G2 – Metodi Algebrici Medium

Titolo nota 09/09/2010

1. Coord. cartesiane (str. di calcolo)
2. Complessi (geometria della cfr e del trn)
3. Vettori (comb. lineari convesse) prodotti vettori

[Coord. baricentriche]

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Conche: $p(x,y) = ax^2 + 2bxy + cy^2 + 2dx + 2ey + f$

- $x^2 + y^2 - 1 = 0$
- $x^2 - y^2 - 1 = 0$
- $x^2 - y = 0$

($x^2+y^2+1=0$ è FINTA)

$a>0$

$$\left(\sqrt{a}\cdot x + \frac{b}{\sqrt{a}}y + \frac{d}{\sqrt{a}}\right)^2 = ax^2 + \frac{b^2}{a}y^2 + \frac{d^2}{a} + 2bxy + 2dx + 2\frac{bd}{a}y$$

$$y^2\left(c - \frac{b^2}{a}\right) + 2y\left(e - \frac{bd}{a}\right) + f - \frac{d^2}{a} =$$

$$= \pm \square + k$$

$$c - \frac{b^2}{a} \gtreqless 0$$

$$ac - b^2 \gtreqless 0$$

$$\alpha t^2 + \beta t + \gamma = + \square \pm h^2$$

$+ =$ niente radici

$- =$ 2 radici

$h=0$ 1 radice

$a^2 - b^2 = 0$ ok

$a(x)^2 + b(x)^2 = 0$

$a(x) = b(x) = 0$

$x^2+1 \quad \Delta<0$

$x^2-1 \quad \Delta>0$

$x^2 \quad \Delta=0$

$p(x,y) = ax^2 + 2bxy + cy^2 + \dots$

$\Delta < 0$ ellisse

$\Delta = 0$ parabola

$\Delta > 0$ iperbole

RETTA

Cartesiane: $ax+by+c=0$

Parametriche: $\begin{cases} x = pt+q \\ y = rt+s \end{cases} \quad t \in \mathbb{R}$

CIRCONFERENZA

$\begin{cases} x = \cos\theta \\ y = \sin\theta \end{cases}$

$\begin{cases} x = \dfrac{1-t^2}{1+t^2} \\ y = \dfrac{2t}{1+t^2} \end{cases}$

$t = \tan\dfrac{\theta}{2}$

$\cos^2\dfrac{\theta}{2} =$

$x = \tan\alpha$

$x^2 = \dfrac{\operatorname{sen}^2\alpha}{\cos^2\alpha}$

$\cos^2\alpha = \dfrac{1}{x^2+1}$

$x^2+1 = \dfrac{\cos^2\alpha + \operatorname{sen}^2\alpha}{\cos^2\alpha} = \dfrac{1}{\cos^2\alpha}$

$\cos 2\alpha = 2\cos^2\alpha - 1 = \dfrac{2}{x^2+1} - 1 = \dfrac{2-x^2-1}{x^2+1} = \dfrac{1-x^2}{1+x^2}$

