Geometria 2 - Medium: Metodi Algebrici

1) Coordinate (geo. analitica)

2) Vettori

3) Numeri complessi

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1) <u>Coniche</u>:

$$\mathcal{C} = \{(x,y) \mid Ax^2 + 2Bxy + Cy^2 + 2Dx + 2Ey + F = 0\}$$

$A, B, C, D, E, F \in \mathbb{R}$ ($A=0, B=0, C=0$ non è una conica)

(i) $x^2 = 0$ Retta doppia

(ii) $x^2 = y^2$ Rette incidenti

(iii) $x^2 = 1$ Rette parallele

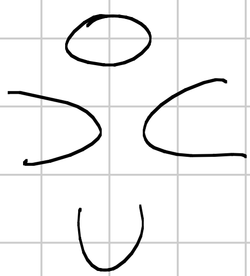
(iv) $x^2 + y^2 = 0$ Punto

(v) $x^2 + y^2 + 1 = 0$ / $x^2 + 1 = 0$ Vuoto

(vi) $x^2 + y^2 = 1$ Ellisse

(vii) $x^2 - y^2 = 1$ Iperbole

(viii) $x^2 = y$ Parabola



• $\sum \square = k$ ha sol. lim. in $\mathbb{R}$

• $\square - \square = k$ ha sol. illim. in $\mathbb{R}$

$\underline{Es}: 10x^2 + \underset{\underset{2\cdot 3xy}{\parallel}}{6xy} + y^2 + 4x + 3 = 0$

$(3x + y)^2$

$(9x^2+y^2+6xy) + x^2 + 4x + 3 = 0$

$(3x+y)^2 + (x+2)^2 = 1$

$$\begin{cases} x' = 3x+y \\ y' = x+2 \end{cases}$$

$\underline{Oss}: ax^2+bx+c = \left(\sqrt{a}\,x + \frac{b}{2\sqrt{a}}\right) + c'$

$\underline{Oss}$: Se $B^2 = AC$

$(\sqrt{A}\,x + \sqrt{C}\,y)^2 + 2Dx + 2Ey + F = 0$

$\Rightarrow$ parabola

$\underline{Oss\ 2}$: Se $B^2 > AC$

$(\sqrt{A}\,x + Hy)^2$

$Ax^2 + \underbrace{2\sqrt{A}Hy}_{\underset{2By}{\shortparallel}} + \underbrace{H^2y^2}$

$H = \frac{B}{\sqrt{A}} > \sqrt{C}$

$\Updownarrow$

$H^2 > C$

$-L^2y^2$

$(\sqrt{A}x + Hy)^2 - (Ly + \ldots)^2$

$\Rightarrow$ iperbole

$\underline{Oss\ 3}$: $B^2 < AC \Rightarrow$ ellisse

Classif. Affine $\to \begin{cases} y' = ax + by + c \\ x' = dx + ey + f \end{cases}$

Classif. Metrica $\rightsquigarrow \begin{cases} x' = x + x_0 \\ y' = y + y_0 \end{cases}$ Trasl $\quad \begin{cases} x' = -x \\ y' = y \end{cases}$ rifl.

$$\begin{cases} x' = \cos\theta\, x + \operatorname{sen}\theta\, y \\ y' = -\operatorname{sen}\theta\, x + \cos\theta\, y \end{cases}$$

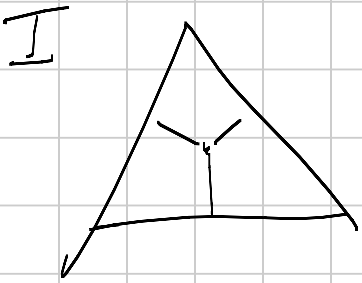
Rotaz. (di un angolo θ)

Altre coordinate

in coord. cart. → usare un'affinità $(0,0) - (1,0) - (0,1)$

→ fissare 2 punti $(0,0) - (1,0) - (a,b)$

I

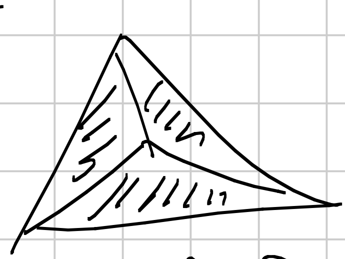


dist. dai lati

↓

Unica

II



aree staccate sui lati

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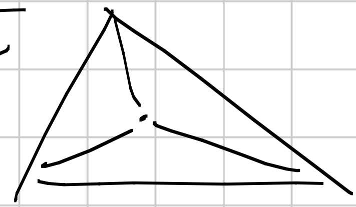
Area di PAB

$\shortparallel$

$\frac{1}{2} AB \cdot d(P, AB)$

Unica!

