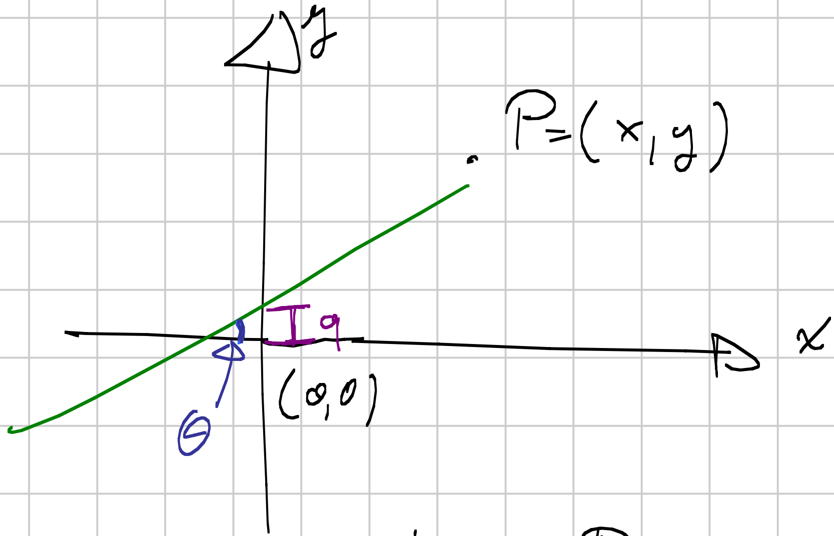
G2 - Metodi algebrici - Basic

Titolo nota 08/09/2010

1 - Coordinate cartesiane

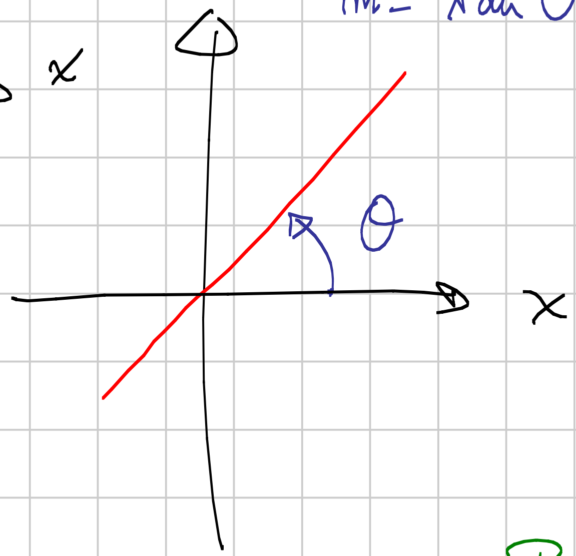
2 - Vettori

3 - Complessi



$$y = mx + q$$

$$m = \tan\theta$$

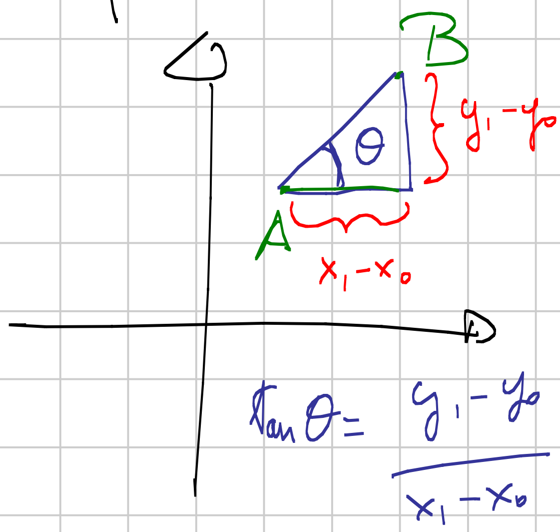


$a, b, c \in \mathbb{R}$

$$\boxed{ax + by + c = 0}$$

$A = (x_0, y_0) \qquad B = (x_1, y_1)$

$$y = \frac{y_0 - y_1}{x_0 - x_1}(x - x_1) + y_1$$



$$\tan\theta = \frac{y_1 - y_0}{x_1 - x_0}$$

Oss: $p(x,y)=0 \qquad \deg p = 3$

$$p(x,y) = (x^2+1)\,y$$

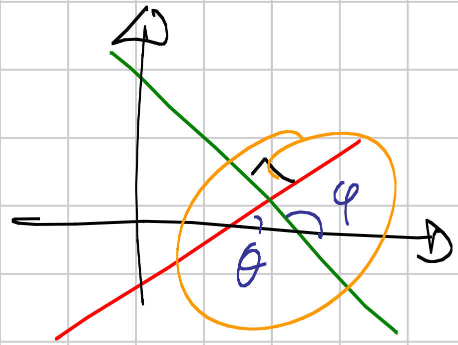
$$p(x,y)=0 \iff (x^2+1)y=0 \iff y=0$$

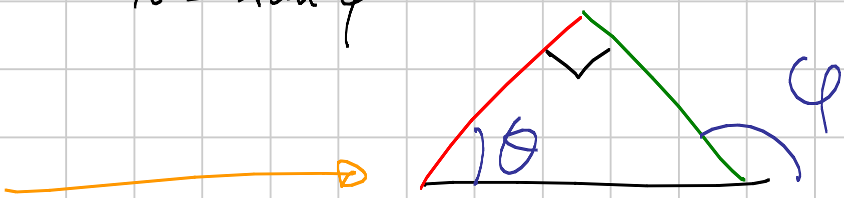
$$p(x,y) = y^2+2xy+x^2 = (x+y)^2$$

$y = mx + q \qquad\qquad mn = -1 \iff$ perpendicolari

$y = nx + r \qquad\qquad m = \tan\theta$

$n = \tan\varphi$



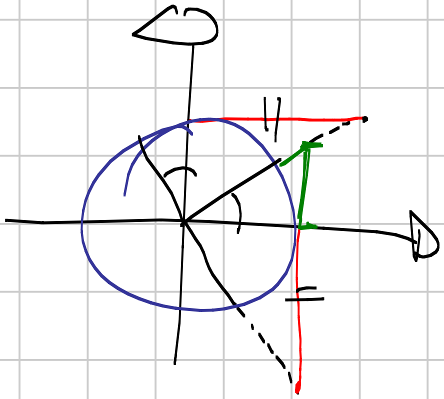


$$\varphi = \frac{\pi}{2}+\theta$$

$$\tan\varphi = -\cot\theta = -\frac{1}{\mathrm{Tg}\,\theta}$$

$$\underset{n}{\tan\varphi}\cdot\underset{m}{\tan\theta} = -1$$

$$n\cdot m = -1$$



parallele $\iff m=n$

C'è una distanza

$$d(A,B) = \sqrt{(x_0 - x_1)^2 + (y_0 - y_1)^2}$$

- bisettrice: bisett. dell'angolo tra r, s

$$\mathcal{L} = \{ P : d(P,r) = d(P,s) \}$$

- circonferenza: $\mathcal{C} = \{ P : d(P,O) = r \}$

$$(x - x_0)^2 + (y - y_0)^2 = r^2$$

- asse di un segmento

$$\mathcal{L} = \{ P : d(P,A) = d(P,B) \}$$

$$\sqrt{(x-x_0)^2 + (y-y_0)^2} = \sqrt{(x-x_1)^2 + (y-y_1)^2}$$

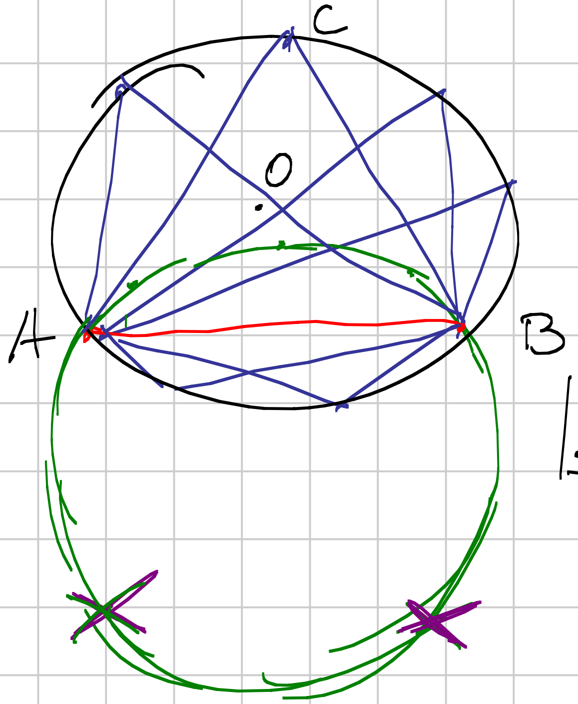
$$(x-x_0)^2 + (y-y_0)^2 = (x-x_1)^2 + (y-y_1)^2$$

$$\cancel{x^2} + x_0^2 - 2xx_0 + \cancel{y^2} + y_0^2 - 2yy_0 = \cancel{x^2} + x_1^2 - 2xx_1 + \cancel{y^2} + y_1^2 - 2yy_1$$

$$\boxed{2x(x_1 - x_0) + 2y(y_1 - y_0) + x_0^2 - x_1^2 + y_0^2 - y_1^2 = 0}$$

pt. medio di $AB = \left(\frac{x_0 + x_1}{2}, \frac{y_0 + y_1}{2} \right)$

Es:

