frac{k^2y_B^2-y_A^2}{1-k^2}$$

Centro $\left(\dfrac{x_A-k^2x_B}{1-k^2},\ \dfrac{y_A-k^2y_B}{1-k^2}\right)$

<u>Solo sul segmento</u>

$PA=k\,PB \qquad k>0,\ k\neq 1$

1) Scrivere un punto sul segmento AB

$$\frac{x-x_A}{x_B-x_A}=\frac{y-y_A}{y_B-y_A}$$

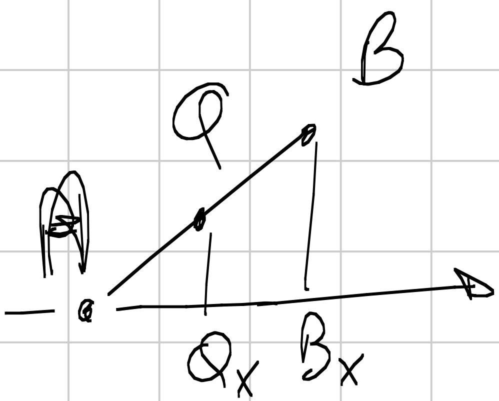
$x_A\le t\le x_B$

$\left(t,\ \dfrac{t-x_A}{x_B-x_A}(y_B-y_A)+y_A\right)$

$(t,0)$

$(0,t)$

(t,t)



$$d(Q,A) + d(Q,B) = d(A,B)$$

$$\sqrt{} + \sqrt{} = \sqrt{}$$

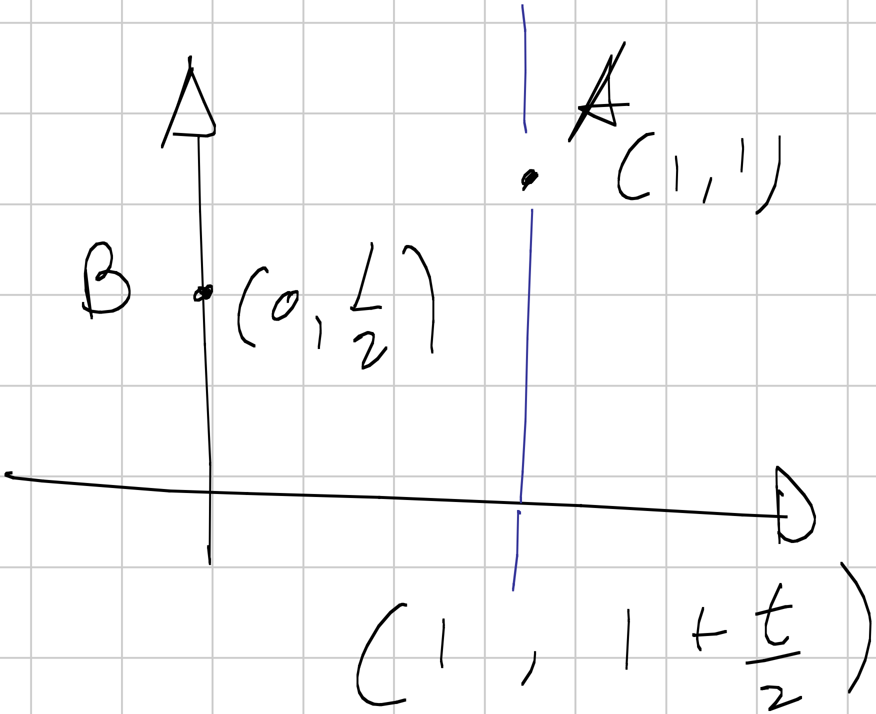
$$\frac{AQ}{AB} = \frac{AQ_x}{AB_x}$$

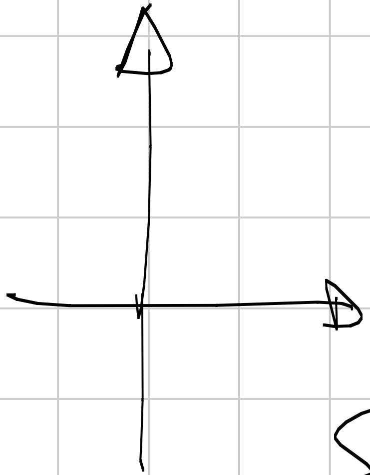
$$(x_A + t\,x_B \,,\; y_A + t\,y_B)$$

$$x_A + t\,x_B = x_B$$

$$x_B t = x_B - x_A$$

$$t = \frac{x_B - x_A}{x_B}$$





$ax = by$

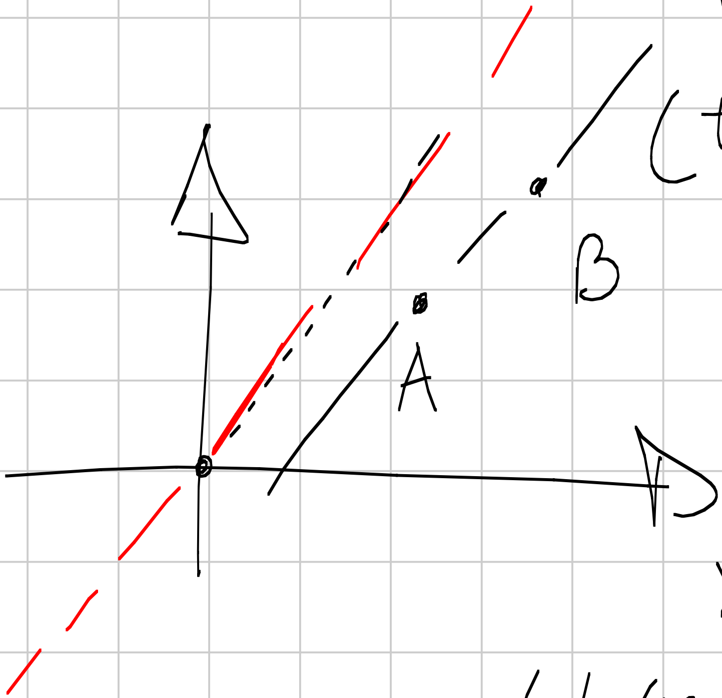
(tb, ta)

$x = tb$

$y = ta$

Al variare di t ho tutti i punti

Se scelgo α, β t.c. $\alpha b = \beta a$ la retta $\alpha x = \beta y$ è la stessa di $ax = by$