III



dist. dai vertici

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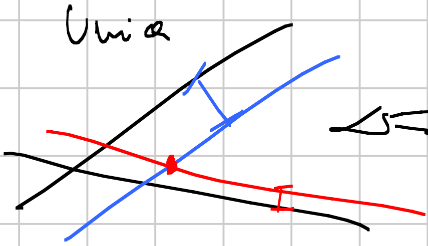
Intersez. di 3 circ. non coassiali $\Rightarrow$ Unica

Oss:

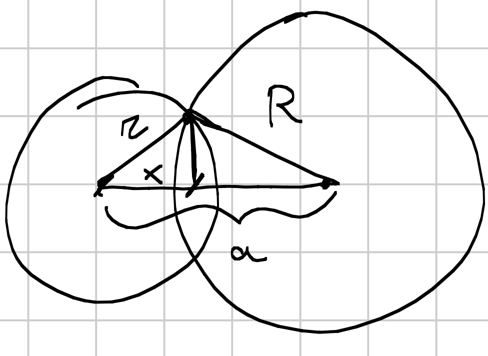


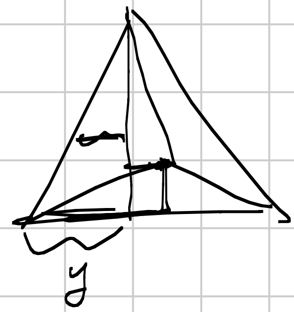
$\Longleftrightarrow$ Area di PAB

II : bisogna avere $[ABP]+[ACP]+[BCP]=[ABC]$

I : $d(P,AB)c + d(P,AC)b + d(P,BC)a = 2[ABC]$

III : (idea)





I - Coord Trilineari

II - Coord baricentriche

III - Orrore.

I - (x, y, z) t.c. $ax+by+cz=1$

$(d\text{-}BC, d\text{-}AC, d\text{-}AB)$

Si usano equazioni omogenee, si intendono le triple (x,y,z) come "cose" omogenee, ovvero

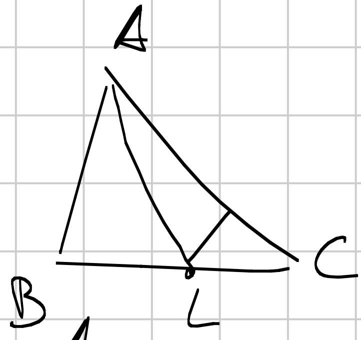
(x,y,z) è lo stesso che $(2x,2y,2z)$

Coord incentro $(r,r,r) \sim (1,1,1)$

Coord A $(h_A, 0, 0) \sim (1,0,0)$

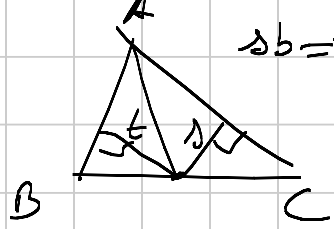
B $(0,1,0)$

C $(0,0,1)$



AL bisett. di $B\hat{A}C$ $(0,1,1)$

Pt. medio di BC $(0,c,b)$



Rette : $px+qy+rz=0$

$$\begin{cases} px+qy+rz=0 \\ sx+ty+uz=0 \end{cases} \qquad (qu-rt,\ rs-pu,\ pt-sq)$$

$$\left(\frac{1}{D}, \frac{3}{D}, \frac{2}{D}\right) \approx$$

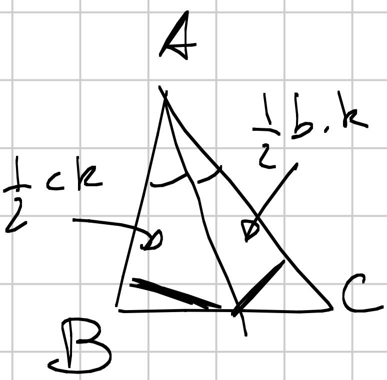
$$\approx (1,3,2)$$

$$\left(1, \frac{2}{D}, \frac{3}{D}\right) \sim (D,2,3)$$

II) Anche qui tutto omogeneo $\Rightarrow$ retta $px+qy+rz=0$

(x,y,z) $\quad x+y+z=1$

A	$(1,0,0)$	G	$(1,1,1)$
B	$(0,1,0)$	I	(a,b,c)
C	$(0,0,1)$		



M pt. med di BC $\quad (0,1,1)$

L piede della bisett. $\quad (0,b,c)$

(x,y,z) coord. baricentriche $\Rightarrow \left(\frac{x}{a}, \frac{y}{b}, \frac{z}{c}\right)$ coord trilin

