Funzioni generatrici

Titolo nota 07/09/2010

$a_0, a_1, \ldots \qquad a_0 + a_1 x + a_2 x^2 + \ldots$

$\binom{n}{0} + \binom{n}{1} x + \binom{n}{2} x^2 + \ldots \qquad \binom{n}{n} x^n = (1+x)^n$

$1 + x + x^2 + x^3 + \ldots = \frac{1}{1-x}$

$a_0 = 0$

$a_{n+1} = 2a_n + 1$

$$S(x) = \underset{\overset{\shortparallel}{0}}{a_0} + a_1 x + a_2 x^2 + \ldots$$

$$\underbrace{a_1 + a_2 x + a_3 x^2 + \ldots}_{\frac{S(x)}{x}} = 2a_0 + 1 + (2a_1 + 1)x + (2a_2 + 1)x^2 + \ldots$$

$$= 2S(x) + \underbrace{1 + x + x^2 + \ldots}_{\frac{1}{1-x}}$$

$$S(x) = 2x S(x) + \frac{x}{1-x}$$

$$S(x) = \frac{x}{(1-x)(1-2x)} = \frac{A}{1-x} + \frac{B}{1-2x}$$

$A = -1 \qquad B = 1$

$$S(x) = \frac{1}{1-2x} - \frac{1}{1-x}$$

$$1 + 2x + 4x^2 + \ldots - 1 + x + x^2 + \ldots = (1-1) + (2-1)x + (4-1)x^2 + \ldots$$

$a_n = 2^n - 1 \qquad a_2$

$$1 + x + x^2 + \ldots = \frac{1}{1-x}$$

$\infty \qquad -1$

$a_0, a_1, a_2, \dots$ $\qquad$ $\mathrm{ogf}(a_i)$

$[x^n]\, \delta(x) =$ coef. di x^n.

$\mathrm{ogf}(a_i) = \mathrm{ogf}(b_i) \iff a_i = b_i \ \forall i$

$\mathrm{ogf}(a_i) + \mathrm{ogf}(b_i) = \mathrm{ogf}(a_i + b_i)$

$\mathrm{ogf}(a_i) \cdot \mathrm{ogf}(b_i) = \mathrm{ogf}\left(\sum_{s=0}^{i} a_s \cdot b_{i-s}\right)$ $\qquad x^k \delta(x)$

$a_k = 0$

$\dfrac{\mathrm{ogf}(a_i)}{\mathrm{ogf}(b_i)} =$ $\qquad [x^0] \neq 0 \to \dfrac{G(x)}{\delta(x)}$

$b_n = 0$ $\qquad [x^0] = 0$ $\qquad \underbrace{[x^0]\, \delta(x) \neq 0}$

$\dfrac{G(x)}{\delta(x)} = G(x) \cdot \boxed{\dfrac{1}{\delta(x)}}$ $\qquad A(x) = \dfrac{G(x)}{\delta(x)}$ $\qquad \cancel{x^n \, \delta(x)}$

$A(x) \cdot \delta(x) = G(x)$

$\qquad\quad \| $

$\qquad x^n V(x)$

$\underbrace{x^n \left[A(x) \cdot \dfrac{\delta(x)}{x^n} \right]}$ = col termine noto.

$\dfrac{A(x)}{B(x)} = A(x) \cdot \dfrac{1}{B(x)}$

$C(x) = \dfrac{1}{B(x)} \iff \overbrace{C(x) \cdot B(x) = 1}$

$\mathrm{ogf}(a_i) \cdot \mathrm{ogf}(b_i) = 1$

$\underbrace{a_0 \cdot b_0}$ $\qquad a_0 = \dfrac{1}{b_0}$

$\mathrm{ogf}\left(\underbrace{\sum_{s=0}^{i} a_s \cdot b_{i-s}}\right)$

$$\sum_{j=0}^{n} a_j b_{n-j} = 0 \quad \overset{a_n}{\Longrightarrow} \quad a_n = \frac{\text{quelcosa}}{b_0}$$