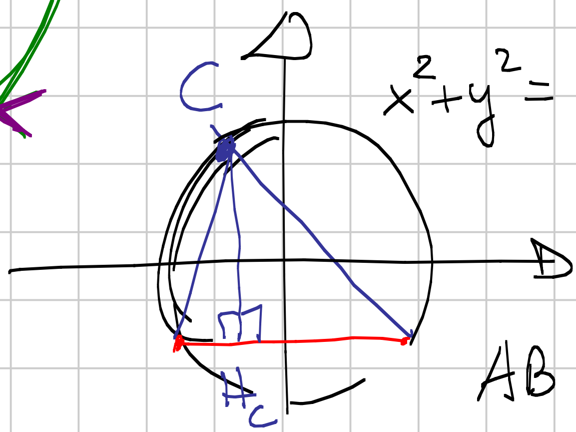


1- Origine in O

2- Origine in A o B

3 - Origine nel pt. medio di AB

$\boxed{\text{Scegliamo } 1}$



$x^2+y^2=1$

$AB \subset \{y=k\}$

$-1<k\leq 0$

Calcolo A, B

$$\begin{cases} y=k \\ x^2+y^2=1 \end{cases} \qquad x^2+k^2=1$$

$$x^2=1-k^2 \longrightarrow x=\pm\sqrt{1-k^2}$$

$A(\sqrt{1-k^2}, k) \qquad B(-\sqrt{1-k^2}, k) \qquad C=(x_0, y_0)$

Dobbiamo trovare H

Altezza da C: $x=x_0$

Lato AC: $y=\underbrace{\dfrac{k-y_0}{\sqrt{1-k^2}-x_0}}_{=m}(x-x_0)+y_0$

$$-\frac{1}{m}=-\frac{\sqrt{1-k^2}-x_0}{k-y_0}=$$

$$=\frac{\sqrt{1-k^2}-x_0}{y_0-k}$$

Altezza da B: $y=\dfrac{\sqrt{1-k^2}-x_0}{y_0-k}(x+\sqrt{1-k^2})+k$

$H=$ alt. da $C \cap$ alt da B

$$H = \begin{cases} x = x_0 \\ y = \dfrac{\sqrt{1-k^2} - x_0}{y_0 - k}\left(x + \sqrt{1-k^2}\right) + k \end{cases}$$

$$y = \frac{\sqrt{1-k^2} - x_0}{y_0 - k}\left(x_0 + \sqrt{1-k^2}\right) + k =$$

$$= \frac{1 - k^2 - x_0^2 + k y_0 - k^2}{y_0 - k}$$

$$x = x_0 \qquad H = \left(x_0, \frac{1 - 2k^2 - x_0^2 + k y_0}{y_0 - k}\right)$$

$$C \in \Gamma \Longrightarrow x_0^2 + y_0^2 = 1 \qquad x_0^2 = 1 - y_0^2$$

$$H = \left(x_0, \frac{\cancel{1} - 2k^2 - \cancel{1} + y_0^2 + k y_0}{y_0 - k}\right) =$$

$$= \left(x_0, \frac{y_0^2 - k^2 - k^2 + k y_0}{y_0 - k}\right) =$$

$$y_0 + k + k$$

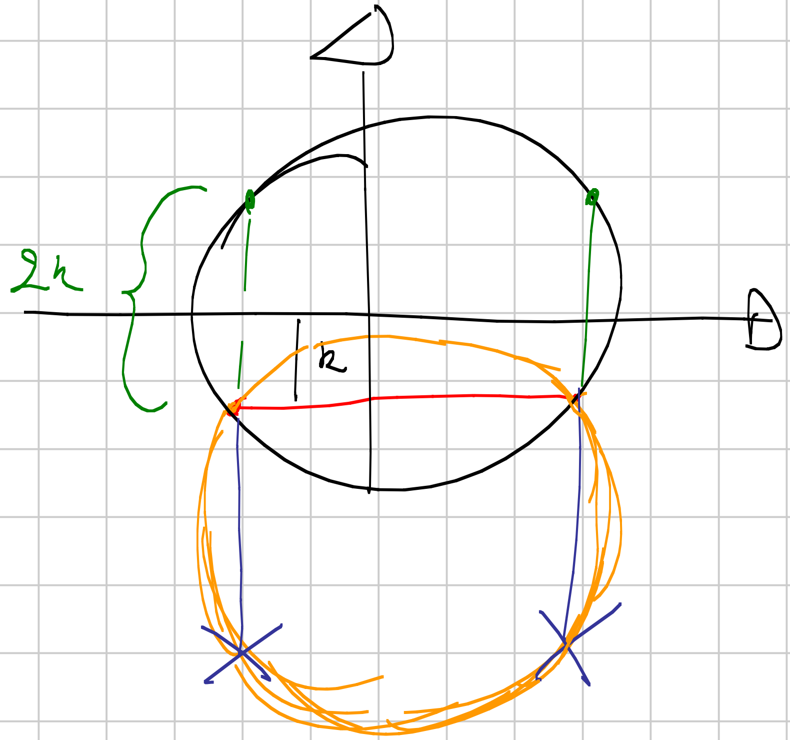
$$= \left(x_0, y_0 + k + k\right) = \left(x_0, y_0 + 2k\right)$$