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2) + 3) Vettori & Numeri complessi

•) Se fisso l'origine nel circocentro di ABC

$$\vec{H} = \vec{A} + \vec{B} + \vec{C} \qquad \left(O, G, H \text{ allineati e } \frac{OG}{OH} = \frac{1}{3} \right.$$

$$\vec{N} = \frac{\vec{A}+\vec{B}+\vec{C}}{2} \qquad \left. \vec{G} = \frac{\vec{A}+\vec{B}+\vec{C}}{3} \right)$$

Es: ABCD quad. $\Rightarrow$ i baricentri di ABC, ABD, BCD, ACD formano un quadrilatero simile.

$$G_D = G_{ABC} = \frac{\vec{A}+\vec{B}+\vec{C}}{3} \qquad G_{BCD} = \frac{\vec{B}+\vec{C}+\vec{D}}{3} = G_A$$

$$G_C = G_{ABD} = \frac{\vec{A}+\vec{B}+\vec{D}}{3} \qquad G_{ACD} = \frac{\vec{A}+\vec{C}+\vec{D}}{3} = G_B$$

$$\vec{G}_X = \frac{\vec{A}+\vec{B}+\vec{C}+\vec{D}-\vec{X}}{3}$$ = simm centrale $(X \to -X)$ + traslazione $(Y \to A+B+C+D+Y)$ + omotetia $\left(Z \to \frac{Z}{3}\right)$

$G_A G_B G_C G_D$ si ottiene da ABCD tramite SIMILITUDINE.

Es: ABCD ciclico $\Rightarrow$ gli ortocentri H_A, H_B, H_C, H_D formano un quad ciclico

$$\vec{H}_A = \vec{A}+\vec{B}+\vec{C}+\vec{D}-\vec{A}$$

$$\vec{H}_X = \vec{A}+\vec{B}+\vec{C}+\vec{D}-X$$

fissando l'origine nel comune circocentro

$$\vec{H}_A = \vec{B}+\vec{C}+\vec{D}$$

$\Rightarrow H_A H_B H_C H_D$ è CONGRUENTE a ABCD (ottenuto tramite ISOMETRIA)

Es: ABC tri, G, H, R = raggio circ. circo.

M = pt. medio di GH.

$$AM^2 + BM^2 + CM^2 = 3R^2$$

origine in O

$$\vec{M} = \left(\frac{\vec{A}+\vec{B}+\vec{C}}{3} + \vec{A}+\vec{B}+\vec{C}\right)\frac{1}{2} = \frac{2}{3}(\vec{A}+\vec{B}+\vec{C})$$

$$AM^2 = \langle \vec{A}-\vec{M}, \vec{A}-\vec{M} \rangle =$$

$$= \left\langle \frac{\vec{A}}{3} - \frac{2}{3}(\vec{B}+\vec{C}), \frac{\vec{A}}{3} - \frac{2}{3}(\vec{B}+\vec{C}) \right\rangle =$$

$$= \frac{\|A\|^2}{9} + \frac{4}{9}\|\vec{B}+\vec{C}\|^2 - \frac{4}{9}\langle \vec{A}, \vec{B}+\vec{C} \rangle =$$

$$= \underbrace{\frac{R^2}{9} + \frac{8}{9}R^2} + \frac{8}{9}\langle \vec{B}, \vec{C} \rangle - \frac{4}{9}\langle \vec{A}, \vec{B} \rangle - \frac{4}{9}\langle \vec{A}, \vec{C} \rangle$$

$$\langle \vec{B}, \vec{C} \rangle = R^2 - \frac{a^2}{2}$$

$$AM^2 + BM^2 + CM^2 = 3R^2.$$

Es: ABCD quad. M, N, P, Q pt. medi dei lati

$MP + NQ =$ semiperimetro $\Longleftrightarrow$ ABCD parallelogramma.

$$\frac{1}{2}\|\vec{A}-\vec{B}\| + \frac{\|\vec{D}-\vec{C}\|}{2} + \frac{\|\vec{B}-\vec{C}\|}{2} + \frac{\|\vec{D}-\vec{A}\|}{2} \geq$$

$$\geq \frac{1}{2}\|\vec{A}+\vec{D}-\vec{B}-\vec{C}\| + \frac{1}{2}\|\vec{B}+\vec{D}-\vec{C}-\vec{A}\|$$

$$\left\| \frac{A+B}{2} - \frac{C+D}{2} \right\| + \left\| \frac{B+C}{2} - \frac{D+A}{2} \right\| =$$ ipotesi

$$= \left\| \frac{B-A}{2} \right\| + \left\| \frac{C-D}{2} \right\| + \left\| \frac{C-B}{2} \right\| + \left\| \frac{D-A}{2} \right\| \geq$$ Disug. triang.