$(t\beta, t\alpha)$

$$\frac{x}{x_B - x_A} = \frac{y}{y_B - y_A}$$

parall x l'origine

$$x(y_B - y_A) = y(x_B - x_A)$$

$$(t(x_B - x_A), t(y_B - y_A))$$

Torniamo ad A $\Rightarrow$ sommiamo x_A, y_A

$$\left(t(x_B - x_A) + x_A,\ t(y_B - y_A) + y_A \right)$$

$$\left(t x_B + (1-t) x_A,\ t y_B + (1-t) y_A \right)$$

$t = 0 \rightarrow A$ $\qquad 0 \leq t \leq 1$

$t = 1 \rightarrow B$

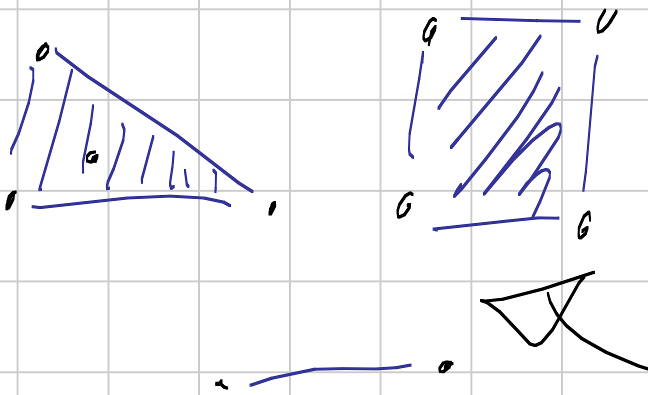
lo scorro sul segmento

$$\left\{ \lambda x + (1-\lambda) y \quad \text{con } \lambda \geq 0,\ 1-\lambda \geq 0 \right.$$

si chiama

COMB. LIN. CONVESSA

di x e y



Sul sg: $\left(t x_B + (1-t) x_A ,\ t y_B + (1-t) y_A \right) = P \quad 0 \leq t \leq 1$

$PA = k PB \qquad k \geq 0,\ k \neq 1$

$$PA^2 = \left(t x_B + (\cancel{1} - t) x_A - \cancel{x_A} \right)^2 + \left(t y_B + (\cancel{1} - t) y_A - \cancel{y_A} \right)^2 =$$

$$= t^2 \left(x_B^2 + x_A^2 - 2 x_B x_A + y_B^2 + y_A^2 - 2 y_B y_A \right) =$$

$$= t^2 (AB^2)$$

$$PB^2 = \left((t-1) x_B + (1-t) x_A \right)^2 + \left((t-1) y_B + (1-t) y_A \right)^2 =$$

$$= (1-t)^2 \left[(x_A - x_B)^2 + (y_A - y_B)^2 \right] = (1-t)^2 (AB^2)$$

$$k^2 = \frac{PA^2}{PB^2} = \frac{t^2}{(1-t)^2}$$

$$k = \frac{t}{1-t} \leftarrow \text{perché } t>0, \; 1-t>0$$

$$(1-t)k = t$$

$$t = \frac{k}{1+k}$$

INTERNO

All'esterno

$$k = -\frac{t}{1-t}$$

$$k = t(k-1)$$

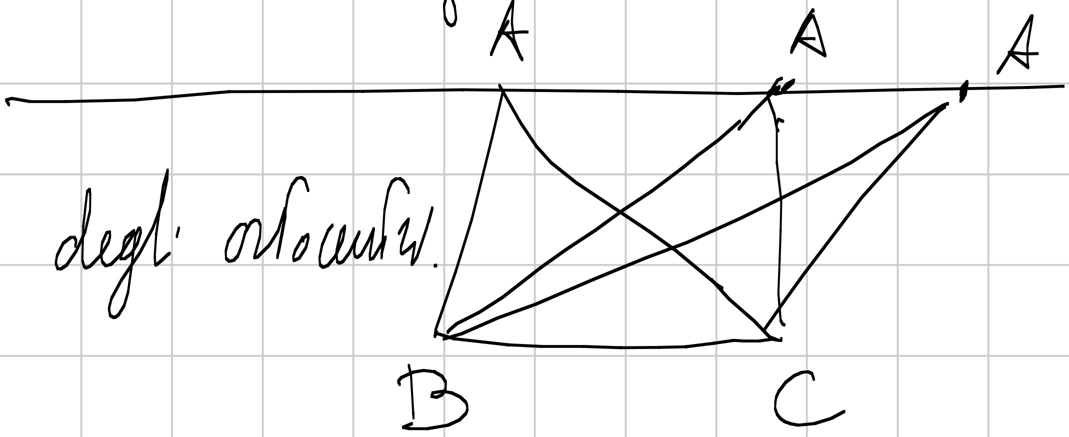
$$t = \frac{k}{k-1} \quad \text{ESTERNO}$$

Centro $\left(\dfrac{x_A - k^2 x_B}{1-k^2}, \dfrac{y_A - k^2 y_B}{1-k^2} \right)$ $\quad t = \dfrac{1}{1-k^2}$

$$\left(x_A \frac{1}{1-k^2} + \frac{(-k^2)}{1-k^2} x_B, \; \frac{1}{1-k^2} y_A + \frac{(-k^2)}{1-k^2} y_B \right)$$

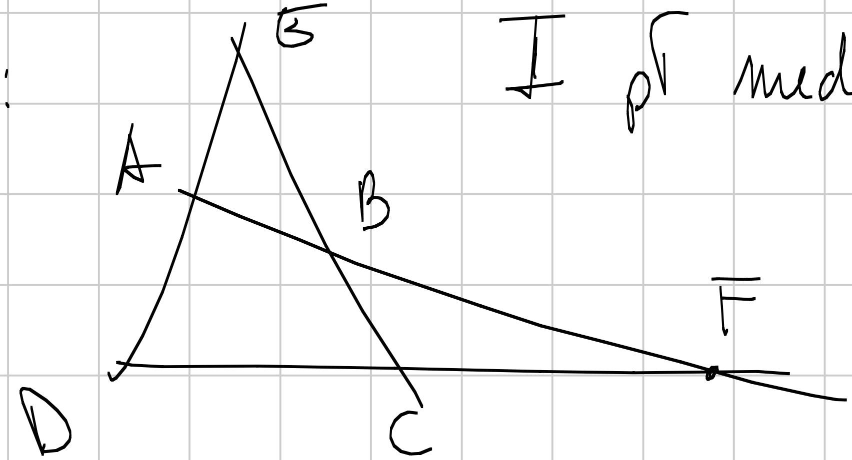
Es x casa: ABC triangolo A varia su r // BC

Trovare il luogo degli ortocentri.



Es istruttivo: Trovare le coord dell'incentro dati i vertici

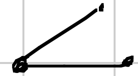
Lemma:



I pt medi di AC, BD, EF sono allineati.

2) Vettori

Piano cartesiano : un punto + due segmenti



un punto.

Vettori

ORIGINE

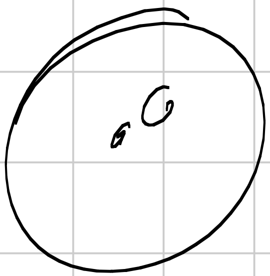
A

Fissato O, A determino

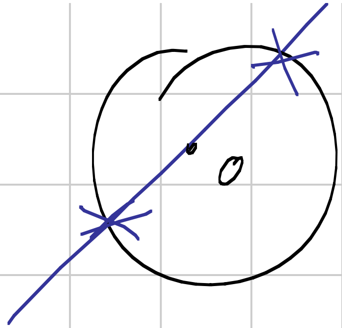
1) Una distanza OA
2) La retta OA
3) Divido la retta del punto 2 in 2 semirette da O, una con A e una senza.