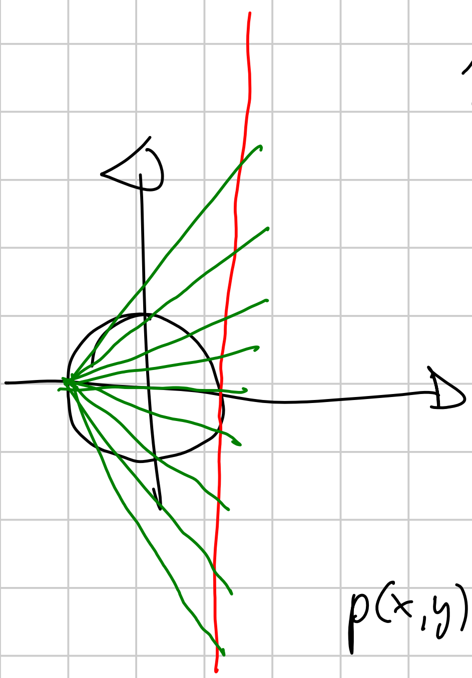
$m, n \in \mathbb{Z}$

$t = \dfrac{m}{n}$

$x = \dfrac{1 - \frac{m^2}{n^2}}{1 + \frac{m^2}{n^2}}$

$(n^2-m^2,\ 2mn,\ n^2+m^2)$

$y = \dfrac{2\frac{m}{n}}{1 + \frac{m^2}{n^2}}$



fascio di rette: $y = m(x+1)$

$\begin{cases} y = m(x+1) \\ x^2+y^2-1=0 \end{cases}$

$p(x,y) =$ di grado 2

$a(x) =$ lineare (1° grado)

$p(x, a(x)) =$ grado 2

$p(x,y)=0$ conica $\qquad (x_0,y_0)\in$ conica

$$\begin{cases} y-y_0=m(x-x_0) \\ p(x,y)=0. \end{cases}$$

Se $p(x,y)\in\mathbb{Q}[x,y]$ (o $\mathbb{Z}[x,y]$), $x_0,y_0\in\mathbb{Q}$ o $\mathbb{Z}$

$m\in\mathbb{Q},\mathbb{Z} \Rightarrow$ altro punto $\in\mathbb{Q},\mathbb{Z}$

$$m = \frac{y_0-y_1}{x_0-x_1}\in\mathbb{Q}\ (\mathbb{Z})$$

$\Uparrow$

$\mathbb{Q}$ (o $\mathbb{Z}$)

$$\begin{cases} x=\dfrac{r_1(t)}{r_2(t)} \\ y=\dfrac{s_1(t)}{s_2(t)} \end{cases} \qquad r_i,s_i\in\mathbb{Z}[t]$$

3° grado = 3 soluzioni

$p(x)\in\mathbb{Q}[x]$ irriducibile su $\mathbb{Q}[x]$

ha radici ζ_1,ζ_2,ζ_3

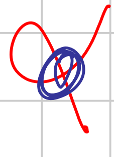
non esiste ($\nexists$) $q(x)\in\mathbb{Q}[x]$

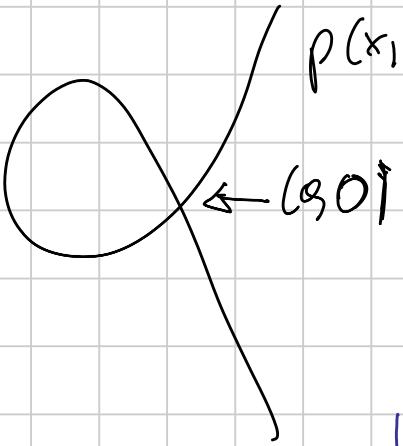
t.c. $q(\zeta_1)=0 \qquad q(\zeta_2)\neq 0.$

Il metodo delle corde funziona se:

1) il 3° grado è finito $\big((x+y)(x^2+y^2-1)=0\big)$

2) la curva $p(x,y)=0$ è fatta così:

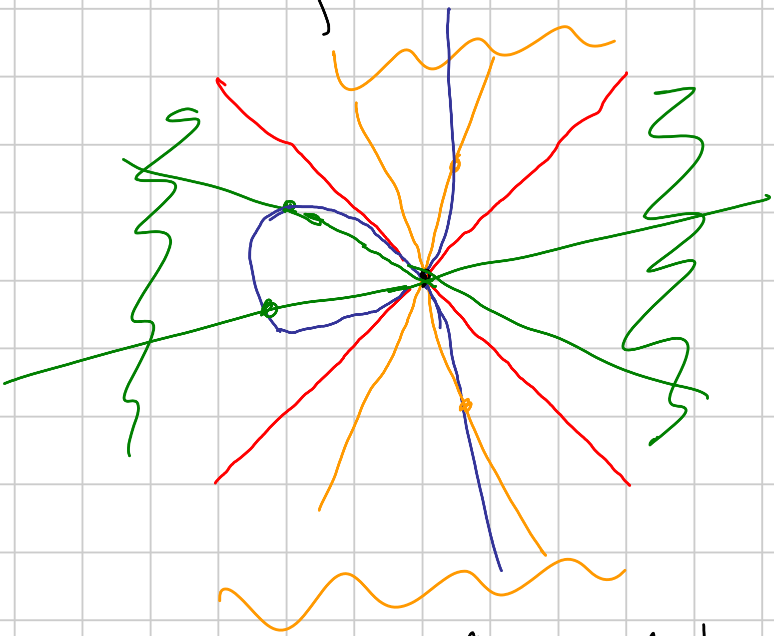


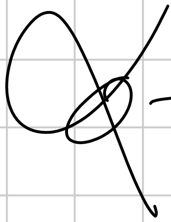


$p(x,y)=0$

$\leftarrow (0,0)$

$$p(x,y)=0$$
$$\parallel$$
$$x^2-y^2+y^3$$





$\rightarrow$ NODO (node)

$$\parallel$$

$$\begin{cases} p(x,y)=0 \\ \dfrac{dp}{dx}=0 \\ \dfrac{dp}{dy}=0 \end{cases}$$

$$\frac{d\, x^n}{dx}=nx^{n-1} \qquad \frac{d\, y^n}{dx}=0$$

$$\frac{d\, y^n}{dy}=ny^{n-1} \qquad \frac{dy^n}{dx}=0$$

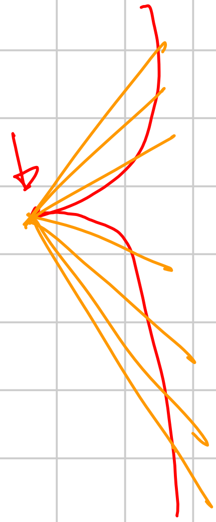
$$\frac{d\, a\cdot f(x,y)}{d\square}=a\cdot\frac{d f(x,y)}{d\square}$$

$$\frac{d\, f(x,y)\cdot g(x,y)}{d\square}=\frac{df}{d\square}\cdot g+f\cdot\frac{dg}{d\square}$$

$$\frac{d\, p(x,y)}{dx}=\frac{d\, x^2-y^2+y^3}{dx}=2x$$

$$\frac{dp}{dy}=-2y+3y^2$$

$x^2 = y^3$



CUSPIDE

Es: $x^2 + y^2 = 2z^2$

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<u>Determinante:</u>

$$\begin{cases} ax + by = 0 \\ cx + dy = 0 \end{cases}$$

esiste sol $\neq (0,0) \iff a = \lambda c$, $b = \lambda d$

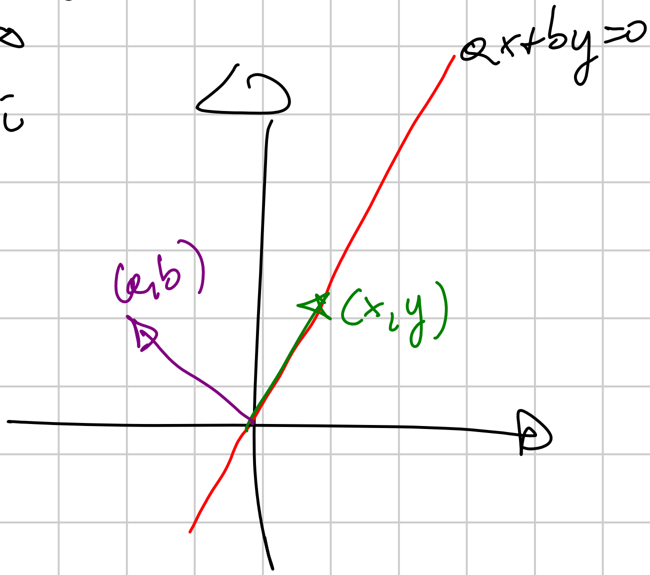
$$\iff \frac{a}{c} = \frac{b}{d} \iff \underbrace{ad - bc = 0}_{\text{determinante del sistema}}$$