$$x_0 + \sqrt{1-k^2} - \sqrt{1-k^2}$$

$$C \in \Gamma \dashrightarrow H = (x_0, y_0 + 2k)$$

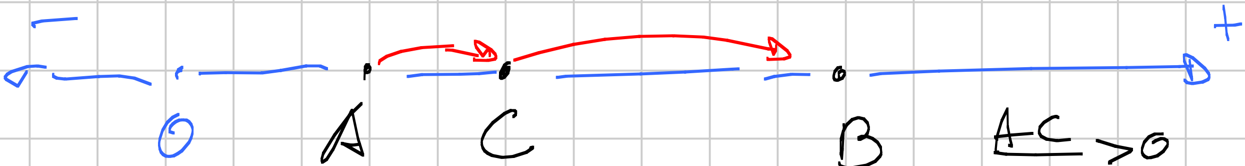
$$\| \; (x_0, y_0)$$

$$f(x,y) = (x, y + 2k)$$

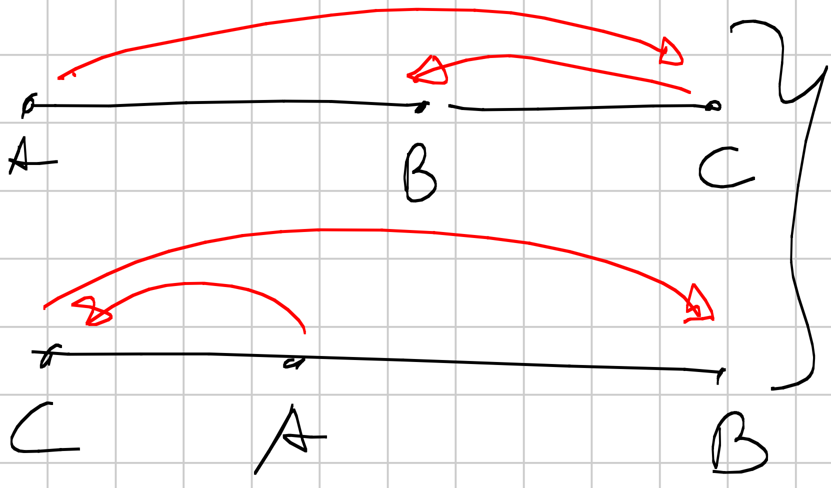


Rapporti: A, B, C allineati

$$\frac{AC}{CB}$$



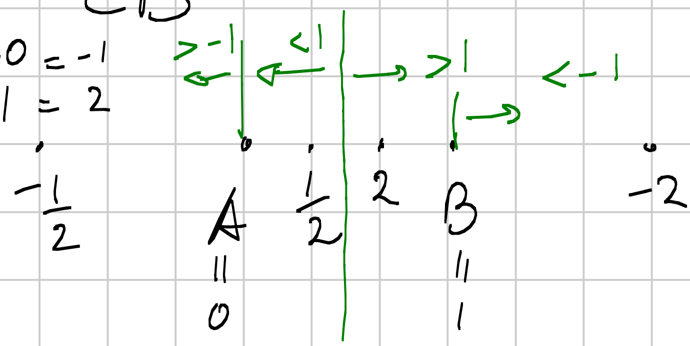
$$\frac{AC}{CB} > 0$$



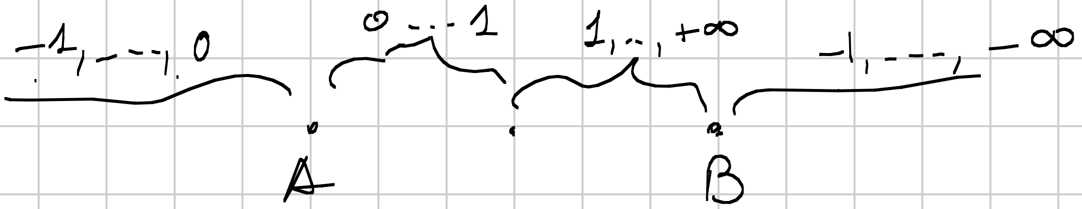
$$\frac{AC}{CB} < 0$$

Es: $\frac{AC}{CB} = 2, \frac{1}{2}, -2, -\frac{1}{2}$

$AC = -1 - 0 = -1$
$CB = 1 + 1 = 2$

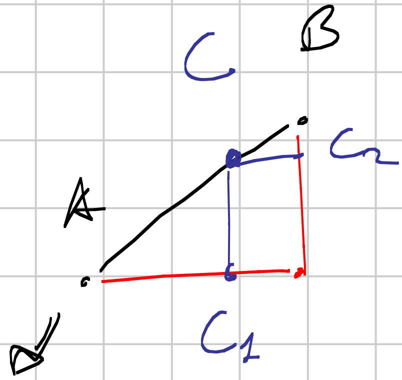


$AC = 2 - 0$
$CB = 1 - 2$
$= -2$



Es: $A(x_0, y_0)$ $B(x_1, y_1)$

Voglio C t.c. $\frac{AC}{CB} = \lambda$



mi riduco a 1 coordinata

A B C

x_0, x_1 x t.c.

$$\frac{AC}{CB} = \frac{x - x_0}{x_1 - x} = \lambda$$

$$x - x_0 = \lambda x_1 - \lambda x$$

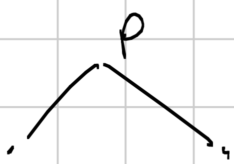
$$x = \frac{\lambda x_1 + x_0}{1 + \lambda} = \frac{1}{1+\lambda} x_0 + \frac{\lambda}{1+\lambda} x_1$$

$$\frac{1}{1+\lambda} + \frac{\lambda}{1+\lambda} = 1$$

$$C = \left(\frac{x_0}{1+\lambda} + \frac{\lambda x_1}{1+\lambda}, \frac{y_0}{1+\lambda} + \frac{\lambda y_1}{1+\lambda} \right)$$

Es: $\mathcal{L} = \{ d(P,A) = k \cdot d(P,B) \}$ $k \in \mathbb{R}^+ - \{0, 1\}$

$k > 0$
$k \neq 1$.



$PA = k \cdot PB$

$A = (x_0, y_0)$ $B = (x_1, y_1)$

Sol: $\sqrt{(x-x_0)^2 + (y-y_0)^2} = k \cdot \sqrt{(x-x_1)^2 + (y-y_1)^2}$

$$(x-x_0)^2 + (y-y_0)^2 = k^2 \left[(x-x_1)^2 + (y-y_1)^2 \right]$$

$$x^2(1-k^2) + y^2(1-k^2) - 2x(x_0 - k^2 x_1) - 2y(y_0 - k^2 y_1) +$$
$$+ x_0^2 + y_0^2 - k^2 x_1^2 - k^2 y_1^2 = 0$$

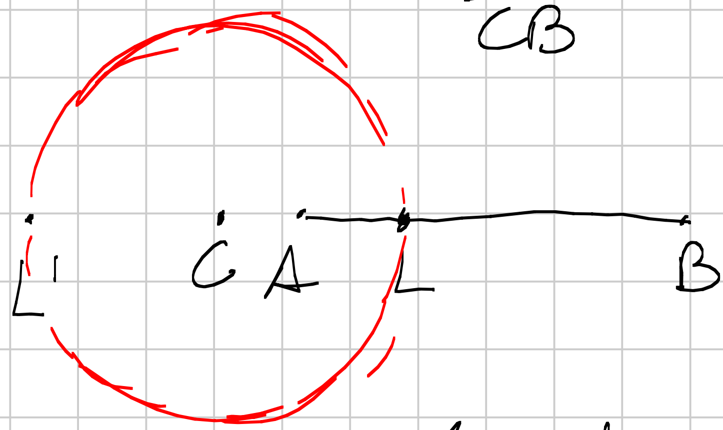
$$x^2+y^2-2x\boxed{\frac{x_0-k^2x_1}{1-k^2}}-2y\boxed{\frac{y_0-k^2y_1}{1-k^2}}+\frac{x_0^2-k^2x_1^2}{1-k^2}+\frac{y_0^2-k^2y_1^2}{1-k^2}=0$$

$$\frac{x_0}{1+(-k^2)}+\frac{(-k^2)\,x_1}{1+(-k^2)}=\frac{x_0}{1+\lambda}+\frac{\lambda x_1}{1+\lambda}$$

$\Rightarrow$ il centro divide AB in rapporto $-k^2$

(centro $= C$)

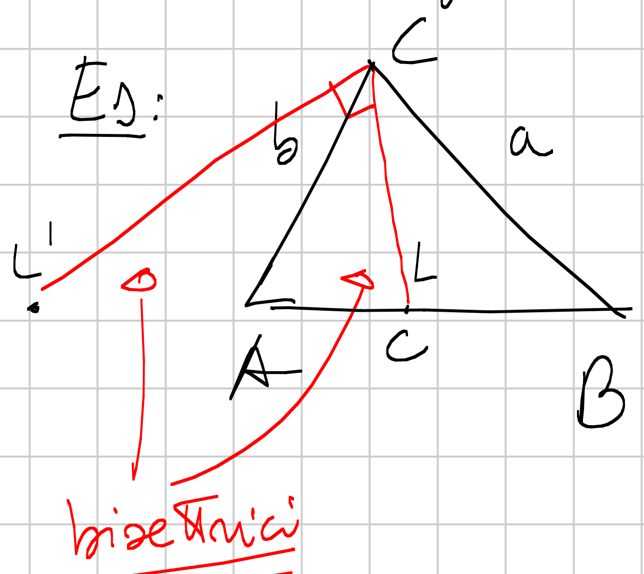
$$\frac{AC}{CB}=-k^2$$



$$\frac{AL}{LB}=k \qquad \frac{AL'}{L'B}=-k \quad \Rightarrow L, L' \in \mathcal{L}$$

$\mathcal{L}$ si chiama $\boxed{\text{cfr. di Apollonio}}$ del seg. AB con parametro k.