Bastano? (1, 2, 3)

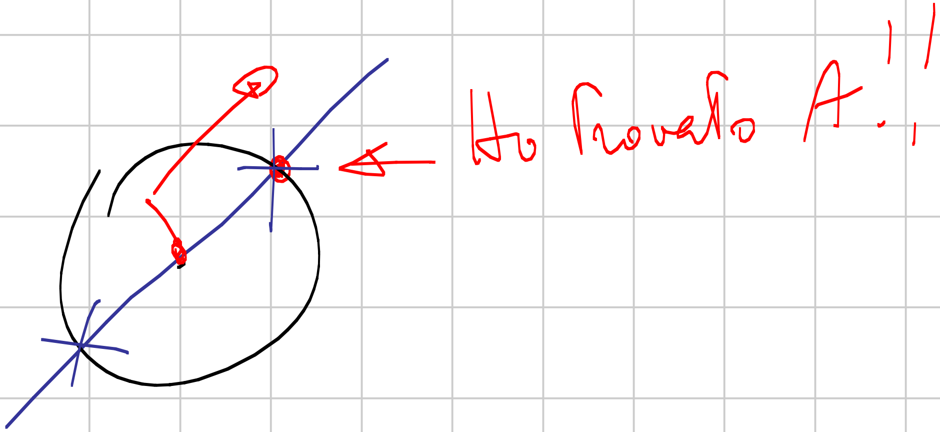
1) 



2 →



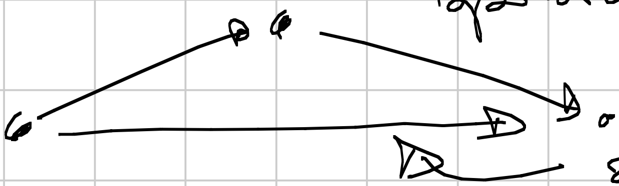
3 →



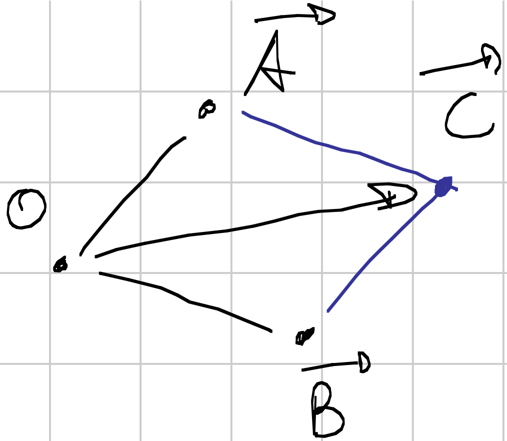
Un vettore è un punto in un piano con fissato un punto speciale.

<u>I vettori si sommano e si sottraggono</u>

spostamenti



somma di spostamento



$\vec{C} = \vec{A} + \vec{B}$

Regola del parallelogramma.
(SOMMA)

Si allungano e si accorciano.

$\vec{OA}$ vett, $K \in \mathbb{R}$ $\qquad K \cdot \vec{OA}$ vett. con ·) direz = direz di $\vec{OA}$

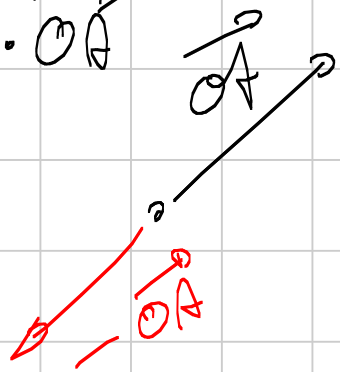
·) lung. = $|K| \cdot$ lung. di $\vec{OA}$

·) verso = verso di $\vec{OA}$ $k>0$

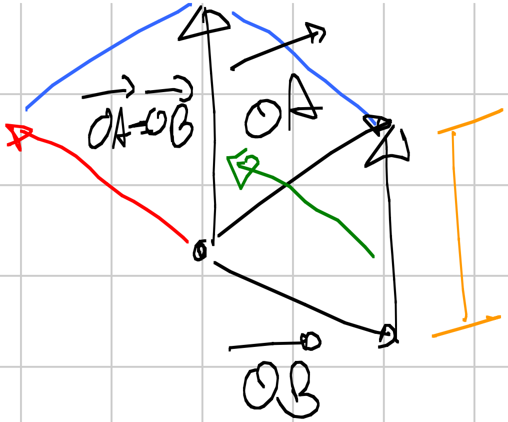
$\neq$ verso di $\vec{OA}$ $k<0$

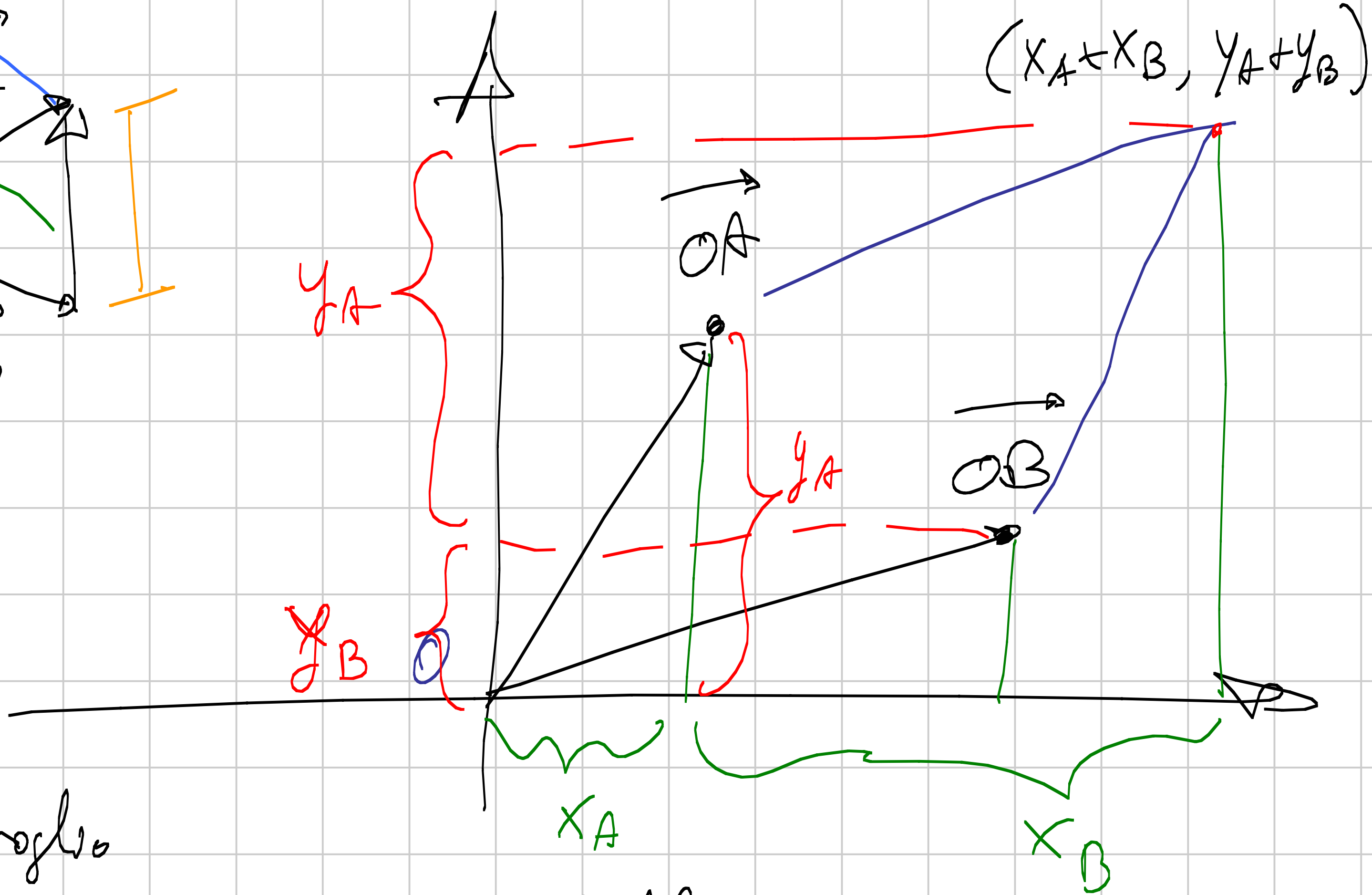
($k=0$ $k\vec{OA}$ collasso)