$$\det\begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - bc$$

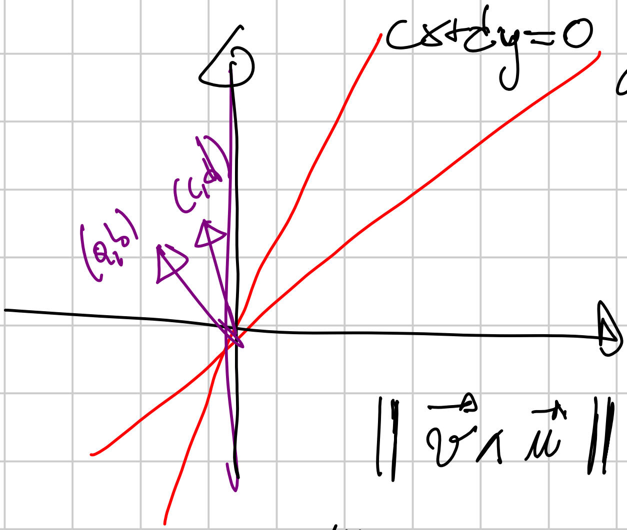
$$0 = ax + by = \langle (a,b), (x,y) \rangle$$

vettori

$\iff (a,b)$ perp. a (x,y)

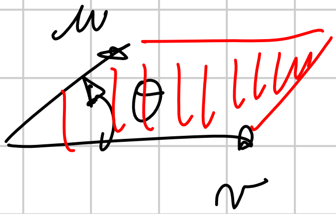


$cx+dy=0$ $\qquad ax+by=0$

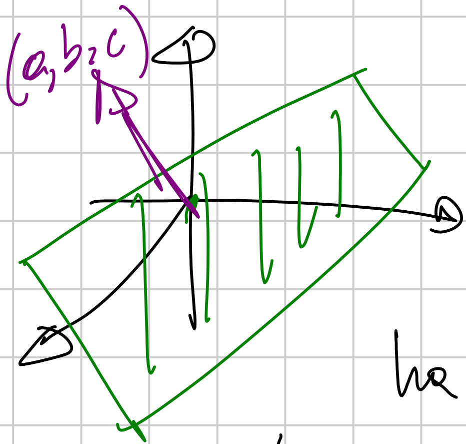


$$(a,b,0)\wedge(c,d,0)=0$$

$$\|\vec{v}\wedge\vec{u}\| = \|\vec{v}\|\cdot\|\vec{u}\|\cdot\sin\hat{\theta}$$

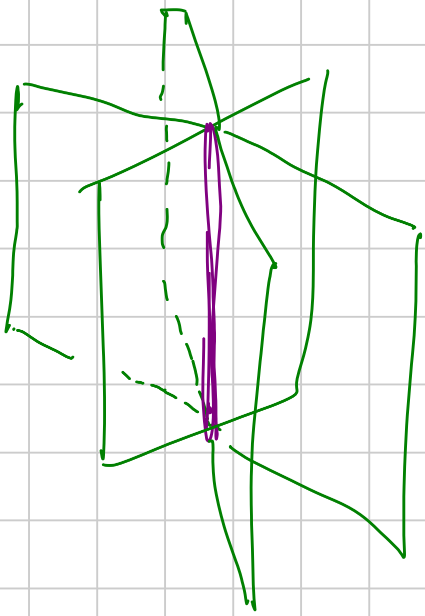


<u>In 3d:</u> $\quad (a,b,c)\cdot(x,y,z)=0$



Se $\begin{cases} ax+by+cz=0 \\ dx+ey+fz=0 \\ gx+hy+jz=0 \end{cases}$

ha altre sol $(\neq(0,0,0))$



c'è una retta in comune
(r)