$$\geq \left\| \frac{B+C}{2} - \frac{A+D}{2} \right\| + \left\| \frac{B+A}{2} - \frac{C+D}{2} \right\|$$

$$|\vec{a}+\vec{b}| = |\vec{a}| + |\vec{b}|$$

Vale l'ug. $\Longleftrightarrow$ $\left.\begin{array}{l} \frac{B-A}{2} \text{ allin. con } \frac{C-D}{2} \\ \frac{C-B}{2} \text{ allin con } \frac{D-A}{2} \end{array}\right\} \Rightarrow$ lati opposti paralleli

Es x casa: ABC scaleno $\Rightarrow$ $G\hat{I}H > 90°$.

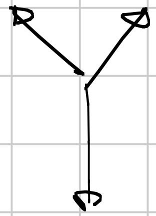
$$\vec{I} = \frac{a\vec{A} + b\vec{B} + c\vec{C}}{a+b+c}$$

Es ABCDE pent. conv. M, N, P, Q, R pt. medi di AB, BC, CD, DE, EA

Se AP, BQ, CR, DM concorrono in X, anche EN ci passa.

$$\vec{x} + \lambda \vec{y} = \vec{z} + \mu \vec{w} \qquad s\vec{a} + t\vec{b} + u\vec{c} = 0$$

non si intersecano bene le rette



Mettiamo l'origine in X?

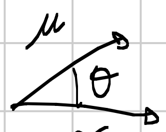
$$P = \frac{C+D}{2} = \lambda_1 A \qquad Q = \frac{D+E}{2} = \lambda_2 B$$

$$R = \frac{E+A}{2} = \lambda_3 C \qquad M = \frac{A+B}{2} = \lambda_4 D$$

Tesi: $N = \dfrac{B+C}{2} = \lambda_5 E$

$$P+Q+R+M = A+D+E+\frac{C}{2}+\frac{B}{2} = \lambda_1 A + \lambda_2 B + \lambda_3 C + \lambda_4 D$$

Non funziona!



$\vec{u} \times \vec{v}$
- lung $= |u||v| \operatorname{sen} \theta$
- dir. $\perp$ piano (u, v)
- verso regola della mano destra

se $u = \lambda v$ $\quad \vec{u} \times \vec{v} = 0$ $\quad (\operatorname{sen} \theta = 0)$

$P \times A = 0 \qquad Q \times B = 0 \qquad R \times C = 0 \qquad M \times D = 0$

$\frac{C+D}{2} \times A = 0 \qquad \frac{D+E}{2} \times B = 0 \qquad \frac{E+A}{2} \times C = 0 \qquad \frac{A+B}{2} \times D = 0$

$C \times A \overset{②}{=} A \times D \qquad D \times B \overset{④}{=} B \times E \qquad E \times C \overset{①}{=} C \times A \qquad A \times D \overset{③}{=} D \times B$

Tesi : $(B+C) \times E = 0 \iff B \times E = E \times C$.

$$E \times C \underset{①}{=} C \times A \underset{②}{=} A \times D \underset{③}{=} D \times B \underset{④}{=} B \times E$$

Es: $\vec{P} = \alpha \vec{A} + \beta \vec{B} + \gamma \vec{C}$ con $\alpha + \beta + \gamma = 1$

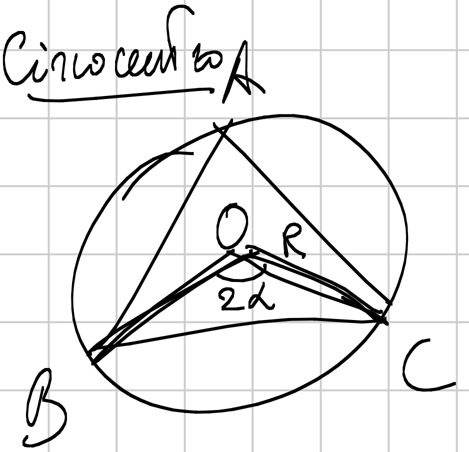
$$\Rightarrow [PBC] = \alpha [ABC]$$
$$[PAC] = \beta [ABC]$$
$$[PAB] = \gamma [ABC]$$