$A(x) = ogf(a_i)$

$B(x) = ogf(b_i)$

$A(B(x)) =$

$$a_0 + a_1\, ogf(b_i) + a_2\, \underbrace{ogf(b_i)^2}_{} + \dots$$

$$\left.\begin{array}{l} A(x) = \frac{1}{1-x} \\ B(x) = \frac{1}{1-x} \end{array}\right\} \quad 1 + (1+x+x^2+\dots) + (1+x+x^2+\dots)^2 + \dots$$

$\log(f(x)) = g(x)$ t.c. $e^{g(x)} = f(x)$

$a_0 + a_1 x + a_2 x^2 + \dots = \delta(x)$

$a_0 + a_1 \cdot s + a_2 \cdot s^2 + \dots$

$\exists R \in \mathbb{R} : \forall |r| < R \quad \delta(r)$ converge; $\forall |r| > R$ si comporta male.

Per $r = R$ o $-R$.

$a_0 + a_1 x + \dots = \delta(x)$

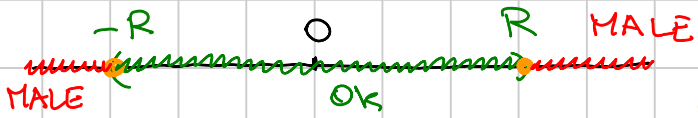
Se converge per $\overset{\text{positivo}}{R}$ $(\delta(R)) \Rightarrow$ converge anche per i positivi minori.

$$B_n^{(x)} = \sum_{i=0}^{n} a_i x^i$$

SERIE DI POTENZE

$$\sum_{n=0}^{\infty} a_n x^n$$

Domanda: per quali $x \in \mathbb{R}$ la serie converge



può anche essere $+\infty$

$R = [0, +\infty]$

SERIE DI NUMERI $\sum_{n=0}^{\infty} b_n$

$\boxed{\text{FATTO 1}}$ $\sum_{n=0}^{\infty} b_n$ converge $\Rightarrow$ $b_n \to 0$

Dim. $S_n = b_0 + b_1 + \dots + b_n$ $\qquad b_n = S_n - S_{n-1}$

$$\underset{\ell}{\downarrow} \quad - \quad \underset{\ell}{\downarrow} \quad = 0$$

Viceversa Se $b_n \not\to 0$, allora di sicuro $\sum b_n$ NON converge

Nella zona rossa $a_n x^n$ NON tende a zero

Esempio $\sum_{n=0}^{\infty} x^n$

$x \geq 1$ DIVERGE A $+\infty$

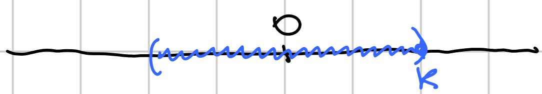
$x < -1$ INDETERMINATA

$x = -1$ $+1 - 1 + 1 - 1 + 1 - 1$

$\boxed{\text{FATTO 2}}$ $\sum_{n=0}^{\infty} |b_n|$ converge $\Rightarrow$ $\sum_{n=0}^{\infty} b_n$ converge

FATTO 2 $\Rightarrow$ Teorema sul raggio di convergenza

Se in k converge, allora converge per gli x con $|x| < k$



Prendo k e prendo $|x|<k$. Dico che in x si ha

$$\sum_{n=0}^{\infty} |a_n x^n| \quad \text{converge}$$

$$= \sum_{n=0}^{\infty} |a_n| \frac{|x|^n}{k^n} k^n = \sum_{n=0}^{\infty} |a_n| k^n \left|\frac{x}{k}\right|^n = (\star)$$