Es:



$$\mathcal{L}=\left\{P:\ PA=\frac{b}{a}\cdot PB\right\}$$

1) $C\in\mathcal{L}$

2) $L, L'\in\mathcal{L}$ $\qquad \frac{AL}{LB}=\frac{b}{a} \qquad \frac{AL'}{L'B}=-\frac{b}{a}$

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ooo

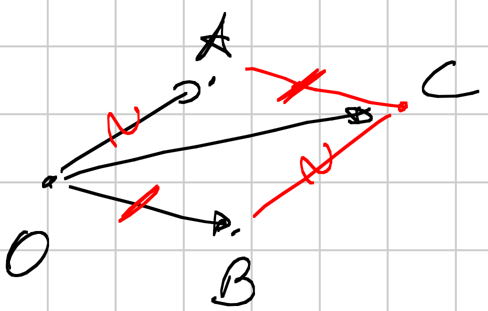
Un vettore è una cosa che si somma e si sottrae, si allunga e si accorcia.

Un vettore è una freccia: un punto di applicazione (origine), una lunghezza, una direzione, un verso.

$\overrightarrow{AB}$

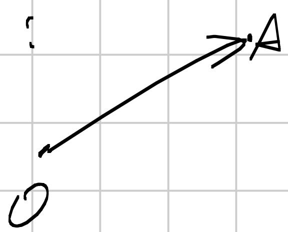
$\vec{A} = \overrightarrow{OA}$ se ho fissato prima O come origine.

Somma:



$\vec{A} + \vec{B} = \vec{C}$

Allungamento/Accorciamento:



$k \in \mathbb{R}$

$k \cdot \vec{A} = \vec{B}$

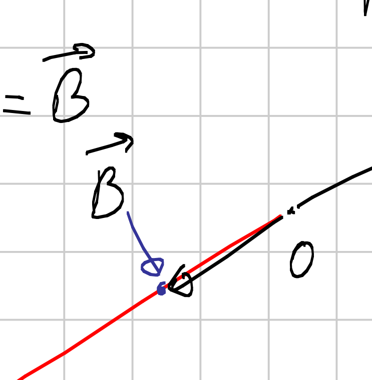
t.c. i) $\overline{OB} = |k| \cdot \overline{OA}$

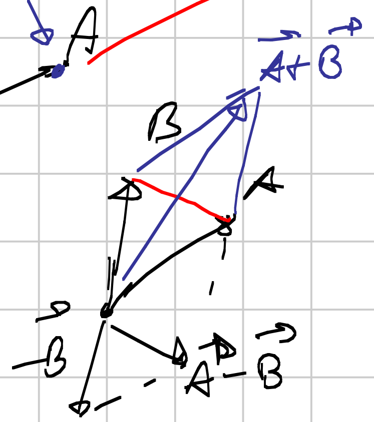
ii) O, A, B sono allineati

iii) se $k > 0$ $\vec{B}$ ha lo stesso verso di $\vec{A}$

se $k < 0$ $\vec{B}$ ha verso opposto ad $\vec{A}$

Es: $-\vec{A} = (-1) \cdot \vec{A} = \vec{B}$





Es: $\vec{A} - \vec{B} = \vec{A} + (-1) \cdot \vec{B}$

Notazione: $\|\vec{A}\| = OA$ $\qquad \|\cdot\| =$ norma

$|\vec{A}| = OA$ $\qquad$ la lunghezza del vettore

Es: $\|k\cdot\vec{A}\| = |k|\cdot\|\vec{A}\|$

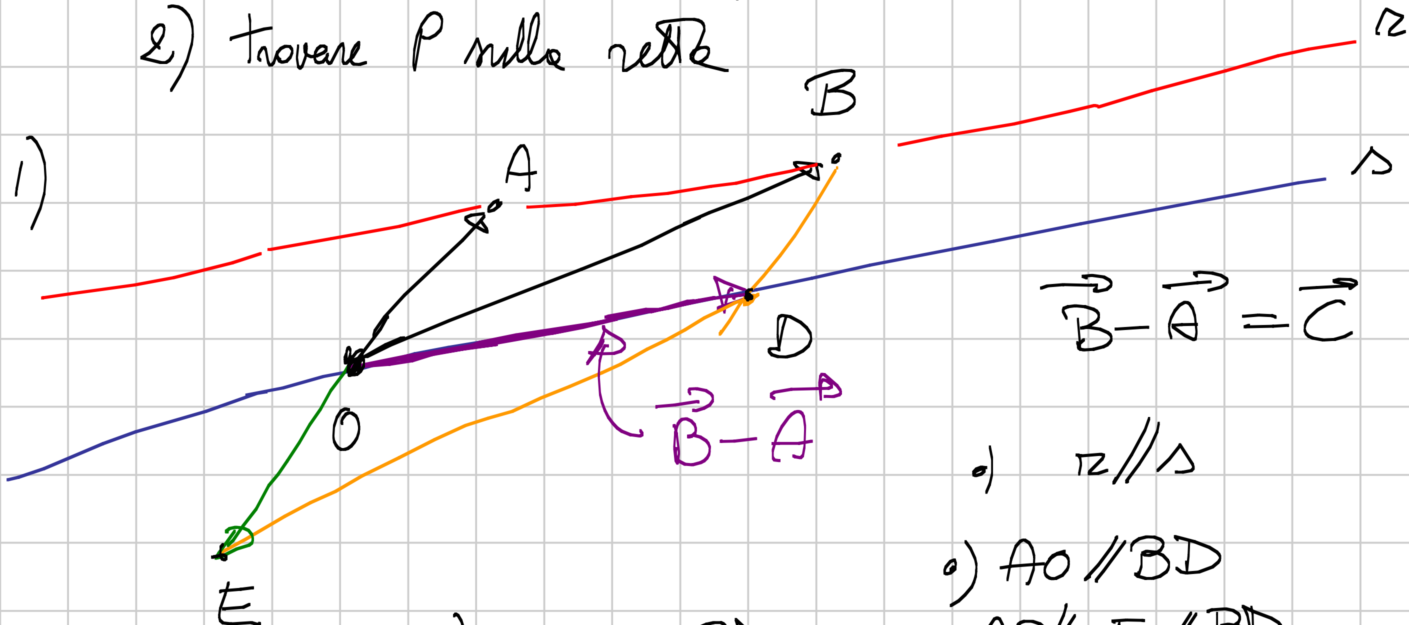
Es: $\mathcal{L} = \{ k\cdot\vec{A} \,;\, k\in\mathbb{R} \} =$ retta per A e O $=$

$=$ direzione di $\vec{A}$

Es: P t.c. $\dfrac{AP}{PB} = \lambda$

1) descrivere la retta per A, B
2) trovare P sulla retta

1)



$\vec{B} - \vec{A} = \vec{C}$

•) $r \,//\, s$

•) $AO \,//\, BD$

$AO \,//\, OE \,//\, BD \Leftarrow OEDB$ parall.

•) $AO = OE = BD$

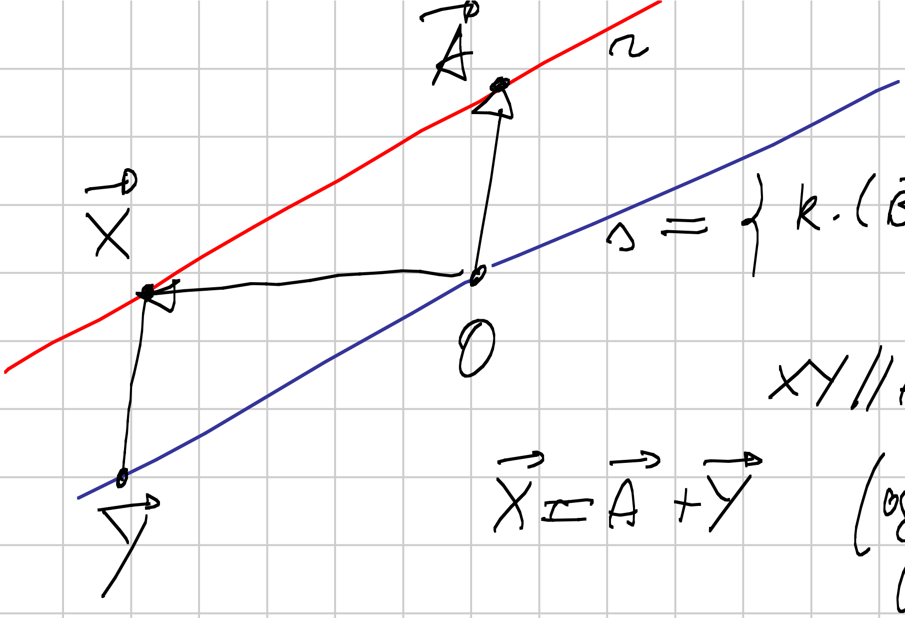
$\Downarrow$

$AODB$ parall.