$$[ABC] = \frac{1}{2}(B-A) \times (C-A) = \frac{1}{2}(C-B) \times (A-B) = \frac{1}{2}(A-C) \times (B-C)$$
$$= \frac{1}{2} B \times C - \frac{1}{2} B \times A - \frac{1}{2} A \times C$$
$$= \frac{1}{2} A \times B + \frac{1}{2} B \times C + \frac{1}{2} C \times A$$

$$[PBC] = \frac{1}{2}(B-P) \times (C-P) = \frac{1}{2}\Big((\alpha+\gamma)B - \alpha A - \gamma C\Big) \times \Big((\alpha+\beta)C - \alpha A - \beta B\Big)$$
$$= \frac{1}{2}\Big[(\alpha+\gamma)(\alpha+\beta)\, B \times C - \alpha(\alpha+\gamma)\, B \times A - \alpha(\alpha+\beta)\, A \times C + \alpha\beta\, A \times B$$
$$+ \gamma\alpha\, C \times A + \gamma\beta\, C \times B\Big] =$$
$$= \frac{1}{2}\Big[A \times B(\alpha^2 + \overset{\alpha}{\alpha\gamma + \alpha\beta}) + B \times C(\alpha^2 + \overset{\alpha}{\alpha\gamma + \alpha\beta} + \cancel{\gamma\beta} - \cancel{\gamma\beta}) +$$
$$+ C \times A(\alpha^2 + \overset{\alpha}{\alpha\beta + \alpha\gamma})\Big] =$$

$$= \frac{\alpha}{2}\left[A\times B + B\times C + C\times A\right] = \alpha\,[ABC]$$

Es: Circocentro



$$[OBC] = \frac{1}{2}R^2 \sin 2\alpha = $$
$$= R^2 \sin\alpha\cos\alpha$$
$$[ABC] = \frac{1}{2} bc \sin\alpha$$
$$\frac{[OBC]}{[ABC]} = \frac{2R^2\cos\alpha}{bc} = $$
$$= \frac{\cos\alpha}{2\sin\beta\sin\gamma}$$

$$\vec{O} = \frac{\cos\alpha}{2\sin\beta\sin\gamma}\vec{A} + \frac{\cos\beta}{2\sin\alpha\sin\gamma}\vec{B} + \frac{\cos\gamma}{2\sin\alpha\sin\beta}\vec{C}$$

3) Numeri Complessi

Es: Quanto vale il prodotto di lati e diagonali di un poligono regolare di n lati inscritto nella cf. unitaria?

$A_1, \dots, A_n$ punti su Γ = cf. di raggio 1,

$$\sqrt{\prod_{i\neq j} A_iA_j} = ??$$

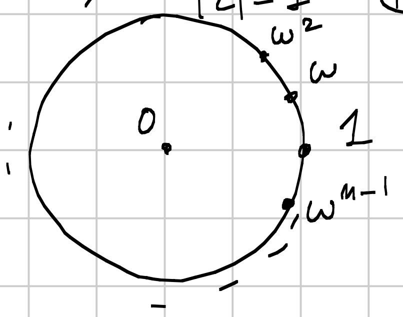
Es: $z_1, z_2, z_3 \in \mathbb{C}$ t.c. $|z_1| = |z_2| = |z_3| = R$

e $z_2 \neq z_3$

dim che $\min_{a\in\mathbb{R}} |az_2 + (1-a)z_3 - z_1| = \frac{1}{2R}|z_1 - z_2|\cdot|z_1 - z_3|$

1) $\prod_{i\neq j} A_iA_j = \left(\prod_{i\neq 1} A_iA_1\right)^n \qquad \prod_{i\neq 1} A_iA_1 = ?$

$|z|=1 \quad \mathbb{C}$



$z^n - 1$

$$\prod_{j=1}^{n-1} |\omega^j - 1| =$$

$$= \left| \prod_{j=1}^{n-1} (\omega^j - 1) \right| = |p(1)|$$

$$\prod_{j=1}^{n-1} (\omega^j - z) = p(z) \qquad \deg p(z) = n-1$$

$$p(\omega^j) = 0 \qquad j = 1, \dots, n-1$$

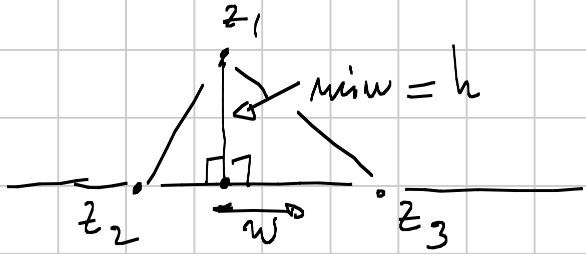
$$\Rightarrow p(z) \mid z^n - 1 \qquad p(z) = \frac{z^n-1}{z-1} = 1+z+\dots+z^{n-1}$$