Sapendo che $\sum a_n k^n$ converge, FATTO 1 $\Rightarrow a_n k^n \to 0$

$\Rightarrow \exists M$ t.c. $|a_n| \cdot k^n \le M \quad \forall n \in \mathbb{N}$

$$(\star) \le M \underbrace{\sum_{n=0}^{\infty} \left|\frac{x}{k}\right|^n}_{\text{converge}}$$

(Serie del tipo $\sum_{n=0}^{\infty} a^n$ con a fisso <1)

$\boxed{\text{FATTO 3}}$ $\sum_{n=0}^{\infty} a^n$ converge $\Leftrightarrow |a|<1$ e la somma è $\frac{1}{1-a}$

$D \, ogf(e_i) = ogf((n+1)\, e_{n+1})$

$$D \underset{f(x)}{\underset{\shortparallel}{\sum_{n=0}^{\infty} a_n x^n}} = \sum_{n=0}^{\infty} D a_n x^n = \sum_{n=0}^{\infty} n a_n x^{n-1}$$

$$D \frac{e^x}{1-x} = D \text{ serie di potenze}$$

$$\int \sum_{n=0}^{\infty} a_n x^n = \sum_{n=0}^{\infty} \frac{a_n}{n+1} x^{n+1}$$

$\delta(x) = ogf(a_i)$ $\qquad a_0 + a_1 x + \dots + a_k x^k + a_{k+1} x^{k+1} + \dots$

$A(x) = ogf(a_{i+k})$

$a_k + a_{k+1} x + a_{k+2} x^2 + \dots$

$$\frac{\delta(x) - (a_0 + a_1 x + \dots + a_{k-1} x^{k-1})}{x^k} = A(x)$$

$B(x) = \mathrm{ogf}(a_{i-k}) \to$

$\underbrace{0 + 0 + \cdots}\, a_0 x^k + a_1 x^{k+1} + a_2 x^{k+2} + \cdots$

$B(x) = S(x) \cdot x^k$

$\mathrm{ogf}(n \cdot a_n) = x \cdot D\, \mathrm{ogf}(a_n)$

$\mathrm{ogf}\left(\frac{a_n}{n}\right) = \int \frac{\mathrm{ogf}(a_n) - a_0}{x}$

Moltiplicare o dividere per $1-x$

$\mathrm{ogf}(a_i) = A(x)$ $\to$ n-esimo

Quanto fa $A(x) \cdot (1-x)$? $= B(x)$

$B(x) = \mathrm{ogf}\left(\sum_{j=0}^{k} a_0 \cdot 0 + a_1 \cdot 0 + \cdots - a_{n-1} + a_n\right) = \mathrm{ogf}(a_n - a_{n-1})$

Dividere per $1-x$ = moltiplicare per $\frac{1}{1-x}$ =

$\mathrm{ogf}(1)$ $\quad A(x) \cdot \frac{1}{1-x} = \mathrm{ogf}\left(\sum_{j=0}^{n} a_j\right)$

$\mathrm{Egf}(a_i) = \mathrm{ogf}\left(\frac{a_i}{i!}\right)$

$\mathrm{Egf}(a_i) \cdot \mathrm{Egf}(b_i) = \mathrm{ogf}\left(\frac{a_i}{i!}\right) \cdot \mathrm{ogf}\left(\frac{b_i}{i!}\right) = \mathrm{ogf}\Big($

$\sum_{j=0}^{n} a_j \cdot b_{n-j} \cdot \frac{1}{j!\,(n-j)!}\Big) = \mathrm{Egf}\left(\sum_{j=0}^{n} \binom{n}{j} \cdot a_j \cdot b_{n-j}\right)$