$-\vec{OA} = -1 \cdot \vec{OA}$



$\vec{OA} - \vec{OB} = \vec{OA} + (-\vec{OB})$





Dati A, B voglio

il vettore associato al p.to medio di AB

$\frac{1}{2}(\vec{OA}+\vec{OB})$ × mille motivi

1- fede
2- coordinate
3- parallelogramma
etc.

$$\overrightarrow{OP_\lambda} = \lambda \overrightarrow{OA} + (1-\lambda)\overrightarrow{OB}$$ vettore verso il punto sulla retta AB

se $0 \le \lambda \le 1$ è interno

<u>Norma di un vettore</u>: $\|\overrightarrow{OA}\|$ = lunghezza di $\overrightarrow{OA}$ =
= dist. tra O e A.

$$\|\lambda \overrightarrow{OA}\| = |\lambda| \cdot \|\overrightarrow{OA}\|$$

$$\|\overrightarrow{OA} + \overrightarrow{OB}\| \le \|\overrightarrow{OA}\| + \|\overrightarrow{OB}\|$$

$$\|\overrightarrow{OA}\| = 0 \iff A \equiv O$$

<u>Es</u>: $\mathrm{dist}(P_\lambda, A) = \|\overrightarrow{OP_\lambda} - \overrightarrow{OA}\| = \|\lambda \overrightarrow{OA} + (1-\lambda)\overrightarrow{OB} - \overrightarrow{OA}\| =$

$= \|(1-\lambda)[\overrightarrow{OB} - \overrightarrow{OA}]\| = |1-\lambda| \cdot \|\overrightarrow{OB} - \overrightarrow{OA}\| =$

$= |1-\lambda| \cdot AB$

$$\operatorname{dist}(P_\lambda, B) = \| \overrightarrow{OP_\lambda} - \overrightarrow{OB} \| = \| \lambda \overrightarrow{OA} + (1-\lambda)\overrightarrow{OB} - \overrightarrow{OB} \| =$$

$$= \| \lambda [\overrightarrow{OA} - \overrightarrow{OB}] \| = |\lambda| \cdot \| \overrightarrow{OA} - \overrightarrow{OB} \| =$$

$$= |\lambda| \cdot AB$$

<u>Es</u>: $ABCD$ quadrilatero

M pt. medio di AC
N pt. medio di BD

$$\left.\begin{matrix} E \\ F \\ G \\ H \end{matrix}\right\} \text{pt. med. di } \left\{\begin{matrix} AB \\ BC \\ CD \\ DA \end{matrix}\right.$$

$\Rightarrow M, N, EG \cap FH$ sono allineati.

<u>OSS</u>: 3 punti X, Y, Z sono allineati se uno è comb. lin. convessa degli altri due.

Non va bene: $\|\vec{OA}+\vec{OB}\| = \|\vec{OA}\| + \|\vec{OB}\|$

questo vuol dire O, A, B allineati.

Sol: Fisso O da qualche parte. Considero $\vec{OA}, \vec{OB}, \vec{OC}, \vec{OD}$.

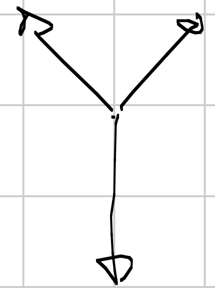
$$\vec{OM} = \frac{1}{2}(\vec{OA}+\vec{OC}) \qquad \vec{ON} = \frac{1}{2}(\vec{OB}+\vec{OD})$$

$$\vec{E} = \frac{1}{2}(\vec{A}+\vec{B}) \quad \vec{F} = \frac{1}{2}(\vec{B}+\vec{C}) \quad \vec{G} = \frac{1}{2}(\vec{C}+\vec{D}) \quad H = \frac{1}{2}(\vec{D}+\vec{A})$$

$$\left.\begin{aligned} &\text{pt. med. di } EG = \frac{1}{4}(\vec{A}+\vec{B}+\vec{C}+\vec{D}) \\ &\text{pt med di } FH = \frac{1}{4}(\vec{A}+\vec{B}+\vec{C}+\vec{D}) \end{aligned}\right\} EG \cap FH$$

pt med di $MN = \frac{1}{4}(\vec{A}+\vec{B}+\vec{C}+\vec{D}) \Longrightarrow$ Fine!!

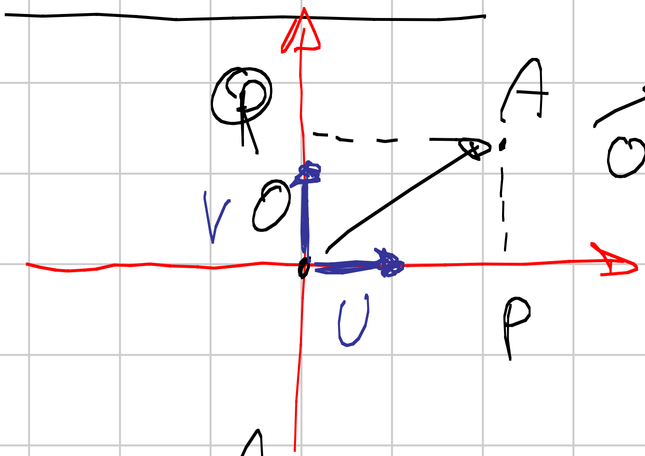
$s\vec{A} + t\vec{B} + k\vec{C} = 0 \;\cancel{\Longrightarrow}\; s = t = k = 0.$



X Casa: $t\vec{OA} + s\vec{OB} = 0$

??

Prodotto scalare



$\vec{OA}, \vec{OU}$

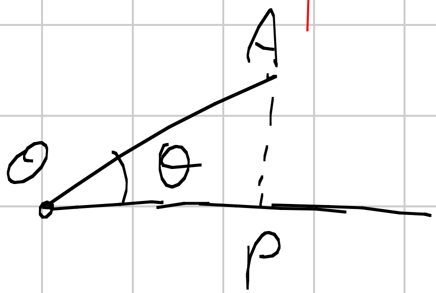
$$\vec{OP} + \vec{OQ} = \vec{OA}$$

$$\parallel \qquad \parallel$$

$$x_A \cdot \vec{OU} + y_A \cdot \vec{OV}$$

Prodotto scalare

x_A



$OP = OA \cos\theta$

$\langle \vec{OX}, \vec{OY} \rangle = \vec{OX} \cdot \vec{OY} = (\vec{OX}, \vec{OY}) =$

$$= \|\vec{OX}\| \cdot \|\vec{OY}\| \cdot \cos(\widehat{XOY}) = x_1 x_2 + y_1 y_2$$

se $X = (x_1, y_1)$
$Y = (x_2, y_2)$

$$\langle \vec{OX}, \vec{OX} \rangle = \|\vec{OX}\|^2$$

$$\boxed{\langle \vec{OX}, \vec{OY} \rangle \leq \|\vec{OX}\| \cdot \|\vec{OY}\|}$$

Cauchy-Schwarz.