$\Rightarrow D \in s \Rightarrow s$ è la direzione di $\vec{B} - \vec{A}$

$(D = C)$

$s = \{ k\cdot(\vec{B} - \vec{A}) \mid k\in\mathbb{R} \}$



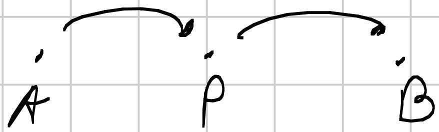
$$s = \{ k\cdot(\vec B - \vec A) \mid k \in \mathbb{R} \}$$

$XY // AO$

$\vec X = \vec A + \vec Y$ (ogni punto di r è la somma di $\vec A$ e di un ~~vettore~~ su s)

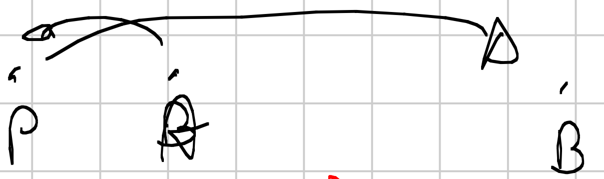
$$r = \{ k\cdot(\vec B - \vec A) + \vec A \mid k \in \mathbb{R} \}$$

2) $\dfrac{AP}{PB} = \lambda$ $\qquad \lambda > 0$



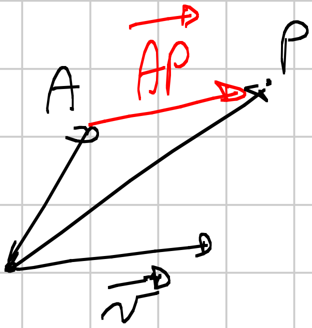
$\Downarrow$

$$\vec{AP} = \lambda \cdot \vec{PB}$$



$$\vec P - \vec A = \vec v$$

$\vec P = \vec{OP}$, $\vec A = \vec{OA}$



$\|\vec v\| = \|\vec{AP}\|$

$\vec v // \vec{AP}$

hanno lo stesso verso.

$$\vec{AP} = \lambda \cdot \vec{PB} \qquad \vec P - \vec A = \lambda(\vec B - \vec P)$$

$$\vec P = k\cdot(\vec B - \vec A) + \vec A$$

$$k\cdot(\vec B - \vec A) + \cancel{\vec A} - \cancel{\vec A} = \lambda \vec B - \lambda k(\vec B - \vec A) - \lambda \vec A$$

$$k(\vec{B}-\vec{A}+\lambda\vec{B}-\lambda\vec{A})=\lambda\vec{B}-\lambda\vec{A}$$

$$k(\lambda+1)(\vec{B}-\vec{A})=\lambda(\vec{B}-\vec{A})$$

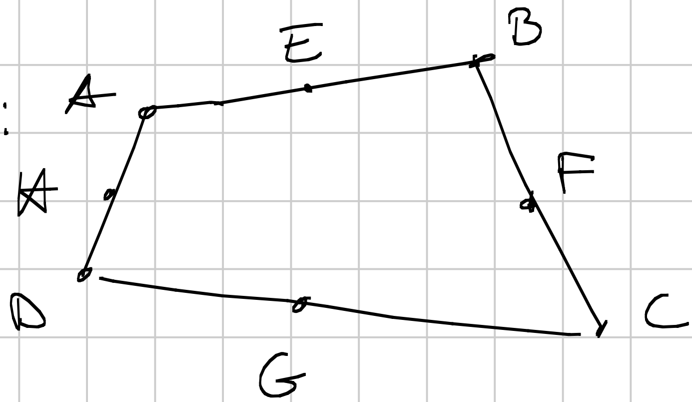
$$k(\lambda+1)=\lambda \qquad k=\frac{\lambda}{\lambda+1}$$

$$\vec{P}=\frac{\lambda}{\lambda+1}(\vec{B}-\vec{A})+\vec{A}=\frac{\lambda}{\lambda+1}\vec{B}+\frac{1}{\lambda+1}\vec{A}$$

<u>Es</u>: $\vec{A},\vec{B}$ $\quad M$ pt. medio $\quad \frac{AM}{MB}=1$

$$\vec{M}=\frac{\vec{A}+\vec{B}}{2}$$

<u>Es</u>:



$$\vec{E}=\frac{\vec{A}+\vec{B}}{2}$$
$$\vec{F}=\frac{\vec{B}+\vec{C}}{2}$$
$$\vec{G}=\frac{\vec{C}+\vec{D}}{2}$$
$$\vec{H}=\frac{\vec{D}+\vec{A}}{2}$$

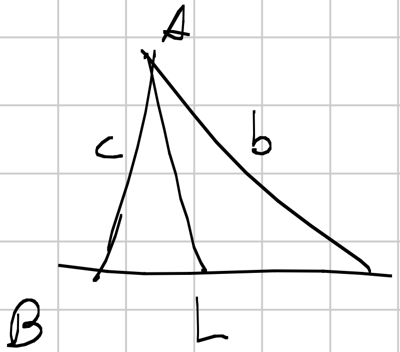
pt. medio di $EG=\frac{\vec{A}+\vec{B}+\vec{C}+\vec{D}}{4}$

pt. medio di $FH=\frac{\vec{A}+\vec{B}+\vec{C}+\vec{D}}{4}$

pt. medio di $AC=\frac{\vec{A}+\vec{C}}{2} = M$ $\qquad$ pt. medio di $BD=\frac{\vec{B}+\vec{D}}{2} = N$

pt. medio di $MN=\frac{\vec{A}+\vec{B}+\vec{C}+\vec{D}}{4}$

Es: $\overset{\Delta}{ABC}$ Triangolo AL bisettrice

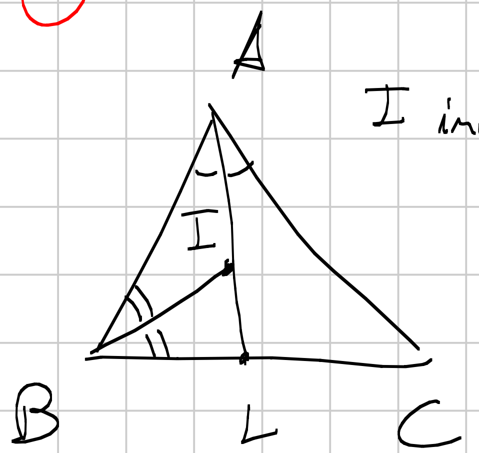


Teo bisett.: $\dfrac{BL}{LC} = \dfrac{AB}{AC} = \dfrac{c}{b}$

$$\vec{L} = \frac{\frac{c}{b}\cdot\vec{C} + \vec{B}}{1 + \frac{c}{b}} = \frac{c\cdot\vec{C} + b\cdot\vec{B}}{b+c}$$

$$\frac{BL}{LC} = k \implies \vec{L} = \frac{k\cdot\vec{C} + \vec{B}}{1+k}$$

Es: I incentro teo bisett $\Rightarrow \dfrac{AI}{IL} = \dfrac{AB}{BL} = \dfrac{c}{BL}$



$$\|\vec{L} - \vec{B}\| = \left\| \frac{c\vec{C} + b\vec{B}}{c+b} - \vec{B} \right\| =$$

$$= \left\| \frac{c\vec{C} - c\vec{B}}{c+b} \right\| =$$

$$= \frac{c}{c+b}\|\vec{C} - \vec{B}\| = \frac{ac}{c+b}$$

$$\frac{AI}{IL} = \frac{\cancel{c}}{\frac{a\cancel{c}}{c+b}} = \frac{c+b}{a}$$

$$\vec{I} = \frac{\frac{c+b}{a}\vec{L} + \vec{A}}{1 + \frac{c+b}{a}} = \frac{(c+b)\vec{L} + a\cdot\vec{A}}{a+b+c} =$$

$$= \frac{\frac{c\vec{C} + b\vec{B}}{c+b}\cdot(c+b) + a\vec{A}}{a+b+c} = \frac{a\vec{A} + b\vec{B} + c\vec{C}}{a+b+c}$$