$\mathrm{Egf}(a_{i+k}) = D^k\, \mathrm{egf}(a_i)$

$\mathrm{Egf}(a_{i-k}) = \int^k \mathrm{egf}(a_i)$

Mac Laurin

$A(x) = ogf(a_i)$ $\qquad$ $[x^k]\, ogf(a_i)$

$[\cancel{x}^0]\, D^k\, ogf(a_n) = \underbrace{a_k \cdot k!}$

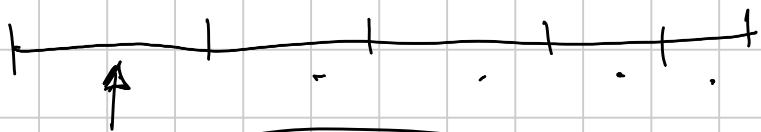
$$[x^k]\, ogf(a_i) = \frac{\left[D^k\, ogf(a_i)\right]_0}{k!}$$

$$ogf(a^n) = \frac{1}{1-ax}$$

$$ogf\left(\binom{k}{i}\right) = (x+1)^k$$

$$ogf\left(\binom{i+k}{k}\right) = \binom{k}{k} + \binom{k+1}{k}x + \binom{k+2}{k}x^2 + \dots$$

Partizionare n in $i+1$ parti si può fare in $\binom{n+i}{i}$.



$i+1$

$$(\ x^k\ )(\ x^n\ )(\ x^j\ )\dots(\ x^k\ )$$

$$[x^n](1+x+x^2+x^3+\dots)(1+x+x^2+x^3+\dots)\dots(1+x+x^2+x^3+\dots)$$

CORRETTO DOPO VIDEO

$$\frac{1}{(1-x)^{\boxed{k+1}}} \qquad \binom{n+i}{i}$$

$$(1+x+x^2+\dots)^{k+1} = ogf\left(\binom{n+k}{k}\right)$$

$$\lim_{k\to\infty} \underbrace{\sum_{i=0}^{\infty} \frac{\binom{i+k}{k}}{(k+2)^i}}$$

$\frac{1}{k+2}$ nella formula di $\mathrm{ogf}\left(\binom{n+k}{k}\right)$

$$\lim_{n\to\infty} \frac{1}{\left(1-\frac{1}{k+2}\right)^{\boxed{k+1}}} = \left(\frac{1}{\frac{k+2-1}{k+2}}\right)^{\boxed{k+1}} = \left(\frac{k+2}{k+1}\right)^{\boxed{k+1}} = \left(1+\frac{1}{k+1}\right)^{\boxed{k+1}}$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots$$

$$a_{n+k} = 80\, a_n + 57\, a_{n+1} \; - - - - \; + 29\, a_{n+k-1}$$

$$a_{n+1} = \underbrace{\sum_{i=0}^{n} a_i \cdot b_{n-i}} \qquad A(x) = \mathrm{ogf}(a_n)$$

$$\underset{\displaystyle \frac{\mathrm{ogf}(a_n) - a_0}{x}}{\mathrm{ogf}(a_{n+1})} = \mathrm{ogf}(a_n) \cdot \mathrm{ogf}(b_n)$$

$$\frac{A(x) - a_0}{x} = A(x)\,B(x) \Rightarrow A(x) = \frac{a_0}{1 - xB(x)}$$

Catalan

Il numero di modi in cui ordinare n parentesi aperte e n chiuse nel modo solito.

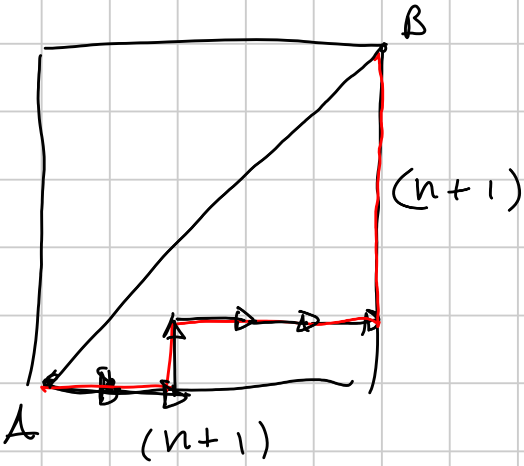
(( ) (( ) ( ))) $\qquad C_5$

$2n$

$$\overbrace{\Big|\; (\; (\; - \; \underset{\text{MINIMA}}{-} \;\Big|}^{C_{k-1}} \; \overbrace{\qquad) \;)}^{C_{n-k}}$$