Proprietà:

$$\langle \vec{OA}, \vec{OB} \rangle = \langle \vec{OB}, \vec{OA} \rangle$$

$$\langle \vec{OA} + \vec{OC}, \vec{OB} \rangle = \langle \vec{OA}, \vec{OB} \rangle + \langle \vec{OC}, \vec{OB} \rangle$$

$$\langle \lambda \vec{OA}, \vec{OB} \rangle = \lambda \langle \vec{OA}, \vec{OB} \rangle$$

Es: $\langle \lambda \vec{OA}, \lambda \vec{OB} \rangle = \lambda^2 \langle \vec{OA}, \vec{OB} \rangle$.

$$\|\vec{OA} + \vec{OB}\|^2 = \langle \vec{OA} + \vec{OB}, \vec{OA} + \vec{OB} \rangle = \langle \vec{OA}, \vec{OA} + \vec{OB} \rangle + \langle \vec{OB}, \vec{OA} + \vec{OB} \rangle$$

$$= \langle \vec{OA}, \vec{OA} \rangle + \langle \vec{OA}, \vec{OB} \rangle + \langle \vec{OB}, \vec{OA} \rangle + \langle \vec{OB}, \vec{OB} \rangle =$$

$$= \|\vec{OA}\|^2 + \|\vec{OB}\|^2 + 2 \langle \vec{OA}, \vec{OB} \rangle.$$

Oss: Il baricentro di $\overset{\triangle}{ABC}$ è $\dfrac{\vec{A}+\vec{B}+\vec{C}}{3}$

(rispetto a ogni origine)

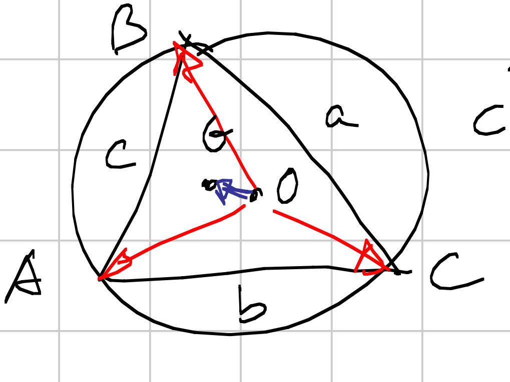
Se l'origine è il circocentro O, $\|\vec{A}\| = \|\vec{B}\| = \|\vec{C}\|$

(e 3 qualunque punto del genere fanno centri.)

Es: $OG = ?$

$$OG^2 = \left\langle \frac{\vec{A}+\vec{B}+\vec{C}}{3}, \frac{\vec{A}+\vec{B}+\vec{C}}{3} \right\rangle = \frac{1}{9}\Big(\underbrace{\|\vec{A}\|^2}_{=R^2} + \underbrace{\|\vec{B}\|^2}_{=R^2} + \underbrace{\|\vec{C}\|^2}_{=R^2} +$$

$$+ 2\langle \vec{A}, \vec{B} \rangle + 2\langle \vec{B}, \vec{C} \rangle + 2\langle \vec{C}, \vec{A} \rangle \Big)$$



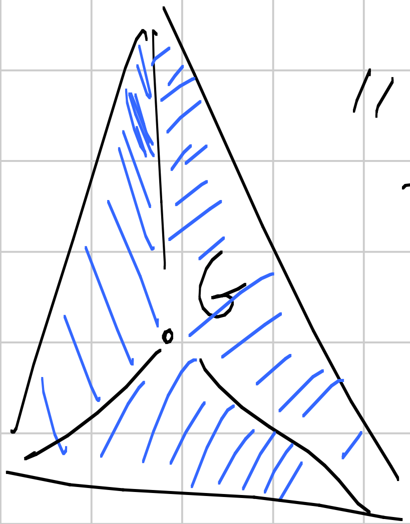
$$c^2 = AB^2 = \langle \vec{A}-\vec{B}, \vec{A}-\vec{B} \rangle =$$

$$= \underbrace{\|\vec{A}\|^2}_{=R^2} + \underbrace{\|\vec{B}\|^2}_{=R^2} - 2\langle \vec{A}, \vec{B} \rangle$$

$$2\langle \vec{A}, \vec{B}\rangle = 2R^2 - c^2$$
$$2\langle \vec{B}, \vec{C}\rangle = 2R^2 - a^2$$
$$2\langle \vec{C}, \vec{A}\rangle = 2R^2 - b^2$$

$$OG^2 = \frac{1}{9}\left(3R^2 + 6R^2 - a^2 - b^2 - c^2\right) =$$
$$= R^2 - \frac{a^2+b^2+c^2}{9}$$

Oss:



$$\Longrightarrow \quad G = \frac{1}{3}\vec{A} + \frac{1}{3}\vec{B} + \frac{1}{3}\vec{C}$$

$$\vec{P} = \alpha\vec{A} + \beta\vec{B} + \gamma\vec{C}$$

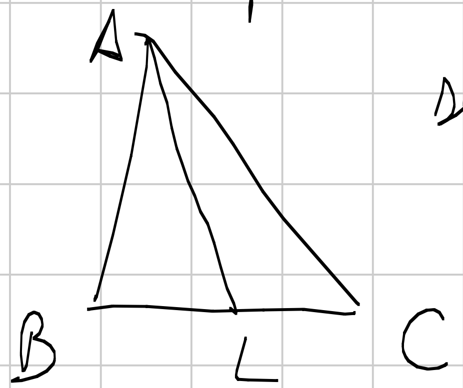
con $\alpha+\beta+\gamma = 1$.

se $\alpha, \beta, \gamma > 0$

Chi ha voglia per casa può provare a dim che

$$\frac{S(PAB)}{S(ABC)} = \gamma \qquad \frac{S(PBC)}{S(ABC)} = \alpha \qquad \frac{S(PAC)}{S(ABC)} = \beta$$

Cosa: Trovare l'espr. vett. per l'incentro ricordando che:



se AL bisett.

$\Updownarrow$

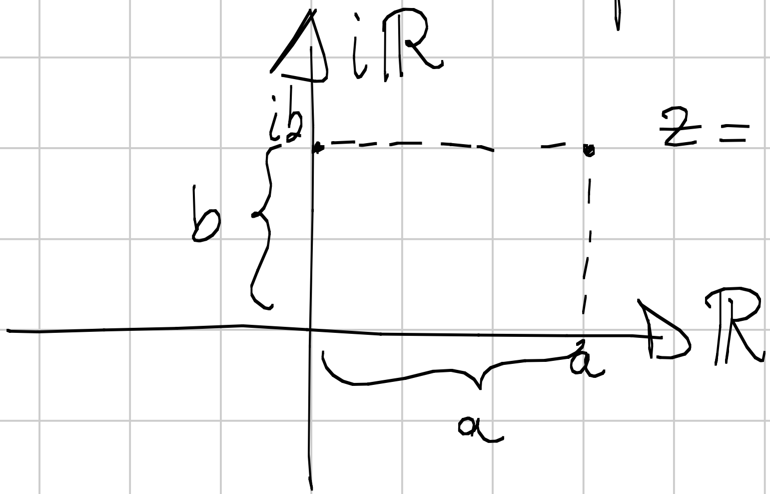
$$\frac{BL}{LC} = \frac{AB}{AC}$$

$$\frac{a\vec{A} + b\vec{B} + c\vec{C}}{a+b+c} = \vec{I}$$ con qualunque origine.