$p(1) = n \Rightarrow$ il prodotto di tutti i lati e le diag fa $n^{\frac{n}{2}}$

2) $$\min_{a\in\mathbb{R}} |az_2 + (1-a)z_3 - z_1| = \frac{1}{2R}|z_1-z_2|\cdot|z_1-z_3|$$

$|w - z_1|$ = dist fra w e z_1

$w = az_2 + (1-a)z_3 \Rightarrow$ punto su z_2, z_3



$$h = \frac{2\,\text{Area}}{|z_2-z_3|} = 2\,\frac{\frac{abc}{4R}}{|z_2-z_3|} =$$

$$= \frac{1}{2R}\,\frac{|z_1-z_2|\,\cancel{|z_2-z_3|}\,|z_3-z_1|}{\cancel{|z_2-z_3|}} \quad \underline{\text{ok}}.$$

Disug. di Tolomeo

$a, b, c, d \in \mathbb{C}$

$$(a-b)(c-d)+(c-b)(d-a)=$$
$$=\cancel{ac}-ad-bc+\cancel{bd}+cd-\cancel{ac}-\cancel{bd}+ab=$$
$$=cd+ab-ad-bc=(c-a)(d-b)$$

Passando ai moduli + disug. triang.

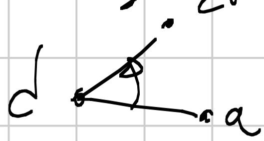
$$|a-b|\cdot|c-d|+|c-b|\cdot|d-a| \geq |c-a||d-b|$$
$$(AB\cdot CD+CB\cdot DA \geq CA\cdot DB)$$

Vale l'uguaglianza se e solo se

$$\arg\left(\frac{(a-b)(c-d)}{(c-b)(d-a)}\right)=0, \pi$$
$$\arg\left(\frac{a-b}{c-b}\right)+\arg\left(\frac{c-d}{a-d}\right)=0, \pi$$

se $=\pi$ ok. (ciclicità)

$=0$ non può essere se gli arg. sono $\neq 0$.



$$|z|+|w|=|z+w|$$
sse
$$z/w \in \mathbb{R}$$
$$\Updownarrow$$
$$\arg\left(\frac{z}{w}\right)=0, \pi$$
$$\Updownarrow$$
$$z\bar{w} \in \mathbb{R}$$
$$(a+ib)(c-id)=$$
$$=ac+bd+i(bc-ad) \in \mathbb{R}$$
$$bc=ad \qquad \frac{a}{c}=\frac{b}{d}$$

Geom. analitica con i complessi

Quando $a, b, z \in \mathbb{C}$ sono allineati?

$$\frac{b-a}{z-a} \in \mathbb{R} \qquad \frac{z-\bar{z}}{2i}=\mathrm{Im}(z)$$

$\mathrm{Im}(z)=0 \Leftrightarrow z=\bar{z}$ $\qquad \frac{z+\bar{z}}{2}=\mathrm{Re}(z)$

a, b, z allineati sse $\qquad \frac{\bar{z}-\bar{a}}{\bar{b}-\bar{a}}=\frac{z-a}{b-a}$

$(\bar{z}-\bar{a})(b-a)=(\bar{b}-\bar{a})(z-a)$ eq. della retta per a, b.

<u>Oss</u>: Se $|a|=|b|=1$ $\qquad \bar{a}=\frac{1}{a}$

$$\left(\bar{z}-\frac{1}{a}\right)\cancel{(b-a)}=\left(\frac{\overset{-1}{\cancel{a-b}}}{ba}\right)(z-a)$$

$$ab\bar{z}-b=a-z$$

$$\boxed{ab\bar{z}+z=a+b}$$ corde nelle cfr. unitaria (vertici a, b)

$$\boxed{w^2\bar{z}+z=2w}$$ tang. nel punto w alla cfr. unit.