Es: G = baricentro $\overset{\Delta}{ABC}$ Triangolo M pt medio di BC

$$\vec{M} = \frac{\vec{B} + \vec{C}}{2} \qquad \frac{AG}{GM} = 2$$

$$\vec{G} = \frac{2\vec{M} + \vec{A}}{3} = \frac{\cancel{2}\,\frac{\vec{B}+\vec{C}}{\cancel{2}} + \vec{A}}{3} = \frac{\vec{A}+\vec{B}+\vec{C}}{3}.$$

Es: centro della cf. di Apollonio nel Tr. $\overset{\triangle}{ABC}$ sul lato AB

N_1 t.c. $\frac{AN_1}{N_1B} = -k^2 = -\left(\frac{AC}{CB}\right)^2 = -\frac{b^2}{a^2}$

$$\vec{N}_1 = \frac{\vec{B}\left(-\frac{b^2}{a^2}\right) + \vec{A}}{1 - \frac{b^2}{a^2}} = \frac{a^2\vec{A} - b^2\vec{B}}{a^2 - b^2}$$

$$BC \ni \vec{N}_2 = \frac{b^2\vec{B} - c^2\vec{C}}{b^2 - c^2} \qquad \underset{AC}{\underset{\uparrow}{N_3}} = \frac{c^2\vec{C} - a^2\vec{A}}{c^2 - a^2}$$

$$\vec{N}_1 - \vec{N}_2 = \left(a^2b^2\vec{A} - a^2c^2\vec{A} - b^4\vec{B} + b^2c^2\vec{B} + a^2b^2\vec{B} - b^4\vec{B} - a^2c^2\vec{C} + b^2c^2\vec{C}\right) \cdot \frac{1}{(b^2-c^2)(a^2-b^2)} =$$

$$= \frac{1}{(\ )(\ )}\left(\vec{A}(a^2b^2 - a^2c^2) + \vec{B}(a^2b^2 + b^2c^2 - 2b^4) + \vec{C}(-a^2c^2 + b^2c^2)\right) =$$

$$= \frac{1}{(\ )(\ )}\left(a^2\vec{A}(b^2-c^2) + b^2\vec{B}(a^2+c^2-2b^2) + c^2\vec{C}(-a^2+b^2)\right)$$

Es: (noioso) trovare k t.c. $k(\vec{N}_1 - \vec{N}_2) + \vec{N}_1 = \vec{N}_3$.

Prodotto scalare: è un coso che mangia due vettori e sputa un numero. (Tra vett. con la stessa origine)

$$\langle \vec{A}, \vec{B} \rangle \qquad (\vec{A}, \vec{B}) \qquad \vec{A} \bullet \vec{B}$$

(i) $\langle \vec{A}, \vec{B} \rangle = \langle \vec{B}, \vec{A} \rangle$

(ii) $\langle \vec{A}+\vec{B}, \vec{C} \rangle = \langle \vec{A}, \vec{C} \rangle + \langle \vec{B}, \vec{C} \rangle$

(iii) $\langle \lambda \vec{A}, \vec{B} \rangle = \lambda \cdot \langle \vec{A}, \vec{B} \rangle$

(iv) $\langle \vec{A}, \vec{B} \rangle = 0 \iff OA \perp OB$

(v) $\langle \vec{A}, \vec{A} \rangle = \|\vec{A}\|^2$

$$\boxed{\langle \vec{A}, \vec{B} \rangle = OA \cdot OB \cdot \cos(A\hat{O}B)}$$

$O = (0,0)$

$\vec{A} = (x_0, y_0)$

$\vec{B} = (x_1, y_1)$

$\langle \vec{A}, \vec{B} \rangle = $
$= x_0 x_1 + y_0 y_1.$

Es: $\langle \vec{A}-\vec{B}, \vec{A}-\vec{B} \rangle = \langle \vec{A}, \vec{A}-\vec{B} \rangle - \langle \vec{B}, \vec{A}-\vec{B} \rangle =$

$\|\vec{A}-\vec{B}\|^2 \qquad = \langle \vec{A}, \vec{A} \rangle - \langle \vec{A}, \vec{B} \rangle - \langle \vec{B}, \vec{A} \rangle + \langle \vec{B}, \vec{B} \rangle =$

$\|$

$AB^2 \qquad = \|\vec{A}\|^2 + \|\vec{B}\|^2 - 2\langle \vec{A}, \vec{B} \rangle =$

$= OA^2 + OB^2 - 2\, OA \cdot OB \cdot \cos B\hat{O}A$

T. di Carnot

Es: $\langle \vec{A}-\vec{B}, \vec{A}+\vec{B} \rangle = \|\vec{A}\|^2 - \|\vec{B}\|^2$

Oss: $\langle \vec{A} \cdot \vec{B} \rangle^2 = OA^2 \cdot OB^2 \cdot \cos^2 B\hat{O}A \leq OA^2 \cdot OB^2 = \|\vec{A}\|^2 \bullet \|\vec{B}\|^2$

$$|\langle \vec{A}, \vec{B} \rangle| \leq \|\vec{A}\| \cdot \|\vec{B}\|$$

disug. di Cauchy-Schwarz.

$$|x_0x_1 + y_0y_1| \le \sqrt{(x_0^2+y_0^2)(x_1^2+y_1^2)}$$

Es: Se l'origine è il circocentro di $\overset{\triangle}{ABC}$ allora $\vec{H} = \vec{A}+\vec{B}+\vec{C}$ è l'ortocentro.

Dim:
$$HA \perp BC \iff \langle \vec{A}-\vec{H}, \vec{C}-\vec{B} \rangle = 0$$
$$HB \perp AC \iff \langle \vec{B}-\vec{H}, \vec{C}-\vec{A} \rangle = 0$$
$$HC \perp AB \iff \langle \vec{C}-\vec{H}, \vec{B}-\vec{A} \rangle = 0$$

$\|\vec{B}\| = OB = R$
$\|\vec{C}\| = OC = R$

$$\langle \vec{A}-\vec{H}, \vec{C}-\vec{B} \rangle = \langle -\vec{B}-\vec{C}, \vec{C}-\vec{B} \rangle =$$
$$= \langle \vec{B}+\vec{C}, \vec{B}-\vec{C} \rangle = \|\vec{B}\|^2 - \|\vec{C}\|^2 = R^2 - R^2 = 0$$

$$\langle \vec{B}-\vec{H}, \vec{A}-\vec{C} \rangle = \langle \vec{A}+\vec{C}, \vec{C}-\vec{A} \rangle = \|\vec{A}\|^2 - \|\vec{C}\|^2 = R^2 - R^2 = 0$$

idem per l'altra $\Rightarrow \vec{A}+\vec{B}+\vec{C}$ è l'ortocentro. $\square$

Es: O origine e circocentro $\Rightarrow \vec{H} = \vec{A}+\vec{B}+\vec{C}$

inoltre sempre $\vec{G} = \dfrac{\vec{A}+\vec{B}+\vec{C}}{3}$ $\qquad \vec{G} = \dfrac{1}{3}\cdot\vec{H}$

$\Rightarrow O, G, H$ allineati e $\dfrac{OG}{GH} = \dfrac{1}{2}$

retta di Eulero.