+ Difficile: Trovare OI e GI

3) Numeri Complessi

$\mathbb{C}$ è un piano con un punto speciale (lo ZERO)



$z = a + ib \longrightarrow$ Punto (a, b) nel piano cartesiano

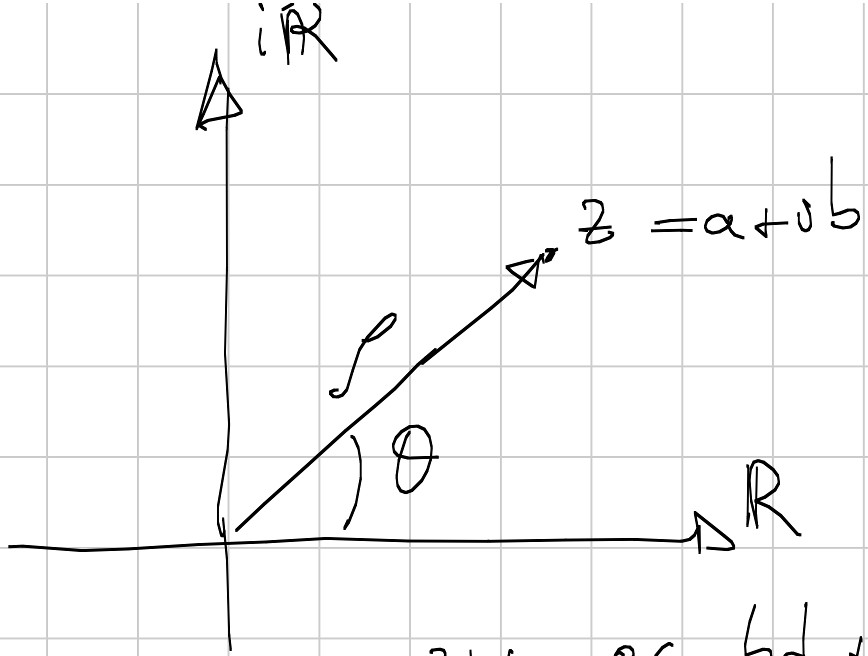
$z = a + ib$
$w = c + id$

$z + w = (a+c) + i(b+d)$

la somma dei vettori.

$$\operatorname{dist}(z, 0) = \sqrt{a^2 + b^2} = |z| = \sqrt{z \cdot \bar{z}}$$

$\lambda \cdot z = \lambda a + i \lambda b \quad \lambda \in \mathbb{R} \qquad |\lambda z| = |\lambda| \cdot |z|$



$$z \longleftrightarrow (a, b)$$

$$z \longleftrightarrow (\rho, \theta)$$

$$w = c + id$$

$$(r, \alpha)$$

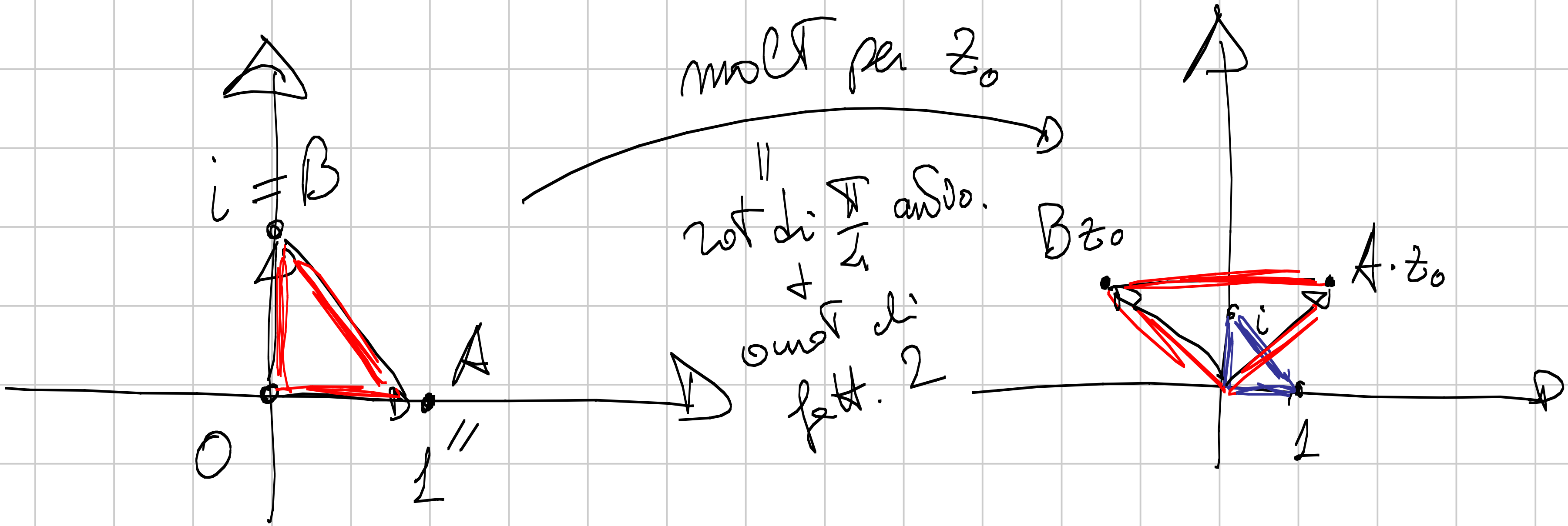
$$zw = ac - bd + i(ad + bc)$$

$$\updownarrow$$

$$(\rho r, \alpha + \theta)$$

$\Rightarrow$ Se moltiplico tutti i numeri complessi per

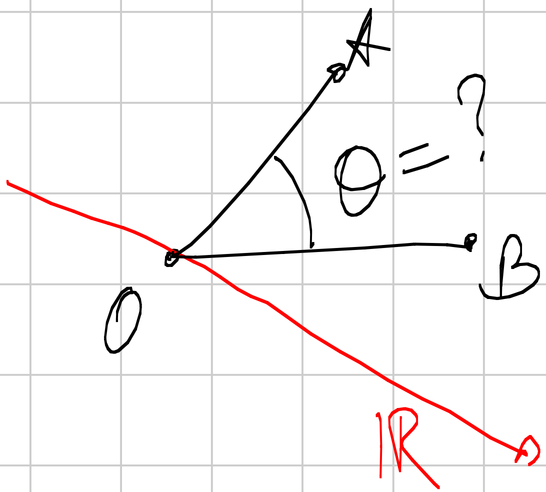
$$z_0 = \frac{2}{\sqrt{2}} + i\frac{2}{\sqrt{2}} \qquad |z_0| = \sqrt{\frac{4}{2} + \frac{4}{2}} = \sqrt{\frac{8}{2}} = \sqrt{4} = 2$$



$|z_0| = 2 \qquad \arg(z_0) = \frac{\pi}{4}$

Coniugato = Simm. risp. all'asse reale.