<u>Eq delle cfr</u>: centro a, raggio R

$$|z-a|=R$$

$$(z-a)(\bar{z}-\bar{a})=R^2$$

<u>Asse radicale</u>: $\begin{cases}(z-b)(\bar{z}-\bar{b})=r^2\\(z-a)(\bar{z}-\bar{a})=R^2\end{cases}$

$$(z-b)(\bar{z}-\bar{b})-(z-a)(\bar{z}-\bar{a})=r^2-R^2$$

$$\cancel{z\bar{z}}-b\bar{z}-\bar{b}z+b\bar{b}-\cancel{\bar{z}z}+a\bar{z}+\bar{a}z-a\bar{a}=r^2-R^2$$

$$\bar{z}(a-b)+z(\bar{a}-\bar{b})+b\bar{b}-a\bar{a}-r^2+R^2=0$$

$$(z-z_0)(\bar{z}_1-\bar{z}_0) = (\bar{z}-\bar{z}_0)(z_1-z_0)$$

$$\boxed{z(\bar{z}_1-\bar{z}_0) - \bar{z}(z_1-z_0) - z_0\bar{z}_1 + \bar{z}_0 z_1 = 0}$$

$pz+q\bar{z}+r=0$ è una retta solo se $p=-\bar{q}$

•) $z+2\bar{z}=0$

$z=-2\bar{z} \Rightarrow |z| = 2|\bar{z}| = 2|z|$

$\Rightarrow |z|=0 \Rightarrow z=0.$

•) $z+2\bar{z}=1$ $\qquad x+iy+2x-2iy=1$

$3x=1$

$-y=0$

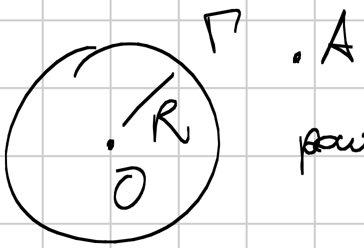
$$\begin{cases} Re = Re \\ Im = Im \end{cases}$$

•) $iz-i\bar{z}+1=0$ $\qquad i(x+iy)-i(x-iy)=-1$

$$\begin{cases} ix-ix=0 \ ✓ \\ -y-y=-1 \end{cases} \qquad \begin{cases} 0=0 \\ -2y=-1 \end{cases}$$

$\Rightarrow y=\frac{1}{2}$

Potenze:

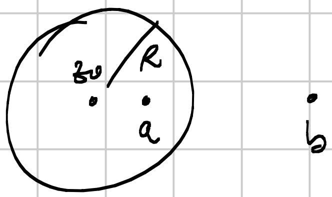


$pow_\Gamma(A) = OA^2 - R^2$

$pow_\Gamma(z) = |z-z_0|^2 - R^2 =$

Γ centro z_0 raggio R $\qquad = (z-z_0)(\bar{z}-\bar{z}_0) - R^2$

Inversione:



$|a-z_0|\cdot|b-z_0| = R^2$

a, b, z_0 allineati con z_0 non tra a e b

$$\arg\left(\frac{a-z_0}{b-z_0}\right)=0$$

$$\arg(a - z_0) = \arg(b - z_0) \qquad \arg(\overline{b} - \overline{z_0}) = -\arg(b - z_0) = \arg\left(\frac{1}{b - z_0}\right)$$

$$a - z_0 = \frac{R^2}{\overline{b} - \overline{z_0}} \qquad \arg(a - z_0) = \arg\left(\frac{1}{\overline{b} - \overline{z_0}}\right)$$

$$a = \frac{R^2}{\overline{b} - \overline{z_0}} + z_0$$

se $z_0 = 0$, $R = 1$

$$z \to \frac{1}{\overline{z}}$$

<u>Retta determinata da punto + direzione</u>

$z_0 \in \mathbb{C}$ $\quad |\alpha| = 1$

$$\frac{z - z_0}{\alpha} = \frac{\overline{z} - \overline{z_0}}{\overline{\alpha}}$$

$$z - z_0 = \frac{\alpha}{\overline{\alpha}}(\overline{z} - \overline{z_0})$$

$$z = \alpha^2(\overline{z} - \overline{z_0}) + z_0$$

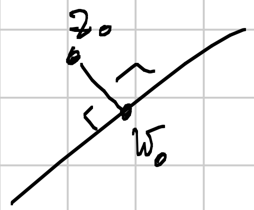
$$z - z_0 = \frac{\alpha}{\overline{\alpha}}(\overline{z} - \overline{z_0})$$

$$z - z_1 = \frac{\beta}{\overline{\beta}}(\overline{z} - \overline{z_1})$$

$\perp$ perpend. $\begin{cases} \beta = \alpha i \\ \beta = -\alpha i \end{cases}$

$//$ parallele $\{ \beta = \pm\alpha$

<u>Es</u>: Determinare la proiez. di un punto su una retta.