Es: $GH = ???$

$$\|\vec{H}-\vec{G}\|^2 = \langle \vec{H}-\vec{G}, \vec{H}-\vec{G} \rangle = \text{(origine nel circocentro)}$$

$$= \left\langle \vec{A}+\vec{B}+\vec{C} - \left(\frac{\vec{A}+\vec{B}+\vec{C}}{3}\right), \vec{A}+\vec{B}+\vec{C} - \left(\frac{\vec{A}+\vec{B}+\vec{C}}{3}\right) \right\rangle =$$

$$= \left\langle \frac{2}{3}(\vec{A}+\vec{B}+\vec{C}), \frac{2}{3}(\vec{A}+\vec{B}+\vec{C}) \right\rangle =$$

$$= \frac{4}{9}\left[\underbrace{\|\vec{A}\|^2+\|\vec{B}\|^2+\|\vec{C}\|^2}_{3R^2} + 2\langle\vec{A},\vec{B}\rangle + 2\langle\vec{B},\vec{C}\rangle + 2\langle\vec{C},\vec{A}\rangle\right]$$

$$c^2 = AB^2 = \langle\vec{A}-\vec{B},\vec{A}-\vec{B}\rangle = \underbrace{\|\vec{A}\|^2+\|\vec{B}\|^2}_{2R^2} - 2\langle\vec{A},\vec{B}\rangle$$

$$\boxed{2R^2 - c^2 = 2\langle\vec{A},\vec{B}\rangle}$$

$$= \frac{4}{9}\left[3R^2 + 2R^2 - c^2 + 2R^2 - a^2 + 2R^2 - b^2\right] =$$

$$= \frac{4}{9}\left[9R^2 - a^2 - b^2 - c^2\right] =$$

$$= 4R^2 - \frac{4}{9}(a^2+b^2+c^2) = GH^2$$

Oss: $OH^2 = 9R^2 - (a^2+b^2+c^2)$

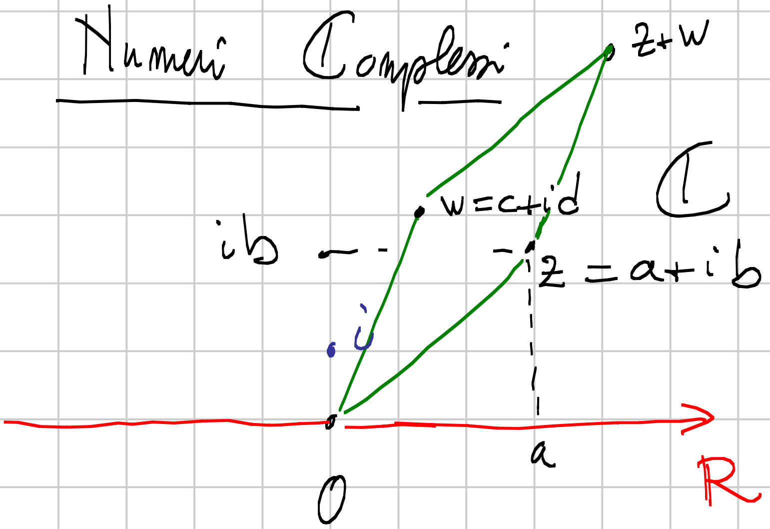
Oss: $OH^2 \geq 0 \qquad 9R^2 \geq a^2+b^2+c^2$

$$9\cancel{R^2} \geq 4\cancel{R^2}\sin^2\alpha + 4\cancel{R^2}\sin^2\beta + 4\cancel{R^2}\sin^2\gamma$$

$$\frac{9}{4} \geq \sin^2\alpha + \sin^2\beta + \sin^2\gamma \geq 0$$

Oss: $OI^2 = R^2 - 2Rr = R(R-2r) \qquad R \geq 2r$

Numeri Complessi



$z+w=(a+c)+i(b+d)$

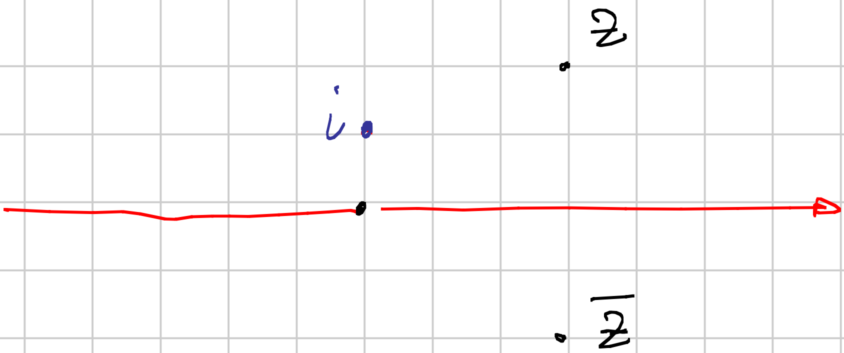
Somma tra complessi

||

Somma tra vettori con origine nello ZERO

||

Traslazioni.

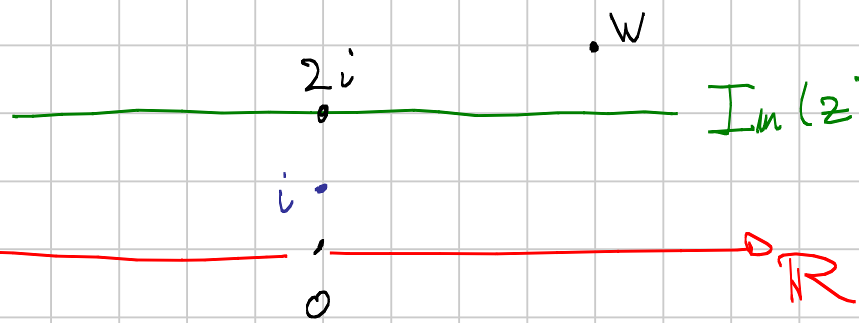


z

$\downarrow$ simmetria rispetto all'asse reale

$\bar{z}$

Es:



$\mathrm{Im}(z) = 2$

Simm di w rispetto alla retta verde $= \bar{w} + 4i$

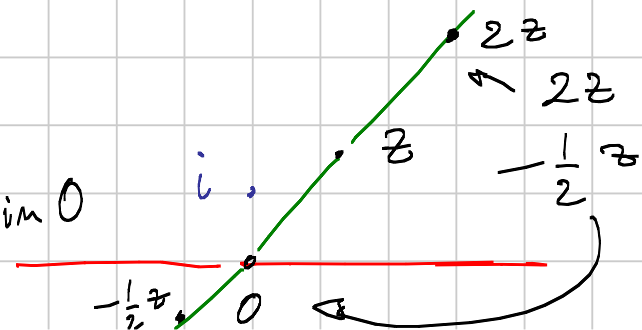
$\zeta \longrightarrow \zeta - 2i$

$w \longrightarrow w - 2i$ simm. risp. all'asse reale $= \overline{w-2i} =$
$= \bar{w} + 2i$

$\zeta' \longrightarrow \zeta' + 2i$ $\bar{w} + 4i$

Oss: $k \in \mathbb{R}$ $k \cdot z$

$k \cdot \vec{A}$ $\leftrightarrow$ omotetia in O



Es: z omotetia di fattore $-\frac{1}{3}$ rispetto al punto $1+i$

$$z \longrightarrow z-1-i \longrightarrow -\frac{1}{3}(z-1-i) \longrightarrow -\frac{1}{3}(z-1-i)+1+i$$

$$\parallel$$

$$-\frac{1}{3}z+\frac{2}{3}(1+i)$$

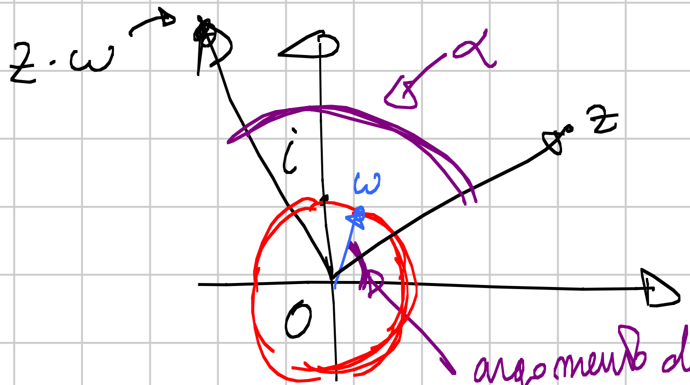
Es: $\omega \in S^1 \quad \|\omega\|=1.$

$$\|w\| = \sqrt{w\cdot\overline{w}} = \sqrt{a^2+b^2}$$

$w = a+ib$

$$\parallel$$

$$\sqrt{(a+ib)(a-ib)} = \sqrt{a^2-(ib)^2} = \sqrt{a^2+b^2}$$



$\|z\cdot\omega\| = \|z\|\cdot\|w\| = \|z\|$

argomento di ω = $\arctan\frac{b}{a} = \alpha$

$z = \rho(\cos\theta + i \operatorname{sen}\theta)$ $\quad$ ρ = modulo, norma

$\omega = \cos\alpha + i\operatorname{sen}\alpha$ $\quad$ θ = argomento

$$z w = \rho\left(\cos\theta\cos\alpha - \operatorname{sen}\theta\operatorname{sen}\alpha + i\left(\cos\alpha\operatorname{sen}\theta + \cos\theta\operatorname{sen}\alpha\right)\right) =$$

$$= \rho\left(\cos(\alpha+\theta) + i\operatorname{sen}(\alpha+\theta)\right)$$

$\Longrightarrow$ $\arg(zw) = \arg(z) + \arg(\omega)$ = rotazione di $\underset{\arg(\omega)}{\underset{\shortparallel}{\alpha}}$