c'è una 2k-stringa buona

$$C_n = \sum_{i=1}^{n} C_{i-1} \cdot C_{n-i}$$



$\rightarrow$ $\uparrow$

C_n

$$\rightarrow = (\qquad \uparrow =) .$$

$$C(x) = ogf\,(C_n)$$

$$C_{n+1} = \sum_{i=0}^{n} C_i \cdot C_{n-i} \Rightarrow C(x) = \frac{C_0}{1 - x\,C(x)}$$

$$C(x) = \frac{1 \pm \sqrt{1-4x}}{2x} \qquad \frac{1 \cancel{+} \sqrt{1-4x}}{2x} \qquad \frac{1 - \sqrt{1-4x}}{2x}$$

$$\frac{(1-\sqrt{1-4x})\,\overline{(1+\sqrt{1-4x})}}{2x\;\underset{\uparrow}{(1+\sqrt{1-4x})}} = \frac{1-(1-4x)}{2x\,(\text{ROBA senza } x)} = \frac{2}{1+\sqrt{1-4x}}$$

$$\boxed{\text{Snake oil method}}$$

$$\underbrace{a_i = \sum^{i} \overset{b_i}{\text{BRUTTO}}}$$

$$b_n = \sum_{j=0}^{n} \binom{2j}{j}$$

$$B(x) = \sum_{n=0}^{\infty} x^n \overbrace{\sum_{j=0}^{n} \binom{2j}{j}}$$

$$B(x) = \sum_{j=0}^{\infty} \sum_{n=j}^{\infty} x^n \binom{2j}{j} = \sum_{j=0}^{\infty} \binom{2j}{j} \cdot \sum_{n=j}^{\infty} x^n$$

$\underline{b_0}$ $\boxed{j=0}$ $\mathring{O}$ $\downarrow$

b_1 $\boxed{j=0}$ $\boxed{j=1}$

b_2 $\boxed{j=0}$ $\boxed{j=1}$ $\boxed{j=2}$

$$\sum_{j=0}^{\infty} \binom{2j}{j} \cdot \frac{x^j}{1-x} = \frac{1}{(1-x)\sqrt{1-4x}}$$

<u>Figo.</u>

$$\forall n \in \mathbb{N} \quad \sum_{i=1}^{n} \binom{n+i-1}{\underbrace{2i-1}} = F_{2n}$$

$$\sum_{n=0}^{\infty} x^n \sum_{i=1}^{n} \binom{n+i-1}{2i-1} = \sum_{i=1}^{\infty} \left[\underbrace{\sum_{n=i}^{\infty} x^n \binom{n+i-1}{2i-1}} \right]$$

$$\operatorname{ogf}\left(\binom{n+k}{k}\right) \qquad \binom{(n+i)+i-1}{2i-1} \qquad x^i \sum_{n=0}^{\infty} x^{n-i} \binom{(n-i)+2i-1}{2i-1}$$

$k = \boxed{2i-1}$

$\operatorname{ogf}\binom{n+2i-1}{2i-1}$ shiftate di $-i$

$$x^i \sum_{n=0}^{\infty} x^n \binom{n+2i-1}{2i-1} = x^i \frac{1}{(1-x)^{2i}} = \left[\frac{x}{(1-x)^2}\right]^i$$

$$\sum_{i=1}^{\infty} \left(\frac{x}{(1-x)^2}\right)^i = \frac{1}{1-\left(\frac{x}{(1-x)^2}\right)} - 1$$

$$F_{2n} = \qquad F_{2n+4} = 3F_{2n+2} - F_{2n}$$

Roots of unity filter

$A(x) = ogf(a_i)$ $\qquad \underline{ogf(a_{pi})}$ con $p \in \mathbb{P}$

$\omega \Rightarrow 1 + \omega + \omega^2 + \dots\dots \omega^{p-1} = 0$

$$\frac{\sum_{i=0}^{p-1} A(\omega^i x)}{p} = a_0 + a_p x^p + a_{2p} x^{2p} + \dots$$