<u>Quando due triangoli sono simili?</u>

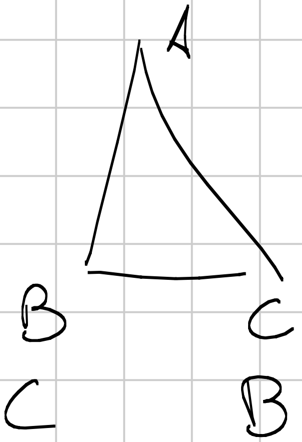
abc, def sono tri simili?

$$\frac{b - a}{c - a} = \frac{e - d}{f - d}$$

$\to$ 1- stesso argomento $\Rightarrow \sphericalangle BAC = \sphericalangle EDF$

$\to$ 2- stesso modulo $\dfrac{BA}{AC} = \dfrac{ED}{FD}$

$1+2 \Rightarrow$ simile.



$$\text{Area}(z_1 z_2 z_3) = \frac{1}{2}\,\mathrm{Im}\left(\bar{z}_1 z_2 + \bar{z}_2 z_3 + \bar{z}_3 z_1\right) =$$
orientata
$$= \frac{i}{4}\det\begin{pmatrix} z_1 & \bar{z}_1 & 1 \\ z_2 & \bar{z}_2 & 1 \\ z_3 & \bar{z}_3 & 1 \end{pmatrix}$$

Teo (Roth) $A_1A_2A_3$ M_i: $M_i \in A_jA_k$ $\quad \dfrac{A_jM_i}{M_iA_k} = \lambda_i$

$$\Rightarrow \quad \frac{[M_1M_2M_3]}{[A_1A_2A_3]} = \frac{1+\lambda_1\lambda_2\lambda_3}{(1+\lambda_1)(1+\lambda_2)(1+\lambda_3)}$$

$$m_1 = \frac{a_2 + \lambda_1 a_3}{1+\lambda_1} \qquad m_2 = -- \qquad m_3 = --$$

Triangolo di vertici $z_1, z_2\, z_3$

$\Downarrow$

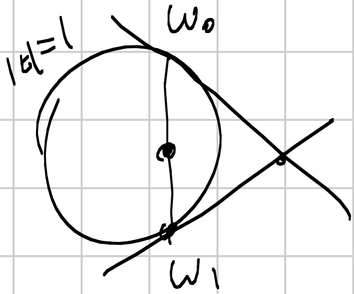
$$\text{circocentro} = \frac{z_1\bar{z}_1(z_2-z_3) + z_2\bar{z}_2(z_3-z_1) + z_3\bar{z}_3(z_1-z_2)}{(\bar{z}_2 z_3 - \bar{z}_3 z_2 + z_1\bar{z}_3 - \bar{z}_1 z_3 + \bar{z}_1 z_2 - z_1\bar{z}_2)}$$

$$= \frac{\det\begin{pmatrix} 1 & 1 & 1 \\ z_1 & z_2 & z_3 \\ |z_1|^2 & |z_2|^2 & |z_3|^2 \end{pmatrix}}{\det\begin{pmatrix} 1 & 1 & 1 \\ \bar{z}_1 & \bar{z}_2 & \bar{z}_3 \\ z_1 & z_2 & z_3 \end{pmatrix}}$$

$$\det\begin{pmatrix} 1 & 1 & 1 \\ a & b & c \\ d & e & f \end{pmatrix} =$$
$$= (bf - ce + dc - af + ae - bd) =$$

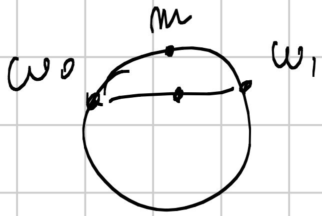
$$= -\det\begin{pmatrix} a-b & c-b \\ d-e & f-e \end{pmatrix}$$

Es :



$$\frac{w_0+w_1}{2} \to \frac{2}{\overline{w_0}+\overline{w_1}} =$$

$$\overline{w_0} = \frac{1}{w_0} \qquad = \frac{2 w_0 w_1}{w_0+w_1}$$



$$m = \frac{w_0+w_1}{2} \cdot \left| \frac{2}{w_0+w_1} \right| =$$

$$= \frac{w_0+w_1}{1\cancel{2}} \cdot \sqrt{\frac{\cancel{2}}{w_0+w_1} \overset{1}{\cdot} \frac{\cancel{2}}{\overline{w_0}+\overline{w_1}}} =$$

$$= \frac{w_0+w_1}{|w_0+w_1|}$$