Es: Voglio ruotare z di $30°$ attorno a $2-\frac{i}{2}$

$$z \longrightarrow z-2+\frac{i}{2} \longrightarrow \left(z-2+\frac{i}{2}\right)(\cos 30° + i\operatorname{sen}30°) \Longrightarrow$$

$$\Longrightarrow \left(z-2+\frac{i}{2}\right)\left(\frac{\sqrt{3}}{2}+\frac{i}{2}\right) \longrightarrow \left(z-2+\frac{i}{2}\right)\left(\frac{\sqrt{3}}{2}+\frac{i}{2}\right)+2-\frac{i}{2}=$$

$$= z\left(\frac{\sqrt{3}}{2}+\frac{i}{2}\right)+\left(2-\frac{i}{2}\right)\left(1-\frac{\sqrt{3}}{2}+\frac{i}{2}\right)$$

Es: Ruoto z di $45°$ attorno a $1+i$ e poi faccio la simmetria rispetto a $Im(z)=-2i$

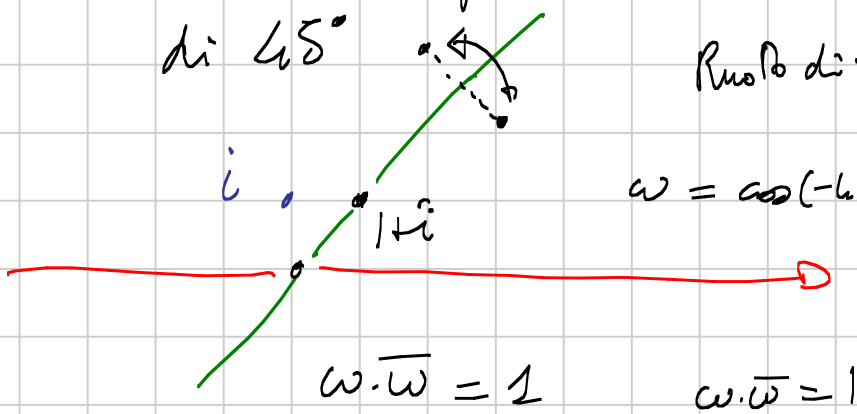
$$z \longrightarrow (z-1-i) \longrightarrow (z-1-i)\left(\frac{1}{\sqrt{2}}+\frac{i}{\sqrt{2}}\right) \longrightarrow (z-1-i)\left(\frac{1+i}{\sqrt{2}}\right)+1+i$$

$$\longrightarrow (z-1-i)\left(\frac{1+i}{\sqrt{2}}\right)+1+i+2i \longrightarrow (\bar{z}-1+i)\left(\frac{1-i}{\sqrt{2}}\right)+1-3i$$

$$\longrightarrow (\bar{z}-1+i)\left(\frac{1-i}{\sqrt{2}}\right)+1-3i-2i = (\bar{z}-1+i)\left(\frac{1-i}{\sqrt{2}}\right)+1-5i$$

Es: Simmetria rispetto alla retta per $1+i$ inclinata di $45°$

Ruoto di $-45°$ $\longrightarrow$ simmetria risp. a $\mathbb{R}$ $\longrightarrow$ Ruoto di $45°$



$$\omega = \cos(-45°)+i\,\mathrm{sen}(-45°) = \frac{1}{\sqrt{2}}-\frac{i}{\sqrt{2}}$$

$$\omega\cdot\bar{\omega}=1 \qquad 1 = \|\omega\|^2$$

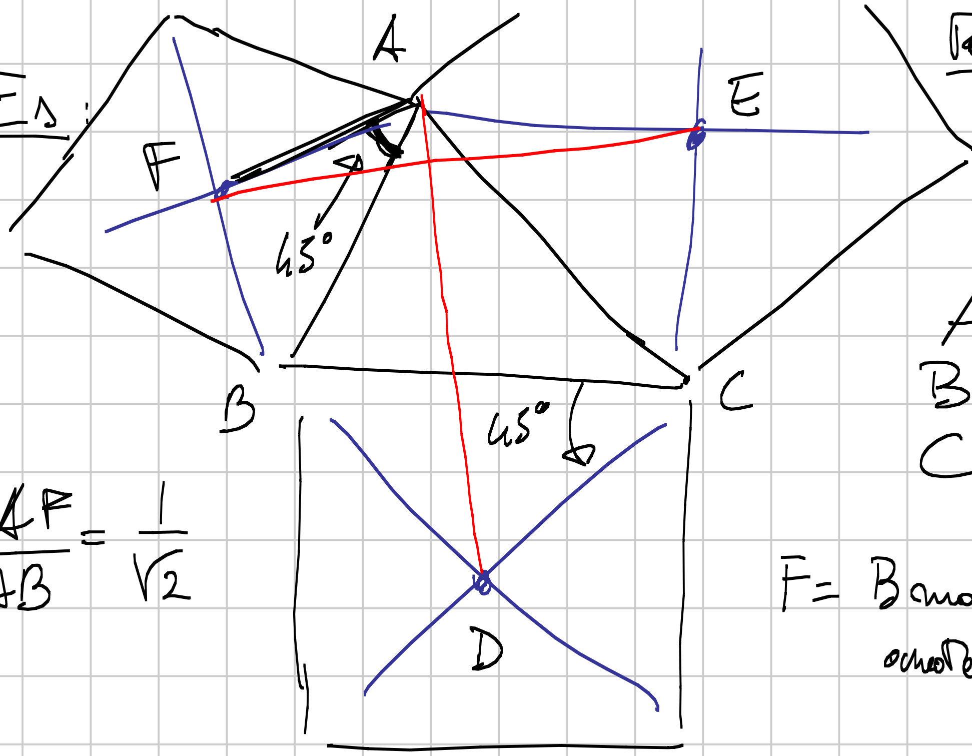
$$\omega\cdot\bar{\omega}=1 \Downarrow \bar{\omega}=\frac{1}{\omega}$$

$$z \xrightarrow{\text{Rot di } -45°} z\left(\frac{1}{\sqrt{2}}-\frac{i}{\sqrt{2}}\right) \xrightarrow{\text{simm. risp a } \mathbb{R}} \bar{z}\left(\frac{1}{\sqrt{2}}+\frac{i}{\sqrt{2}}\right) \xrightarrow{\text{Rot di } 45°} \bar{z}\left(\frac{1}{\sqrt{2}}+\frac{i}{\sqrt{2}}\right)^2$$

Simm risp. a una retta per O inclinata di θ

$$z \longrightarrow \bar{z}(\cos\theta+i\,\mathrm{sen}\theta)^2$$

Es:



Tesi: $AD \perp EF$

$AD = EF$

$A = 0$

$B = b$

$C = c$

$$\frac{AF}{AB} = \frac{1}{\sqrt{2}}$$

$F =$ B ruotato di $-45°$ e omotetizzato di $\frac{1}{\sqrt{2}}$

$$f = b\left(\frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}}\right)\cdot\frac{1}{\sqrt{2}} = b\,\frac{1-i}{2}$$

$$e = c\left(\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}\right)\frac{1}{\sqrt{2}} = c\,\frac{1+i}{2}$$

$$d = (b-c)\left(\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}\right)\cdot\frac{1}{\sqrt{2}} + c =$$

$$= (b-c)\left(\frac{1+i}{2}\right) + c$$

$$\left.\begin{array}{l} AD \perp EF \\ AD = EF \end{array}\right\} \iff d = \pm i\,(e-f)$$

$$(b-c)\left(\frac{1+i}{2}\right) + c = \left[\frac{c-b}{2} + i\,\frac{(c+b)}{2}\right](\pm i)$$

$$\frac{b-c}{2} + c + \frac{i}{2}(b-c) \qquad\qquad \pm i\,\frac{c-b}{2} \mp \frac{(c+b)}{2}$$

$$= \frac{b+c}{2} + \frac{i}{2}(b-c) \quad = \quad -\left(\frac{c-b}{2}\right)i + \left(\frac{c+b}{2}\right)$$