$p \nmid n$

$$\sum_{i=0}^{p-1} a_n x^n \omega^{ni} =$$

$p \mid n$

$$a_n x^n \sum_{i=0}^{p-1} \omega^{in} = \begin{cases} p \mid n = a_n x^n (1+1+1 \dots 1) = p a_n x^n & \text{Somma di uno} \\ p \nmid n \Rightarrow a_n x^n (\text{Permutazione delle radici}) \end{cases}$$

$n \in \mathbb{N}$

$A(n)$ è il numero di 2n-uple $(x_1, x_2, \dots, x_n, r_1, r_2, \dots, r_n)$ $\in \mathbb{Z}_2$

t.c. : $\underline{\sum_{i=1}^{n} x_i r_i \equiv 0 \ (2)}$

$B(n)$ le altre

Calcolare $\frac{A(n)}{B(n)}$

$$x r \in \{\overbrace{\underbrace{(0,1),(1,0)}_{},(0,0)}^{}\}$$

$(1,1)$

(x_i, r_i) $\qquad (x_1, r_1)$

$$[x^n]\left(x+1^{b}+1^{b}+1^{b}\right)^n = \cancel{[x^n]}(x+3)^n = \delta(x)$$

$$\frac{\delta(x)+\delta(-x)}{2}$$

$$A(n) = \frac{(1+3)^n + (-1+3)^n}{2}$$

$$B(n) = \frac{(1+3)^n - (-1+3)^n}{2}$$

$$\frac{4^n+2^n}{4^n-2^n} = \boxed{\frac{2^n+1}{2^n-1}}$$

Quanti sono i numeri $\equiv 0\ (3)$ con n cifre tra queste $\{2, 3, 7, 9\}$.

$$\cancel{[x^n]}\left(x^2+x^3+x^7+x^9\right)^n = P(x)$$

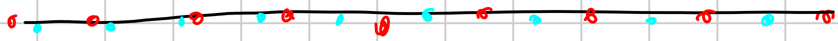
$$N = \frac{P(1)+P(\omega)+P(\omega^2)}{3}$$

$$\frac{4^n+\big(\overbrace{\omega^2+1+\omega}^{0}+1\big)^n+\big(\overbrace{\omega+1+\omega^2}^{0}+1\big)^n}{3} = \frac{4^n+2}{3}$$

Exact covering System

$(a_1,b_1);(a_2,b_2); \dots ;(a_k,b_k)$ di interi con $b_i > 1 \ \forall i$.

$$\boxed{a_i + n b_i}$$



$(1,2)\quad(2,4)\quad(0,4)$.

$$\sum_{m=1}^{k} \frac{1}{b_m} = 1$$

$\exists\, i,j : b_i = b_j$ (Erdős)

$\exists\, i : 2 \mid b_i \ \vee \ \forall i,j \ b_i = b_j$

$a_i + n b_i$ come lo scrivo con generatrici??

$$\sum_{i=1}^{k} \underbrace{x^{a_i} + x^{a_i+b_i} + x^{a_i+2b_i} + x^{a_i+3b_i} + \dots}\ = 1+x+x^2+\dots$$

$$x^{a_i}\sum_{j=0}^{\infty}\left(x^{b_i}\right)^j = \frac{x^{a_i}}{1-x^{b_i}}$$

$$(*)\quad \sum_{i=1}^{k} \frac{x^{a_i}}{1-x^{b_i}} = \frac{1}{1-x} \Longrightarrow \sum_{i=1}^{k} \frac{x^{a_i}}{1+x+x^2+\dots+x^{b_i-1}} = 1$$