Es:



$\longrightarrow$ $a, b \in \mathbb{C}$

$\frac{a}{b}$ ha argomento θ.

Es: Se vogliamo che a, b, c siano allineati, dobbiamo chiedere che ...

l'angolo tra BA e AC sia 0 o π

- Traslliamo A in O $\Rightarrow$ $a - a = 0$

$b - a$

$c - a$

$\frac{b-a}{c-a}$ deve avere $\arg = 0, \pi \Rightarrow \in \mathbb{R}$

$\arg(z) = 0 \Rightarrow z > 0 \in \mathbb{R}$

$\arg(z) = \pi \Rightarrow z < 0 \in \mathbb{R}$

$z = \bar{z} \Leftrightarrow z \in \mathbb{R}$

$$\frac{b-a}{c-a} = \overline{\frac{b-a}{c-a}} = \frac{\bar{b}-\bar{a}}{\bar{c}-\bar{a}}$$

Retta per b, c: $(b - z)(\bar{c} - \bar{z}) = (\bar{b} - \bar{z})(c - z)$

Circonf. di centro a, raggio R :

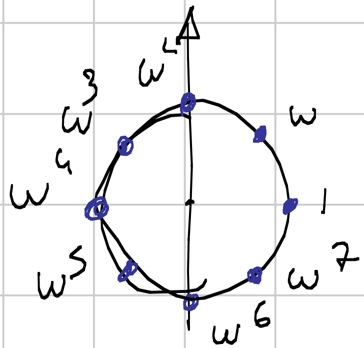
$$|z-a| = R$$

$$(z-a)(\bar{z}-\bar{a}) = R^2$$

Rotazione: se $|z|=1$ molt. per z è rotazione.

$\omega = \frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}$ $\qquad 1, \omega, \omega^2, \omega^3, \omega^4, \omega^5, \omega^6, \omega^7$

$\arg(\omega) = \frac{\pi}{4} \qquad \arg(\omega^2) = \frac{\pi}{2} \qquad \arg(\omega^3) = \frac{3}{4}\pi$



radici 8^e dell'unità.

$\Rightarrow$ formano un 8-agono regolare.

$1 \to \omega$
$\omega \to \omega^2$ ISOMETRIA $\Rightarrow |1-\omega| = |\omega-\omega^2| = |\omega|\cdot|1-\omega| = |1-\omega|$

Generico ottagono regolare

$1\cdot z_0 + z_1$, $\omega\cdot z_0 + z_1$, $\omega^2 z_0 + z_1$, ----

z_1 = centro $|z_0|$ = raggio della circ. circo

$\arg(z_0)$ = angolo tra il raggio che arriva in $1 z_0 + z_1$ e l'asse reale

$$\frac{(1\cdot z_0 + z_1) - (\omega z_0 + z_1) = z_0(1-\omega)}{(\omega z_0 + z_1) - (\omega^2 z_0 + z_1) = z_0(\omega-\omega^2)} = \frac{1}{\omega}$$

<u>Triangolo equilatero</u>: $1, \omega, \omega^2$ radici 3^e dell'unità

$$\omega = -\frac{1}{2} + \frac{i\sqrt{3}}{2} = \cos\left(\frac{2\pi}{3}\right) + i\,\mathrm{sen}\left(\frac{2\pi}{3}\right)$$

$$\frac{1}{\omega} = \omega^2 = \overline{\omega} = -\frac{1}{2} - i\,\frac{\sqrt{3}}{2}$$

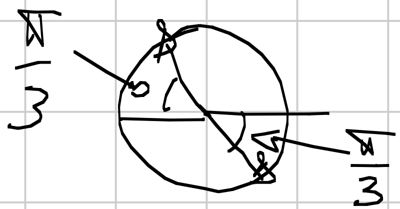
$1, \omega, \omega^2$ $\qquad\qquad$ $z_0+z_1, \ \omega z_0+z_1, \ \omega^2 z_0+z_1$

Quando a, b, c sono un tri equilatero?

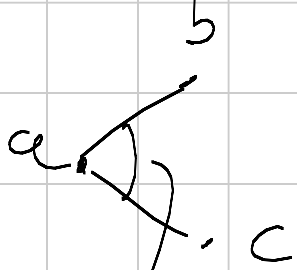
$$a+\omega b+\omega^2 c = 0 \qquad\qquad \bullet)\ 1+\omega+\omega^2 = 0$$

$$\cancel{a}+\omega b+\omega^2 c = \cancel{a}+\omega a+\omega^2 a$$

$$\omega(b-a) = -\omega^2(c-a)$$



$$\frac{b-a}{c-a} = -\omega$$



$\xrightarrow{1}$ $\left|\frac{b-a}{c-a}\right| = |\omega| \Rightarrow |b-a| = |c-a|$

$\xrightarrow{2}$ $\arg\left(\frac{b-a}{c-a}\right) = -\frac{\pi}{3}$

$\Rightarrow$ a, b, c formano un tri equil.

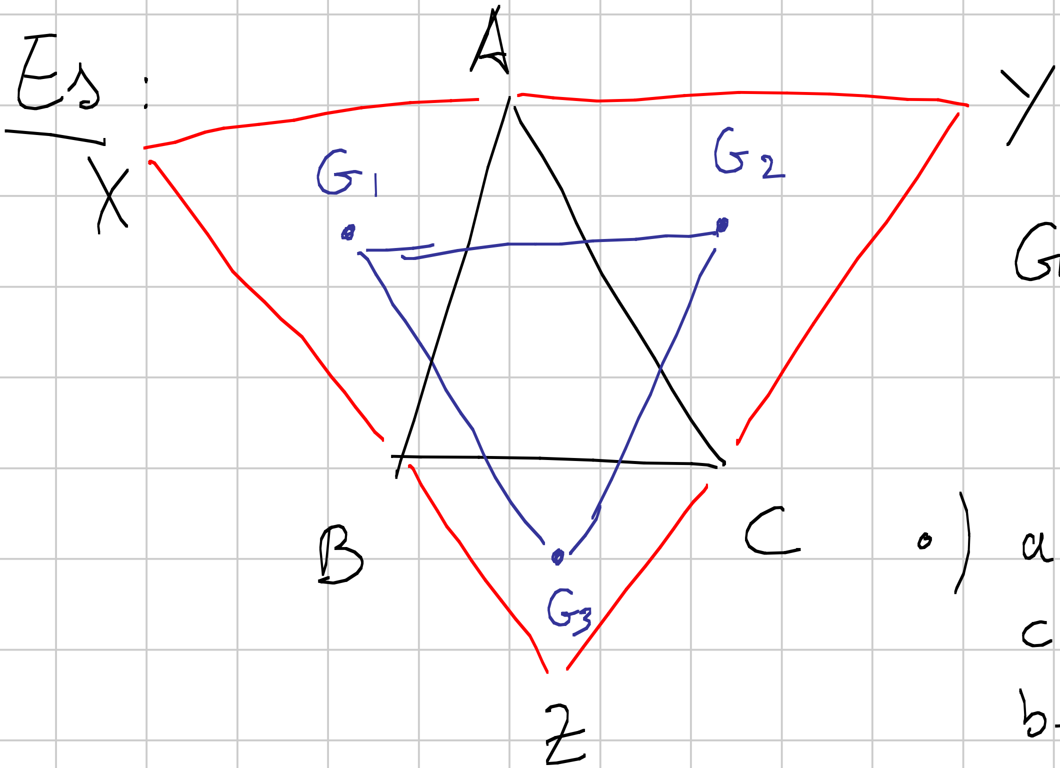
Oss:

$$z_0+z_1+\omega(\omega z_0+z_1)+\omega^2(\omega^2 z_0+z_1)=$$

$$=z_0+\omega^2 z_0+\omega^4 z_0+z_1+\omega z_1+\omega^2 z_1=$$

$$=\underbrace{z_0+\omega^2 z_0+\omega z_0}_{0}+z_1(1+\omega+\omega^2)=0.$$

Es:



AYC, BZC, AXB equilateri

G_1: baricentro $\Downarrow$ $G_1G_2G_3$ equilatero

∘) $a+\omega y+\omega^2 c=0$
$c+\omega z+\omega^2 b=0$
$b+\omega x+\omega^2 a=0$

a) $g_1 = \frac{a+b+x}{3}$ $\quad g_2 = \frac{a+c+y}{3}$ $\quad g_3 = \frac{c+z+b}{3}$

$$\boxed{\begin{array}{ll} \mathrm{I} & a + \omega y + \omega^2 c = 0 \\ \mathrm{II} & c + \omega z + \omega^2 b = 0 \\ & b + \omega x + \omega^2 a = 0 \end{array}}$$

Voglio: $3(g_2 + \omega g_2 + \omega^2 g_3) = 0$

$$\underbrace{\cancel{a}}_{\mathrm{I}} + b + x + \omega a + \omega c + \underbrace{\cancel{\omega y}}_{\mathrm{I}} + \underbrace{\cancel{\omega^2 c}}_{\mathrm{I}} + \omega^2 z + \omega^2 b =$$

$$b + x + \omega a + \omega c + \omega^2 z + \omega^2 b$$

$$(b + \omega^2 z + \omega c) + (x + \omega a + \omega^2 b)$$

$$\omega(\underbrace{\omega^2 b + \omega z + c}_{\substack{\| \\ 0}}) + \omega^2(\underbrace{\omega x + \omega^2 a + b}_{\substack{\| \\ 0}}) = 0$$

II $\quad$ III

$(1, \omega, \omega^2)$

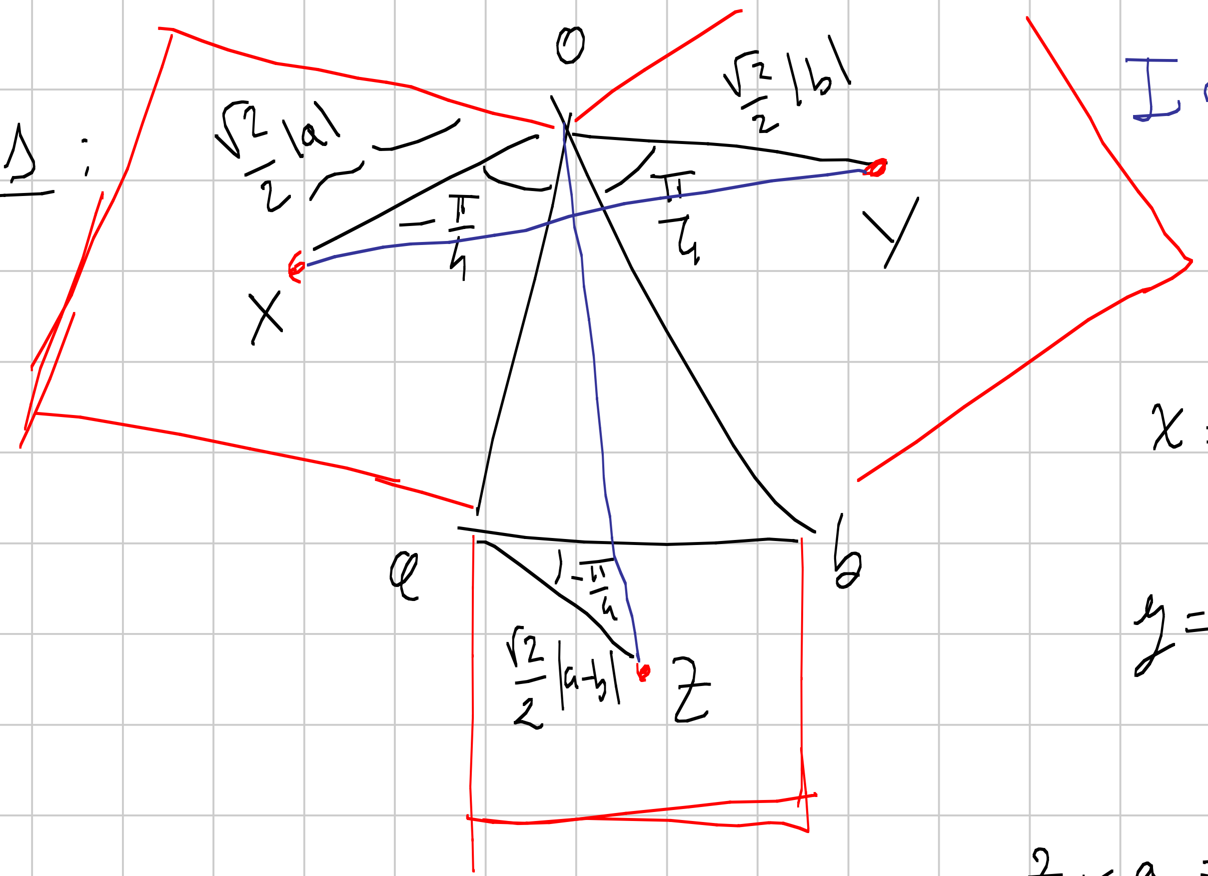
$\downarrow$ molt. per ω

$(\omega, \omega^2, 1)$

$\downarrow$

$(\omega^2, 1, \omega)$

Es:



I segmenti blu sono perp. e uguali.

$$x = a\cdot\frac{\sqrt2}{2}\left(+\frac{1}{\sqrt2}-\frac{i}{\sqrt2}\right)$$

$$y = b\,\frac{\sqrt2}{2}\left(\frac{1}{\sqrt2}+\frac{i}{\sqrt2}\right)$$

$$z-a = (b-a)\,\frac{\sqrt2}{2}\left(\frac{1}{\sqrt2}-\frac{i}{\sqrt2}\right)$$

$$x-y = \frac{\sqrt2}{2}\left(\frac{1}{\sqrt2}(a-b)-\frac{i}{\sqrt2}(a+b)\right)$$

$$z = (b-a)\,\frac{\sqrt2}{2}\left(\frac{1}{\sqrt2}-\frac{i}{\sqrt2}\right)+a =$$

$$= \frac{\sqrt2}{2}\left(\frac{1}{\sqrt2}(b-a+2a)-\frac{i}{\sqrt2}(b-a)\right) =$$

$$= \frac{\sqrt2}{2}\left(\frac{1}{\sqrt2}(b+a)-\frac{i}{\sqrt2}(b-a)\right)$$