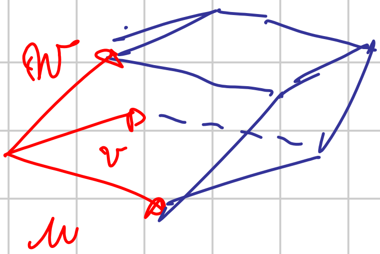
$\Downarrow$

$(a,b,c),\ (d,e,f),\ (g,h,j)$

sono $\perp$ a r

$\Updownarrow$

Vol $=0$



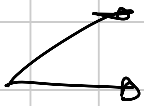
base $=\vec{u}\wedge\vec{v}$ $\qquad \pm$Vol $=(\vec{u}\wedge\vec{v})\cdot\vec{w}$

C'è una sol non nulla $\iff \left[(a,b,c)\wedge(d,e,f)\right]\cdot(g,h,j) = 0$

$$(a,b,c)\wedge(d,e,f) = (bf-ec,\ dc-af,\ ae-bd)$$

$$\pm Vol = (gbf + hdc + jae - gec - hef - jbd) = \det\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & j \end{pmatrix}$$

$$\det\begin{pmatrix} \vec{v} \\ \vec{u} \\ \lambda\vec{v}+\mu\vec{u} \end{pmatrix} = 0$$



$$\begin{cases} ax+by = c \\ dx+ey = f \end{cases}$$

$$D = \det\begin{vmatrix} a & b \\ d & e \end{vmatrix} \qquad D_x = \det\begin{vmatrix} c & b \\ f & e \end{vmatrix}$$

$$x = \frac{D_x}{D} \qquad y = \frac{D_y}{D} \qquad D_y = \det\begin{vmatrix} a & c \\ d & f \end{vmatrix}$$

Oss:

$$\begin{cases} ax+by+cz=0 & \leftarrow \text{piano} \perp (a,b,c) \\ dx+ey+fz=0 & \leftarrow \text{piano} \perp (d,e,f) \end{cases}$$

la retta che risolve il sistema è

$$\lambda\cdot(a,b,c)\wedge(d,e,f) \qquad \lambda\in\mathbb{R}$$

$$\lambda\cdot(bf-ec,\ dc-af,\ ae-bd)$$

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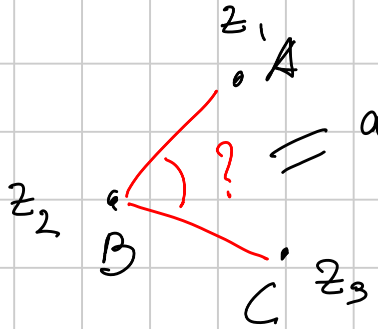

$$= \arg\left(\frac{z_1-z_2}{z_3-z_2}\right)$$

traslare z_2 in 0 + + ruotare la retta tra z_2 e z_3 sull'asse reale.

$$\left|\frac{z_1-z_2}{z_3-z_2}\right| = \frac{AB}{BC}$$

$$\frac{w_1-w_2}{w_3-w_2} = \frac{z_1-z_2}{z_3-z_2} \iff$$ i △ sono (dirett.) simili

(con i coniugati: inv. simili)

$$\Updownarrow$$

$$\det\begin{pmatrix} z_3-z_2 & z_1-z_2 \\ w_3-w_2 & w_1-w_2 \end{pmatrix} = 0$$

$$\Updownarrow$$

$$\det\begin{pmatrix} 1 & 1 & 1 \\ z_1 & z_2 & z_3 \\ w_1 & w_2 & w_3 \end{pmatrix} = 0$$

△ equilatero $\iff \left\|\begin{matrix} 1 & 1 & 1 \\ z_1 & z_2 & z_3 \\ \zeta_3 & \zeta_3^2 & 1 \end{matrix}\right\| = 0$

ζ_3 = rad. 3ª di 1 $\qquad \zeta_3^2 = \overline{\zeta_3}$

$$\Updownarrow$$

$$z_1 + \zeta_3 z_2 + \zeta_3^2 z_3 = 0$$

△ equilatero $\iff$ ABC $\cong$ BCA

(ABC $\cong$ BAC isosc.