$$\sum_{i=1}^{k} \frac{1}{b_i} = 1$$

Considerare il b_i massimo e porre x una "sua" radic dell'unità primitiva. b_3

$$\frac{x^{a_3}}{1-x^{b_3 a}} \text{ minore.}$$

A, B partizione di $\mathbb{N}$ (con 0)

$$\forall n \quad \left| \left\{ x, y \mid (x,y) \in A^2;\ x \neq y;\ x+y=n \right\} \right| = \leftarrow$$

$$\uparrow$$
$$B$$

$$A(x) = x^7 + x^9 + x^{51} + \cdots$$

$B(x) =$ gli altri

$$\begin{cases} A(x) + B(x) = \dfrac{1}{1-x} \\ A(x)^2 - A(x^2) = B(x)^2 - B(x^2) \end{cases}$$

$$B(x) = \frac{1}{1-x} - A(x)$$

$$\cancel{A(x)^2} - A(x^2) = \left(\frac{1}{1-x}\right)^2 + \cancel{A(x)^2} - \frac{2}{1-x} A(x) - \frac{1}{1-x^2} + A(x^2)$$

$$\underbrace{2A(x^2) - \frac{2}{1-x} A(x) = \frac{1}{1-x^2} - \frac{1}{(1-x)^2}}$$

$$[x^k]\ 2A(x^2) - \frac{2}{1-x} A(x) = \text{Definito.}$$

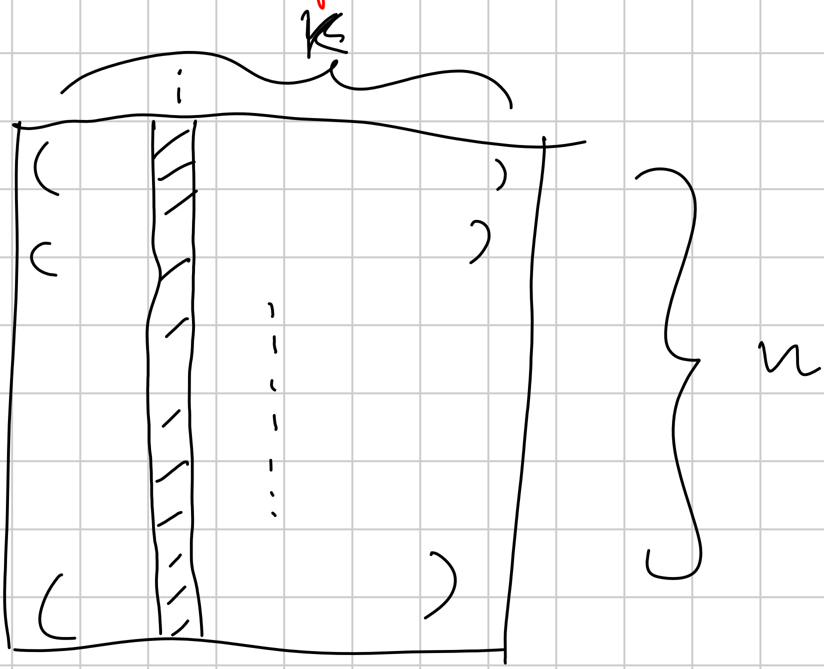
A, B

A, B partizionano in base alla parità del numero di 1 nella rappresentazione binaria.

Una schedina è un insieme non ordinato di k-uple ordinate.

Exponential formula

n-schedina



Schedine vuote: solo o

Giocare una schedina: rimpiazzare gli o con numeri interi.

L'unione di una m-schedina con una n-schedina è una $m+n$-schedina senza colore.

Le schedine hanno un colore

Una (a,b)-scommessa è un insieme di b schedine con somma dei costi a t.c. ogni colonna dell'unione è sia l'insieme $\{1,2,3,\dots,a\}$.

Assumo che esistano d_n colori per le n-schedine.

Definisco $D(x) = \frac{d_0}{0!^k} + \frac{d_1}{1!^k} x + \frac{d_2}{2!^k} x^2 + \dots$

Assumo che di ognuna ne ho infinite vuote. Questo lo chiamo negozio di $D(x)$.

Chiamo $\delta^{D(x)}(a,b)$ numero di (a,b) scommesse diverse creabili con le schedine del negozio giocabile.

Definisco $H^{D(x)}(x,y)$ t.c. $[x^a y^b] H^{D(x)}(x,y) = a!^k \cdot \delta^{D(x)}(a,b)$

enumeratore

$$H^{\frac{|D(x)|}{}}(x,y) = e^{yD(x)}$$

Ponendo $k=1$

Lemma 1: Se un negozio ha solo un tipo di scatolina π la formula vale. $G(x)$

$$G(x) = \frac{x^n}{(n!)^k}$$

Lemma 2: 2 negozi A, B hanno associate $A(x), B(x)$.
Chiamo C il negozio che ha associato $A(x)+B(x) = C(x)$

$$H^{C(x)}(x,y) = H^{A(x)}(x,y) \cdot H^{B(x)}(x,y)$$

Lemma 3: $[x^n]\, e^{yf(x)}$ = se taglio $f(x)$ al coef n-esimo

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$\sigma : (1,2,\dots,2n) \to (1,2,\dots,2n)$

Le ~~~~ 2k-scatoline sono $\left[\,(2k-1)!\,\right]$
Le 2k+1 - " sono 0

$$D(x) = 0 + \frac{x^2 (2-1)!}{2!} + \frac{x^4 (4-1)!}{4!} + \dots = \underbrace{\frac{x^2}{2} + \frac{x^4}{4} + \frac{x^6}{6} + \dots}$$

$$e^{yD(x)} \leadsto e^{D(x)} =$$

$$-x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \cdots\cdots = \log\left(\frac{1}{1-x}\right)$$

$$\frac{\log\left(\frac{1}{1-x}\right) \mp \log\left(\frac{1}{1+x}\right)}{2}$$

$$e \quad = \sqrt{\frac{1}{1-x^2}}$$

$$=$$

$$\sqrt{\frac{1+x}{1-x}}$$

$$\sqrt{\frac{1+x}{1-x}} - \sqrt{\frac{1}{1-x^2}} = x\sqrt{\frac{1}{1-x^2}}$$

$$\sqrt{\frac{1}{1-x^2}} = (2n-1)!!^2$$

$$\frac{\overbrace{\alpha(\alpha-1)(\alpha-2)\cdots\cdots(\alpha-i+1)}}{i!}$$

$$(x+1)^\alpha = \sum_{i=0}^{\infty} x^i \binom{\alpha}{i}$$

$$(1-x^2)^{-\frac{1}{2}} = \cancel{\sum_{i=0}^{\infty}}(-x^2)^i \binom{-\frac{1}{2}}{i} =$$

$$(-1)^n \cdot \cancel{x^{2n}} \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)\left(-\frac{7}{2}\right)\cdots\cdots\left(-\frac{2n-1}{2}\right)}{n!} \cdot 2n!$$

$$\cancel{(-1)^n}(-1)^n \cdot (2n-1)!! \cdot \underbrace{\frac{1}{2^n} \cdot \frac{1}{n!} \cdot (2n)!}_{\substack{\shortparallel \\ (2n-1)!!}} \overset{?}{=} (2n-1)!!^2$$

$$\frac{1}{\sqrt{1-4x}} = \mathrm{ogf}\left(\binom{2n}{n}\right).$$

In quanti modi posso mettere 2n pedoni su una scacchiera $n \times n$ in modo che ce ne siano 2 per riga e per colonna.

Una funzione generatrice è una corda a cui appendere una successione per metterla in mostra

Mr Herbert Wilf